10.3. VARIOUS FORMS OF THE EQUATION OF A LINE

A straight line is a set of points in a plane satisfying some geometrical conditions. The equation of a line is a relationship between the abscissa (x) and ordinate (y) of a general point on the line under some suitable conditions.In this section, we discuss the equation of a line in various forms.

10.3.1 Horizontal and Vertical lines

Equation of a horizontal line (line parallel to x - axis)

Let AB be a line parallel to x - axis at a distance b from it. Then the ordinate (y) of each point on AB is b. Hence the equation of a line parallel to x - axis at a distance b from it is $y = b$

- If the line is parallel and above the x - axis, b is positive
- If the line is parallel and below the x - axis, b is negative
- Equation of x - axis, $y = 0$

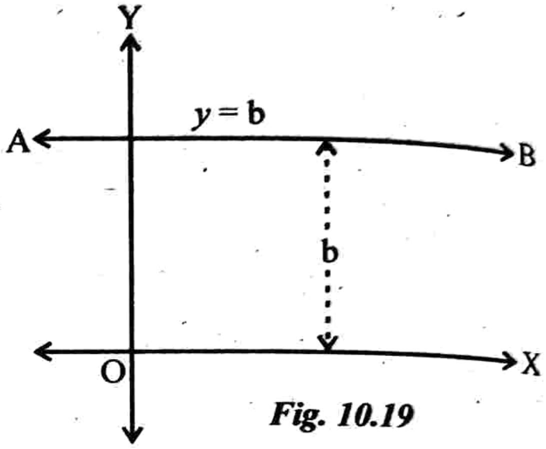


Fig. 10.19

Example 16

Find the equation of a straight line passing through (– 5, 7) and parallel to the x - axis.

(March 2005)

Solution

The given line is passing through the point (– 5, 7) and parallel to the x - axis

∴ Every point on the line has the same ordinate (y coordinate)

Hence the equation of the line is $y = 7$ or $y - 7 = 0$

Equation of a vertical line (line parallel to y - axis)

Let AB be a line parallel to y - axis at a distance 'a' from it. Then the abscissa (x) of each point on AB is 'a'. Hence the equation of a line parallel to y - axis at a distance 'a' from it is $x = a$

- If the line is parallel to y - axis (left of y - axis), a is negative, $x = -a$
- If the line is parallel to y - axis (right y - axis), a is positive, $x = a$
- Equation of y - axis, $x = 0$

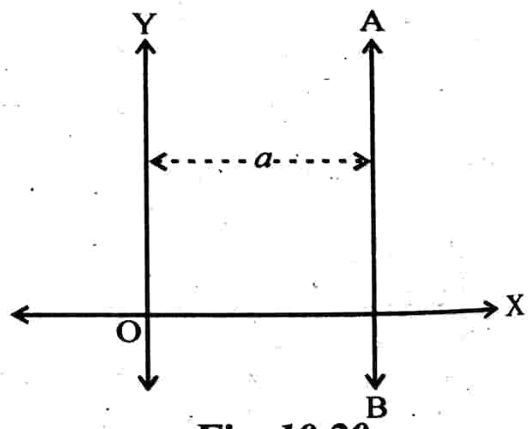


Fig. 10.20

Example 17

Find the equation of the line which is parallel to y - axis and passing through the point $(3, -4)$.

Solution

The given line is passing through the point $(3, -4)$ and parallel to y - axis.

$\therefore$ Every point on the line has the same abscissa (x coordinate).

Hence the equation of the line is $x = 3$

Example 18

Find the equation of the line parallel to y-axis and passing through the point of intersection of $x - 7y + 5 = 0$ and $3x + y - 7 = 0$ **(March 2015)**

Solution

$x - 7y + 5 = 0$ ....(i)

$3x + y - 7 = 0$ ....(ii)

(i) + 7(ii) $\rightarrow 22x - 44 = 0$

$\therefore x = \frac{44}{22} = 2$

(ii) $\rightarrow y = 7 - 3x = 7 - 3 \times 2 = 1$

$\therefore$ Point of intersection = (2, 1)

The required line passes through (2, 1) and parallel to y-axis.

$\therefore$ Equation of the line is $x = 2$

STUDY TIP

The equation of a line

(i) Parallel to x - axis does not contain x and

(ii) Parallel to y - axis does not contain y

10.3.2 Point - slope form

Let P(x, y) be any point on the line l passing through A(x_1, y_1) and having slope m.

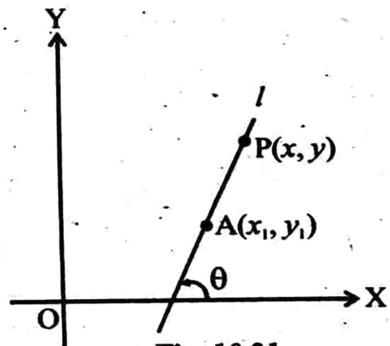


Fig. 10.21

Slope of the line AP = $\frac{y - y_1}{x - x_1}$ ie., $m = \frac{y - y_1}{x - x_1}$

$\therefore$ $y - y_1 = m(x - x_1)$ is the required equation of the line.

