Integrals class-8

Type-6 [Integrals of special functions]

1, $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

2, $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$

3, $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

4, $\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left|x+\sqrt{x^2+a^2}\right|+c$

5, $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x+\sqrt{x^2-a^2}\right|+c$

6, $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)+c$

7, $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+c$

8, $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$

9, $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$

1, Find $\int \frac{1}{x^2+25}\,dx$

$\int \frac{1}{x^2+25}\,dx = \int \frac{1}{x^2+5^2}\,dx = \frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right)+c$

2, $\int \frac{1}{x^2+3}\,dx = \int \frac{1}{x^2+(\sqrt{3})^2}\,dx$

$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+c$

3, $\int \frac{1}{(x+1)^2+16}\,dx = \int \frac{1}{(x+1)^2+4^2}\,dx$

$= \frac{1}{4} \tan^{-1}\left(\frac{x+1}{4}\right)+c$

4, $\int \frac{1}{x^2-16}\,dx = \int \frac{1}{x^2-4^2}\,dx$

$= \frac{1}{8} \log\left|\frac{x-4}{x+4}\right|+c$

5, $\int \frac{1}{\sqrt{x^2-16}}\,dx = \int \frac{1}{\sqrt{x^2-4^2}}\,dx$

$= \log\left|x+\sqrt{x^2-16}\right|+c$

6, $\int \frac{dx}{\sqrt{25-x^2}} = \int \frac{dx}{\sqrt{5^2-x^2}} = \sin^{-1}\left(\frac{x}{5}\right)+c$

7, $\int \frac{dx}{\sqrt{9-25x^2}} = \int \frac{dx}{\sqrt{3^2-(5x)^2}}$

$= \frac{\sin^{-1}\left(\frac{5x}{3}\right)}{5}+c$

8, $\int \frac{dx}{\sqrt{1+4x^2}} = \int \frac{dx}{\sqrt{1^2+(2x)^2}}$

$= \frac{\log|2x+\sqrt{1+4x^2}|+c}{2}$

9, $\int \frac{dx}{\sqrt{(2-x)^2+1}} = \log|2-x+\sqrt{(2-x)^2+1}|+c$

10, $\int \frac{3x^2}{x^6+1}dx = \int \frac{3x^2}{(x^3)^2+1}dx$

$x^3 = u$

$3x^2dx = du$

$= \int \frac{du}{u^2+1}$

$= \tan^{-1}u + c = \tan^{-1}(x^3)+c$

11, $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}dx$

$\tan x = u$

$\sec^2 x dx = du$

$= \int \frac{du}{\sqrt{u^2+2^2}} = \log|u+\sqrt{u^2+4}|+c$

$= \log|\tan x+\sqrt{\tan^2 x+4}|+c$

12, $\int \sqrt{4-x^2}dx = \int \sqrt{2^2-x^2}dx$

$= \frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)+c$

13, $\int \sqrt{1-4x^2}dx = \int \sqrt{1-(2x)^2}dx$

$= \frac{1}{2}\left[\frac{2x}{2}\sqrt{1-4x^2}+\frac{1}{2}\sin^{-1}\left(\frac{2x}{1}\right)\right]+c$