10.3.3 Two - point form

Let P(x, y) be any point on the line l passing through the two points $A(x_1, y_1)$ and $B(x_2, y_2)$

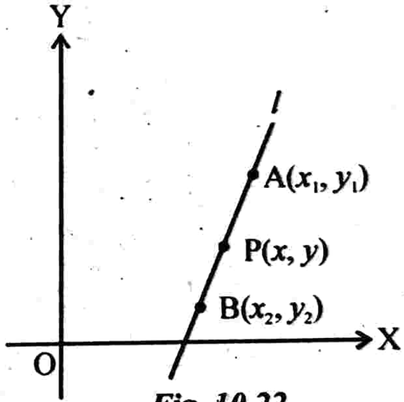


Fig. 10.22

The slope of the lineAB is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

The equation of the line AB is $y - y_1 = m(x - x_1)$

(point - slope form)

$\therefore$ The equation of the line is $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

or $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

Example 24

Write the equation of the line through the points (1, –1) and (3, 5) **(NCERT, August 2014)**

Solution

Let A (1, –1) and B (3, 5) be the points.

Equation of AB is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

i.e., $\frac{y+1}{5+1} = \frac{x-1}{3-1}$

$\Rightarrow \frac{y+1}{6} = \frac{x-1}{2}$

$\Rightarrow \frac{y+1}{3} = x - 1$

$\Rightarrow y + 1 = 3x - 3$

$\Rightarrow 3x - y - 4 = 0$

Example 25

The vertices of ΔABC are A(2, 1), B(–3, 5) and C(4, 3)

i. Write the coordinates of the midpoint of AC.

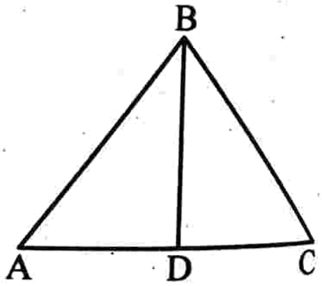
ii. Find the equation of the median through the vertex B. **(September 2012)**

Solution

i. Let D be the midpoint of AC.

$\therefore D = \left(\frac{2+4}{2}, \frac{1+3}{2}\right) = (3, 2)$

ii. Equation of BD is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$.



$\Rightarrow \frac{y-5}{2-5} = \frac{x+3}{3+3}$

$\Rightarrow \frac{y-5}{-3} = \frac{x+3}{6}$

$\Rightarrow 6(y - 5) = -3(x + 3)$

$\Rightarrow 6y - 30 = -3x - 9$

$\Rightarrow 3x + 6y - 21 = 0$

$\Rightarrow x + 2y - 7 = 0$

$\therefore$ Equation of the median through B is $x + 2y - 7 = 0$

10.3.4 Slope - intercept form

Intercepts of a line

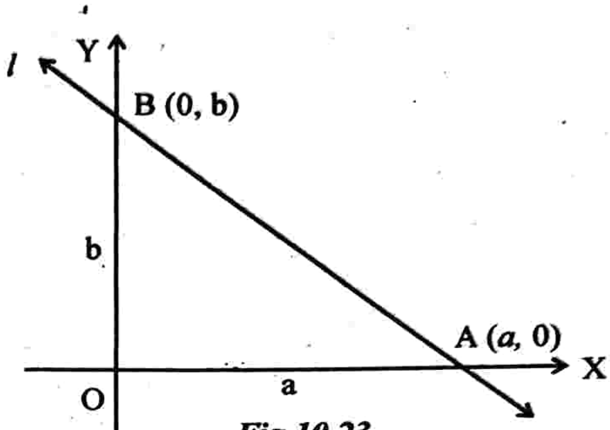


Fig 10.23

Let 'l' be a line (which is not parallel to x - axis and y - axis) intersecting the x - axis at the point A $(a, 0)$ and y - axis at the point B $(0, b)$. Then 'a' is called the x - intercept of l and 'b' is called the y - intercept of l. The line segment AB of the line l is called the portion of the line intercepted between the axes.

If a line 'l' makes y - intercept 'b' and x - intercept 'a' then the line l intersects the y - axis at $(0, b)$ and the x - axis at $(a, 0)$

NOTE

i. The intercept on the x - axis is positive, if measured to the right of the origin and negative, if measured to the left.

ii. The intercept on the y - axis is positive, if it is measured above the origin, and negative if measured below.

iii. Lines passing through the origin has zero intercept.