Example 19

Find the equation of the line passing through the point $(-2, 3)$ with slope -4. **(NCERT)**

Solution

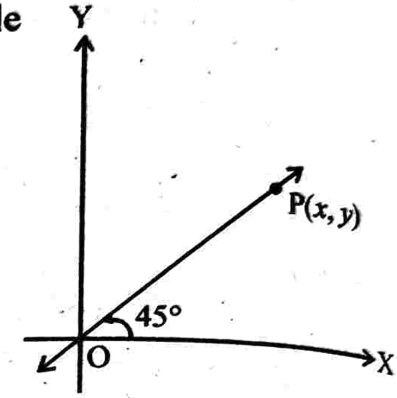
The equation of the line passing through (x_1, y_1) and slope m is $y - y_1 = m(x - x_1)$

The equation of the line passing through the point $(-2, 3)$ with slope -4 is $y - 3 = -4(x - (-2))$

$\therefore$ $4x + y + 5 = 0$ is the required equation.

Example 20

Find the equation of a line through the origin which makes an angle of 45° with the positive direction of x - axis.



Solution

The line passes through the origin

∴ A point on the line is (0, 0).

Slope of the line is $m = \tan 45° = 1$

∴ By point - slope form of the equation of a line,

$y - 0 = 1(x - 0)$

$y = x$, is the required equation of the line

Example 21

Consider the line joining the points (2, – 1) and (6, – 3).

i. Find its slope.

ii. Find the equation of the perpendicular bisector. **(October 2011)**

Solution

i. Let A (2, – 1) and B (6, – 3) be the points.

$$\text{Slope of AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3+1}{6-2} = \frac{-2}{4} = \frac{-1}{2}$$

ii. $$\text{Midpoint of AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2+6}{2}, \frac{-1+-3}{2}\right) = (4, -2)$$

$$\text{Slope of the perpendicular to AB} = \frac{-1}{\text{slope of AB}}$$

$$= \frac{-1}{\left(\frac{-1}{2}\right)} = 2$$

∴ Equation of the perpendicular bisector $y - y_1 = m(x - x_1)$

$y - {}^-2 = 2(x - 4)$

$y + 2 = 2x - 8$

$2x - y - 10 = 0$

Example 22

The vertices of triangle ABC are A (– 2, 3), B (2, – 3) and C (4, 5)

i. Find the slope of BC.

ii. Find the equation of the altitude of triangle ABC passing through A. **(March 2012)**

Solution

i. $$\text{Slope of BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5+3}{4-2} = 4$$

ii. The altitude through A is perpendicular to BC.

Slope of the altitude $= \dfrac{-1}{\text{slope of BC}} = \dfrac{-1}{4}$

Equation of the altitude through A is $y - y_1 = m(x - x_1)$

$y - 3 = \dfrac{-1}{4}(x + 2)$

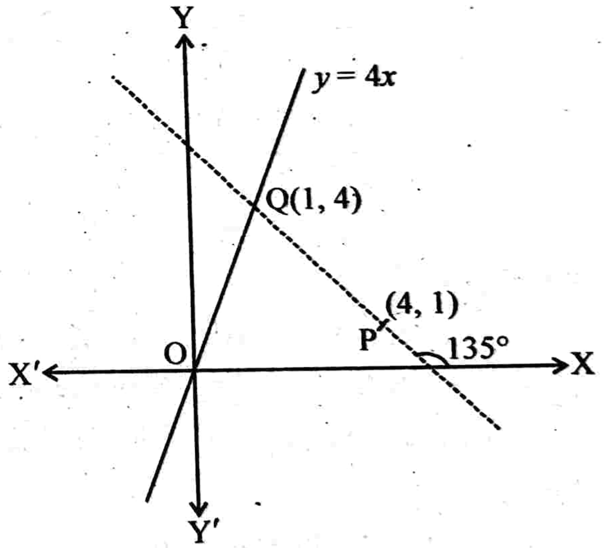
$4y - 12 = -x - 2$

$x + 4y - 10 = 0$

Example 23

Find the distance of the line $4x - y = 0$ from the point P(4, 1) measured along the line making an angle of 135° with the positive x-axis.

(NCERT)



The line $4x - y = 0$ passes through the origin O. The line through P(4, 1) making 135° with x-axis intersect the line $4x - y = 0$ at the point Q.

Slope of the line PQ = tan 135°

$= \tan(180° - 45°)$

$= -\tan 45° = -1$

Equation of PQ is $y - 1 = -1(x - 4)$

$\Rightarrow x + y = 5$ ....(1)

The equation of the line OQ is

$4x - y = 0$ ....(2)

Solving (1) and (2) we get Q = (1, 4)

$\therefore$ Required distance = Distance PQ

$= \sqrt{(1-4)^2 + (4-1)^2}$

$= \sqrt{9+9} = 3\sqrt{2}$ units