10.7 STREAMLINE FLOW

Fluid dynamics deals with the study of fluids in motion. In order to understand the feature of fluid motion, we consider the case of an ideal fluid. An ideal fluid has the following characteristics.

i. The fluid is incompressible

ii. It is non-viscous

iii. Its motion is steady

iv. Fluid moves without turbulence

Motion of a fluid can be streamline, turbulent or both.

If every particle of a fluid, passing a particular point passes with the same velocity and follows the exact path as its predecessor, then the motion of the fluid is called ***streamline or laminar flow***. A streamline flow is regular and steady. A streamline is defined as a curve whose tangent at any point gives the direction of velocity of the fluid at that point.

If the flow rate is high enough, the flow becomes irregular. Such an irregular flow of a fluid is called ***turbulent***. In a turbulent flow there is no steady-state pattern, it changes continuously.

10.7.1 Equation of Continuity

Consider a fluid flowing through a pipe of different cross section. Let us assume that the flow is steady. Let A_1 and A_2 be the area of cross section at any two sections. The fluid enters the pipe through A_1 with a velocity v_1 and leaves out through A_2 with a velocity v_2. The mass of fluid flowing through first face in a small time-interval Δt is $\rho A_1 v_1 \Delta t$; where ρ_1 is the density of the fluid at A_1. Similarly the mass of the fluid at flowing through second face is $\rho_2 A_2 v_2 \Delta t$, where ρ_2 is the density at A_2. Since the flow is steady, the mass of the fluid passing at A_1 and A_2 are equal.

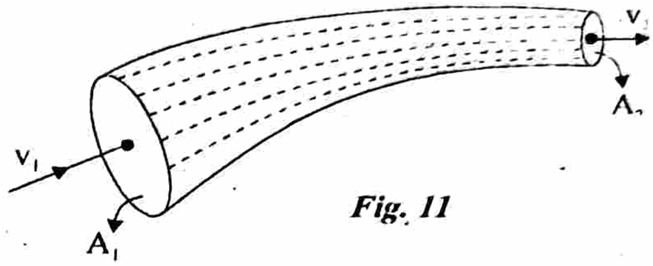

Fig. 11

ie., $\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$ .... (1) This is the equation of continuity.

For an incompressible fluid, $\rho_1 = \rho_2 = \rho$

Then, $A_1 v_1 = A_2 v_2$ or Av = a constant .... (2) (equation of continuity)

ie., $A \propto \frac{1}{v}$ .... (3)

The product 'Av' is called the flow rate. Above expression shows that the speed is high where the area of cross section is small and vice-versa.

10.8 Bernoulli's Principle

When a fluid flows through a pipe of non-uniform cross section and elevation, the pressure will change along the pipe. In 1738, the Swiss physicist, Daniel Bernoulli, developed a relation between the pressure, speed of the fluid and its elevation. The relation is known as Bernoulli's equation and is a consequence of ***energy conservation*** principle.

Bernoulli's theorem *states that the total energy of an ideal fluid (incompressible, non-viscous and isotropic fluid) in a stream lined motion is constant throughout the displacement*

i.e., for ideal fluid in streamline motion.

Kinetic energy + Potential energy + Pressure energy = a constant

To derive Bernoulli's equation, let us assume that the fluid is an ideal fluid.

Consider the flow of such an ideal fluid of density ρ through a pipe of varying cross-section as in fig. 12. Let Δx_1 be the distance through which the fluid moves in a small time Δt. Then work done at the end where the cross section is A_1 is

$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 \Delta V$ .... (1)

where P_1 is the pressure at A_1 and ΔV is the volume of the fluid.

Similarly the work done at the end A_2 is

$W_2 = -F_2\Delta x_2 = -P_2A_2\Delta x_2 = -P_2\Delta V$ .... (2)

The negative sign shows that the force F_2 at A_2 is opposite to the displacement.

Net work done = W = $W_1 + W_2$

= $(P_1 - P_2)\Delta V$ .... (3)

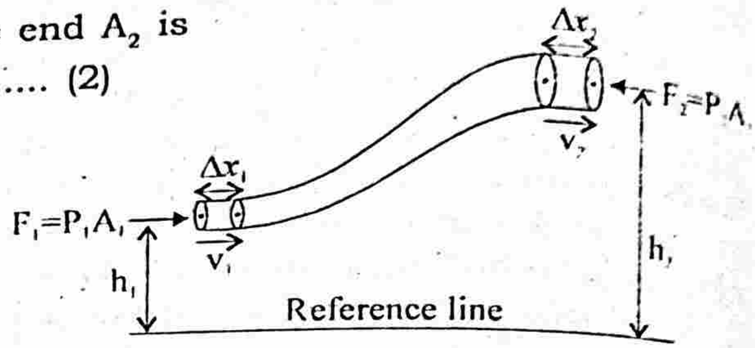

Fig. 12

A portion of this amount of work done goes to change the kinetic energy (ΔE_k) and its remaining portion to change the potential energy (ΔE_p) of the fluid. Let v_1 and v_2 be the velocities of the fluid at A_1 and A_2 respectively. Then change in kinetic energy,

$$\Delta E_k = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2}\rho.\Delta V(v_2^2 - v_1^2) = \frac{1}{2}\rho(v_2^2 - v_1^2)\Delta V \text{ .... (4)}$$

Let h_1 and h_2 be the elevation of A_1 and A_2 from the reference line. Then change in potential energy,

$$\Delta E_p = mg(h_2 - h_1) = \rho g(h_2 - h_1)\Delta V \text{ .... (5)}$$

By work-energy theorem $W = \Delta E_k + \Delta E_p$

i.e., $(P_1 - P_2)\Delta V = \frac{1}{2}\rho(v_2^2 - v_1^2)\Delta V + \rho g(h_2 - h_1)\Delta V$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \text{ ....(6)}$$

This equation is known as Bernoulli's equation. The above eq.6 is also written as

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{a constant} \text{ .... (7)}$$

"The sum of the pressure (P), the kinetic energy per unit volume $\left(\frac{1}{2}\rho v^2\right)$ and potential energy per unit volume $(\rho g h)$ has the same value at all points along a streamline flow".

Another form of the equation is $\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{a constant}$ ......(8)

where $\frac{P}{\rho g}$ is called ***pressure- head,*** $\frac{v^2}{2g}$ is called ***velocity - head*** and 'h' is called ***gravitational - head***

If the flow is through a horizontal pipe, of varying cross section, then effect of gravitational head is neglected (being same).

Then $P + \frac{1}{2}\rho v^2 = \text{a constant}$ or $\frac{P}{\rho g} + \frac{v^2}{2g} = \text{a constant}$ .... (9)