10.4 GENERAL EQUATION OF A LINE

In the earlier section we have discussed various forms of the equation of a line. All the equations are first degree equations in two variables. We also studied that the graph of a first degree equation in two variables is always a straight line. Thus equation of the form $Ax + By + C = 0$, where A and B are not zero simultaneously, is the general equation of the line.

10.4.1 Different forms of $Ax + By + C = 0$

In this section, we discuss the procedure to reduce the general equation of the line $Ax + By + C = 0$ into slope - intercept from, intercept form and normal form.

a. Slope - intercept form

Consider the equation $Ax + By + C = 0$, then $By = -Ax - C$.

If $B \neq 0$, then $y = \frac{-A}{B}x + \frac{-C}{B}$ which is of the form $y = mx + c$

$\therefore$ Slope of the line, $m = \frac{-A}{B} = \frac{-\text{coefficient of } x}{\text{coefficient of } y}$

If $B = 0$, then $x = \frac{-C}{A}$, which is a line parallel to y-axis whose slope is not defined.

Example 37

Reduce the equation of the line $\sqrt{3}x + y - 8 = 0$ in to slope - intercept form

Solution

The equation of the line is $y = -\sqrt{3}x + 8$ which is in the slope - intercept form.

Example 38

Find the angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$

Solution

The lines are $y - \sqrt{3}x - 5 = 0 \Rightarrow y = \sqrt{3}x + 5$ ......(i)

$\sqrt{3}y - x + 6 = 0 \Rightarrow \sqrt{3}y = x - 6$

$$\Rightarrow y=\frac{1}{\sqrt{3}}x-\frac{6}{\sqrt{3}} \quad ......(ii)$$

Let slope of (i) be m_1 and that of (ii) be m_2.

$$\therefore m_1=\sqrt{3},\ m_2=\frac{1}{\sqrt{3}}$$

Let θ be the acute angle between the lines

$$\therefore \tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|=\left|\frac{\frac{1}{\sqrt{3}}-\sqrt{3}}{1+\frac{1}{\sqrt{3}}\times\sqrt{3}}\right|=\left|\frac{\frac{1-3}{\sqrt{3}}}{2}\right|=\left|\frac{1-3}{2\sqrt{3}}\right|=\frac{1}{\sqrt{3}}$$

$$\tan\theta=\frac{1}{\sqrt{3}}\Rightarrow\theta=30^\circ$$

$\therefore$ The angle between the two lines are θ and $180^\circ-\theta$.

i.e., 30° and 150°

b. Intercept form

Consider the equation $Ax+By+C=0$. Then $Ax+By=-C$.

If $C\neq 0$, then $\frac{Ax}{-C}+\frac{By}{-C}=1$

ie., $\frac{x}{\left(\frac{-C}{A}\right)}+\frac{y}{\left(\frac{-C}{B}\right)}=1$,

which is of the form $\frac{x}{x\text{-intercept}}+\frac{y}{y\text{-intercept}}=1$

$$\therefore x\text{ - intercept}=\frac{-C}{A}=\frac{-\text{constant term}}{\text{coefficient of }x}$$

$$y\text{ - intercept}=\frac{-C}{B}=\frac{-\text{constant term}}{\text{coefficient of }y}$$

If $C=0$, then the line passes through the origin and has zero intercepts.

Example 39

Convert the equation of the line $2x-3y+6=0$ into intercept form.

Solution

The equation of the line is $2x-3y+6=0$

i.e., $2x-3y=-6$

Dividing by -6, we get

$$\frac{2}{-6}x - \frac{3}{-6}y = \frac{-6}{-6} \text{ or } \frac{x}{-3} + \frac{y}{2} = 1, \text{ is the intercept form.}$$

Example 40

The equation of a straight line is $3x - 4y + 10 = 0$.