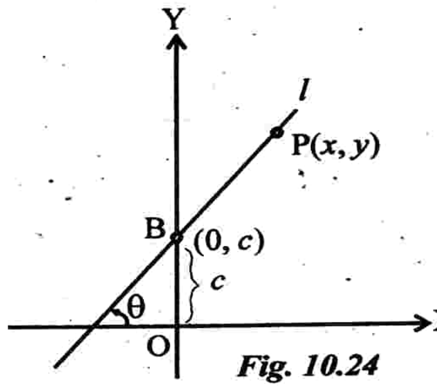


Fig. 10.24

Slope - intercept form of a line

Let 'c' be the y - intercept of a line l having slope m, then the line passes through the point $(0, c)$.

Equation of the line is $y - c = m(x - 0)$ (point - slope form)

i.e., $y = mx + c$, is the equation of the line in the slope - intercept form.

NOTE

- When the equation of a line is written in the form $y = mx + c$, coefficient of x is the slope of the line and the constant term 'c' is the y - intercept.
- If $c = 0$, the line passes through the origin and is of the form $y = mx$.
- The equation of a line with slope m and x - intercept d is $y = m(x - d)$.

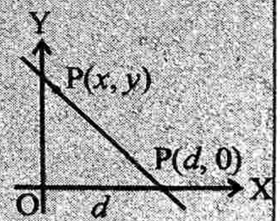


Fig. 10. 25

Example 26

Find the equation of the line with slope 3 and y - intercept -2

Solution

Slope, $m = 3$, y - intercept $= -2$

The equation of the line is $y = 3x - 2$

Example 27

Find the equation of line intersecting the x - axis at a distance of 3 units to the left of the origin with slope -2. **(NCERT)**

Solution

Slope, $m=-2$, x - intercept $d=-3$

$\therefore$ The equation of the line is $y=m(x-d)$

$\Rightarrow y=-2\,(x-(-3))$

$\Rightarrow y=-2\,(x+3)$ $\quad \Rightarrow 2x+y+6=0$

Another Method

Here, point on the x axis is $(-3, 0)$ and slope $=-2$.

$\therefore$ Equation of the line is $y-0=-2(x+3)$

$\Rightarrow 2x+y+6=0$

Example 28

Write the equation of the lines for which $\tan\theta=\frac{1}{2}$, where θ is the inclination of the line and

i. y-intercept is $\frac{-3}{2}$ $\quad$ ii. x-intercept is 4 **(NCERT)**

Solution

Slope of the line $m=\tan\theta=\frac{1}{2}$.

i. Equation of the line with slope $m=\frac{1}{2}$ and y intercept $c=\frac{-3}{2}$ is $y=mx+c$

$\Rightarrow y=\frac{1}{2}x+\frac{-3}{2}$

$\Rightarrow 2y=x-3$

$\Rightarrow x-2y-3=0$

ii. Equation of the line with slope $m=\frac{1}{2}$ and x intercept $d=4$ is

$y=m(x-d)$

$\Rightarrow y=\frac{1}{2}(x-4)$

$\Rightarrow 2y=x-4$

$\Rightarrow x-2y-4=0$

10.3.5 Intercept - form

Let the line l intersect the x - axis at A and y - axis at B having x - intercept 'a' and y - intercept - 'b'.

$\therefore$ Coordinates of the points A and B are $(a, 0)$ and $(0, b)$.

Equation of AB is $y-0=\left(\frac{b-0}{0-a}\right)(x-a)$ (Two-point form)

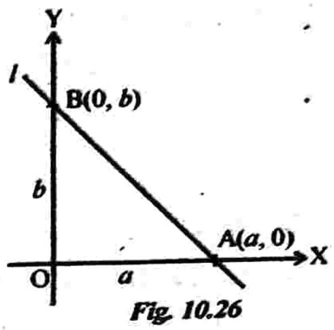


Fig. 10.26

$y=\left(\frac{-b}{a}\right)(x-a) \quad \Rightarrow \frac{y}{b}=\frac{-(x-a)}{a} \quad \Rightarrow \frac{y}{b}=\frac{-x}{a}+1$

$\frac{x}{a}+\frac{y}{b}=1$ is the equation of a line in intercept form

Example 29

Find the equation of the line, which makes intercepts –3 and 2 on the x and y-axes respectively. **(NCERT, March 2013, 2014)**

Solution

Let a and b be the intercepts on the x and y axes.

$\therefore a=-3,\ b=2$

The equation of the line is $\frac{x}{a}+\frac{y}{b}=1$

$$\Rightarrow \frac{x}{-3}+\frac{y}{2}=1$$
$$\Rightarrow 2x-3y=-6$$
$$\Rightarrow 2x-3y+6=0$$

Example 30

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3). **(NCERT)**

Solution

Let the intercepts made by the line on the axes is 'a'

$\therefore$ Equation of the line in intercept form is $\frac{x}{a}+\frac{y}{a}=1$ i.e., $x+y=a$ .... (1)

Since (1) passes through (2, 3), we get $2+3=a$, $\therefore a=5$

Substitute $a=5$ in (1), we get $x+y=5$ is the required equation of the line.

Example 31

Find the equation of the line which passes through the point (3, 4) and whose intercepts on the axes are equal in magnitude but opposite in sign.

Solution

Let each intercepts be 'a' and '$-a$'. Equation of the line is $\frac{x}{a}+\frac{y}{-a}=1$ or $x-y=a$ .... (1)

Since (1) passes through (3, 4), we get $3-4=a$, $\Rightarrow a=-1$

$\therefore$ The required equation is $x-y=-1$ or $x-y+1=0$

Example 32

Find the equation of the line through the point (2, 2) and cutting off intercepts on the axes whose sum is 9 **(NCERT)**

> **STUDY TIP**
> The equation of the line in the intercept form is $\dfrac{x}{x\text{-intercept}}+\dfrac{y}{y\text{-intercept}}=1$

Solution

Let a and b be the intercepts along the x and y axes

Given $a + b = 9$

$\therefore\ b = 9 - a$

Equation of the line in the intercept form is $\dfrac{x}{a}+\dfrac{y}{b}=1$

Since (2, 2) is a point on this line, we get

$$\frac{2}{a}+\frac{2}{b}=1 \Rightarrow 2a+2b=ab$$
$$\Rightarrow 2(a+b)=ab$$
$$\Rightarrow 2(9)=a(9-a)$$
$$\Rightarrow a^2-9a+18=0$$
$$\Rightarrow (a-3)(a-6)=0$$
$$\Rightarrow a=3 \text{ or } a=6$$

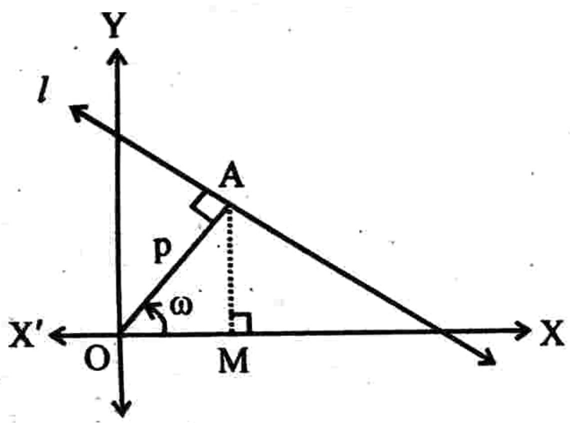
When $a = 3,\ b = 6$ and when $a = 6,\ b = 3$

$\therefore$ Equation of the lines are $\dfrac{x}{3}+\dfrac{y}{6}=1$ or $\dfrac{x}{6}+\dfrac{y}{3}=1$

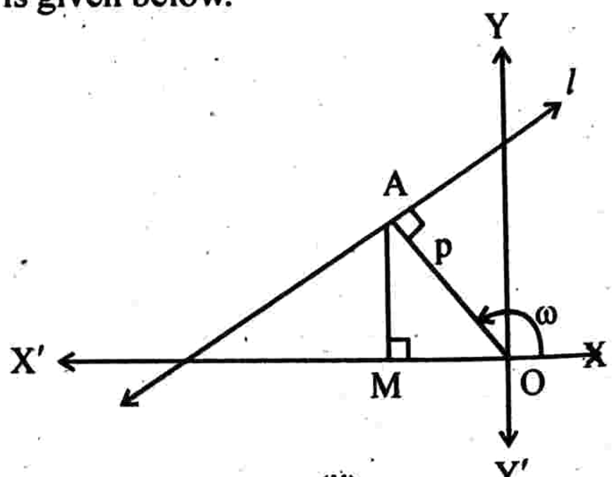
i.e., The equation of the lines are $2x + y = 6$ or $x + 2y = 6$