Find i. slope-intercept form

ii. slope

iii. x and y intercepts

(March 2014)

(March 2010, September 2010, NCERT)

Solution

i. The equation of the line is $3x - 4y + 10 = 0$

i.e., $-4y = -3x - 10$ or $y = \frac{3}{4}x + \frac{10}{4}$ is the slope-intercept form.

ii. The equation of the line is in the form $y = mx + c$ *(from (i))*

$\therefore$ Slope $= \frac{3}{4}$

iii. The equation of the line is $3x - 4y + 10 = 0$

$$x\text{-intercept} = \frac{-\text{constant term}}{\text{coefficient of } x} = \frac{-10}{3}$$

$$y\text{-intercept} = \frac{-\text{constant term}}{\text{coefficient of } y} = \frac{-10}{-4} = \frac{5}{2}$$

STUDY TIP

To get the x - intercept, substitute $y = 0$ in the equation.
To get the y - intercept, substitute $x = 0$ in the equation.

Example 41

Consider the points A(2, 3) and B (4, 5)

i. Find the slope of the line passing through the points A and B.

ii. Find the equation of the line passing through A and B.

iii. Find the x - intercept of the above line.

(March 2011)

Solution

i. Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{4-2} = \frac{2}{2} = 1$

ii. Slope of AB, $m = 1$. A(2,3) is a point on AB.

Equation of AB : $y - y_1 = m(x - x_1)$

$$y - 3 = 1(x - 2)$$

$$x - y + 1 = 0$$

iii. To obtain the x - intercept, put $y = 0$ in the equation of AB.

$\therefore x = -1$ $\quad\therefore x$ - intercept $= -1$

Another Method of (iii)

The equation of the line is $x - y + 1 = 0$

x - intercept $= \dfrac{-\text{constant term}}{\text{coefficient of } x} = \dfrac{-1}{1} = -1$

c. Normal form

Consider the equation $Ax + By + C = 0$, then $-Ax - By = C$.

If C is negative, make C positive by multiplying the equation by -1.

Dividing by $\sqrt{A^2+B^2}$, we get $\dfrac{-A}{\sqrt{A^2+B^2}}x - \dfrac{B}{\sqrt{A^2+B^2}}y = \dfrac{C}{\sqrt{A^2+B^2}}$ ....(1)

Now (coefficient of x)^2 + (coefficient of y)$^2 = 1$

Hence (1) is in the normal form. $x \cos\omega + y \sin\omega = p$,

where $p = \dfrac{C}{\sqrt{A^2+B^2}}$, $\cos\omega = \dfrac{-A}{\sqrt{A^2+B^2}}$, $\sin\omega = \dfrac{-B}{\sqrt{A^2+B^2}}$

Example 42

Reduce the equation of the line $\sqrt{3}x + y - 8 = 0$ into normal form.

Find the values of p and ω **(NCERT, March 2015)**

Solution

We have $\sqrt{3}x + y = 8$

Dividing by $\sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = 2$ we get, $\dfrac{\sqrt{3}}{2}x + \dfrac{1}{2}y = 4$

i.e., $p = 4$ $\cos\omega = \dfrac{\sqrt{3}}{2}$, $\sin\omega = \dfrac{1}{2}$, $\therefore \omega = 30°$

$\therefore$ The equation of the line in the normal form is $x \cos 30° + y \sin 30° = 4$

Condition for parallelism and perpendicularity of lines

$A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ where $B_1, B_2 \neq 0$

Slope of $A_1x + B_1y + C_1 = 0$ is $m_1 = \dfrac{-A_1}{B_1}$, Slope of $A_2x + B_2y + C_2 = 0$ is $m_2 = \dfrac{-A_2}{B_2}$

When the lines are parallel, their slopes are equal.

i.e., $m_1 = m_2 \Rightarrow \dfrac{-A_1}{B_1} = \dfrac{-A_2}{B_2}$, which is $\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2}$.

When the lines are perpendicular product of slopes $= -1$.

$$\left(\frac{-A_1}{B_1}\right)\left(\frac{-A_2}{B_2}\right) = -1 \Rightarrow \frac{A_1A_2}{B_1B_2} = -1 \Rightarrow A_1A_2 + B_1B_2 = 0$$

The lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ are parallel if $\frac{A_1}{A_2} = \frac{B_1}{B_2}$ and perpendicular $A_1A_2 + B_1B_2 = 0$.

STUDY TIP

i. Any line parallel to $Ax + By + C = 0$ is of the form $Ax + By + K = 0$.

ii. Any line perpendicular to $Ax + By + C = 0$ is of the form $Bx - Ay + K = 0$