10.3.6 Normal form

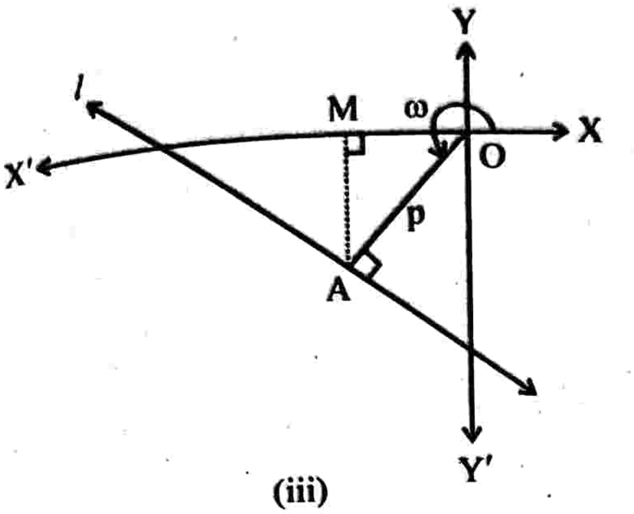
Consider a non - vertical line 'l'. Let 'p' be the perpendicular distance from the origin to the line and ω be the angle made by the perpendicular with the x - axis. Let A be the foot of the perpendicular from the origin to the line 'l'. Therefore OA = p, $\angle$XOA=ω (in fig. 10.27(i)). Then the different positions of the line 'l' in the xy - plane is given below.



(i)



(ii)



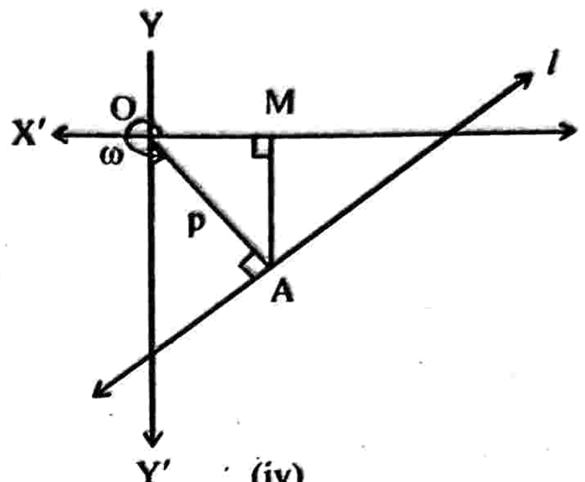


Fig 10.27

Draw AM perpendicular to x - axis

x coordinate of A $= OM = p\cos\omega$ and y coordinate of A $= AM = p\sin\omega$

$\therefore$ A is the point $(p\cos\omega, p\sin\omega)$

Slope of OA $= \tan\omega = \dfrac{\sin\omega}{\cos\omega}$

Since the line l is perpendicular to OA, Slope of line $l = \dfrac{1}{\text{slope of OA}} = \dfrac{-\cos\omega}{\sin\omega}$

$\therefore$ Equation of line l is $y - p\sin\omega = \left(\dfrac{-\cos\omega}{\sin\omega}\right)(x - p\cos\omega)$ *(point - slope form)*

$y\sin\omega - p\sin^2\omega = x\cos\omega + p\cos^2\omega \Rightarrow x\cos\omega + y\sin\omega = p(\sin^2\omega + \cos^2\omega)$

$\Rightarrow x\cos\omega + y\sin\omega = p$

Equation of line in the normal form is $x\cos\omega + y\sin\omega = p$

NOTE

- When the equation of the line is in the normal form, coefficients of x and y are such that (coefficient of x)^2 + (coefficient of y)$^2 = 1$
- p is always positive and ω is such that $0° \leq \omega < 360°$

Example 33

Find the equation of the line which has the length of the perpendicular from origin to the line as 4 units and the perpendicular segment on the line l makes an angle of 30° with the positive direction of x - axis.

Solution

The equation of a line in the normal form is $x\cos\omega + y\sin\omega = p$

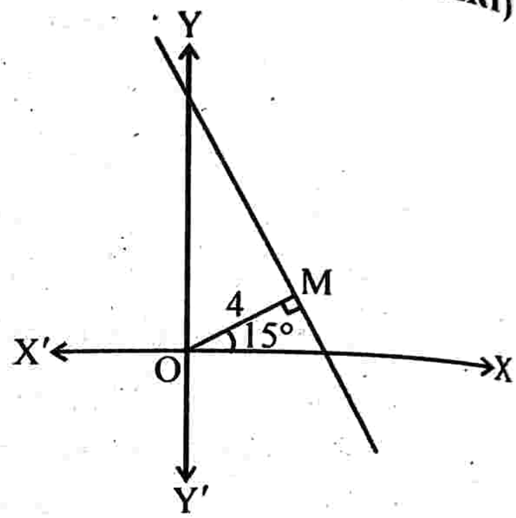
Here $p = 4$ and $\omega = 30°$ $\therefore$ The equation is $x\cos30° + y\sin30° = 4$

i.e., $\frac{\sqrt{3}}{2}x+\frac{1}{2}y=4 \Rightarrow \sqrt{3}x+y-8=0$

Example 34

Find the equation of a line whose perpendicular distance from the origin is 4 units and the angle which the normal makes with the positive direction of x-axis is 15°. **(NCERT)**

Solution



Here $p = 4$, $\omega = 15°$

The equation of the line in the normal form is

$x\cos\omega + y\sin\omega = p$

i.e., $x\cos 15° + y\sin 15° = 4$

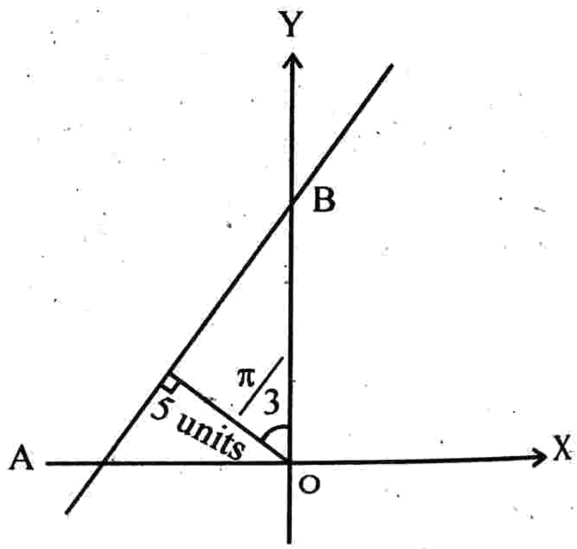
i.e., $\frac{x\left(\sqrt{3}+1\right)}{2\sqrt{2}}+\frac{y\left(\sqrt{3}-1\right)}{2\sqrt{2}}=4$

$\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y-8\sqrt{2}=0$

Example 35

In the figure given below, the equation of the line AB is ...... **(June 2008)**

Solution



Given the perpendicular distance from the origin to AB = 5 units and angle between the perpendicular and positive x - axis (ω) $= \frac{\pi}{2}+\frac{\pi}{3}$

$\therefore$ Equation of AB is $x\cos\omega + y\sin\omega = p$

or $x\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right)+y\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)=5$

or $-x\sin\frac{\pi}{3}+y\cos\frac{\pi}{3}=5$

or $-x\left(\frac{\sqrt{3}}{2}\right)+y\left(\frac{1}{2}\right)=5$

$-\sqrt{3}x+y=10$ or $\sqrt{3}x-y+10=0$

Example 36

By using the concept of equation of a line, prove that the three points (3, 0), (– 2, – 2) and (8, 2) are collinear. **(NCERT)**

Solution

Let A(3, 0), B(–2, –2) and C(8, 2) be the given points

$\therefore$ Equation of AB is $\dfrac{x-3}{-2-3}=\dfrac{y-0}{-2-0}$

$\Rightarrow \dfrac{x-3}{-5}=\dfrac{y}{2} \quad \Rightarrow 2x-5y-6=0$

Substituting the coordinates of C(8,2) in the equation of AB, we get $2(8)-5(2)-6=0$, which is true.

$\therefore$ The point C lies on the line AB. Hence A, B, C are collinear