Class-7
Application of Derivatives

① Find the point at which the tangent to the curve $y = \sqrt{4x-3} - 1$ has its slope $2/3$.

$$y = \sqrt{4x-3} - 1$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4x-3}} \times 4 = \frac{2}{\sqrt{4x-3}}$$

Slope of the tangent $= 2/3$

$$\therefore \frac{dy}{dx} = \frac{2}{3}$$

$$\Rightarrow \frac{2}{\sqrt{4x-3}} = \frac{2}{3}$$

$$\sqrt{4x-3} = 3$$

$$4x-3 = 9$$

$$4x = 12$$

$$x = 3$$

$$y = \sqrt{12-3} - 1 = 2$$

$\therefore$ Point is $(3, 2)$

(2) Find the Point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$

Slope of the tangent $= \frac{dy}{dx} = 3x^2 - 11$

Slope of the tangent $y = x - 11$ is 1

$$\therefore 3x^2 - 11 = 1$$
$$3x^2 = 12$$
$$x^2 = 4$$
$$x = \pm 2$$

When $x = 2$, $y = 8 - 22 + 5 = -9$

When $x = -2$, $y = -8 + 22 + 5 = 19$

$\therefore$ Points are $(2, -9)$ and $(-2, 19)$

But $(-2, 19)$ not satisfy the equation.

$\therefore$ $(2, -9)$ is the Point.

(3) Find a Point on the curve $y = (x-2)^2$ at which the tangent is Parallel to the chord joining the Points $(2, 0)$ and $(4, 4)$.

Slope of the chord $= \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{4}{2} = 2$$

$\therefore$ Slope of the tangent $= 2$

Slope of the tangent $= \frac{dy}{dx} = 2(x-2)$

$\therefore 2(x-2) = 2$

$x - 2 = 1$

$x = 3$

$y = (3-2)^2 = 1$

$\therefore$ Point is $(3, 1)$

(4) Find Points on the curve $y = x^3$ at which slope of the tangent is equal to y co-ordinate of the Point.

$y = x^3$

Slope of the tangent $= \frac{dy}{dx} = 3x^2$

Given slope $= y$

$3x^2 = y$

$3x^2 = x^3$

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$x = 0$ or $x = 3$

When $x = 0$, $y = 0$

When $x = 3$, $y = 27$

$\therefore$ Points are $(0, 0)$ and $(3, 27)$

(5) Find equation of tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$

Slope of the given line is 2

$\therefore$ slope of the tangent $= 2$

slope of the tangent $= \frac{dy}{dx} = 2x - 2$

$\therefore 2x - 2 = 2$

$x = 2$

$\therefore y = 4 - 4 + 7 = 7$

$\therefore$ Point is $(2, 7)$

Eq: of the tangent is

$y - y_1 = m(x - x_1)$

$y - 7 = 2(x - 2)$

$2x - y + 3 = 0$

(6) Find eq: of the tangent line to the curve $y = x^2 - 2x + 7$ which is $\perp^r$ to the line $5y = 15x + 13$

Slope of the given line is 3

$\therefore$ slope of the tangent $= \frac{-1}{3}$

Slope of the tangent $= \frac{dy}{dx} = 2x - 2$

$\therefore 2x - 2 = -\frac{1}{3}$

$2x = 2 - \frac{1}{3}$

$x = 5/6$

$\therefore y = \frac{25}{36} - \frac{5}{3} + 7 = \frac{217}{36}$

$\therefore$ Eq: of the tangent at $\left(\frac{5}{6}, \frac{217}{36}\right)$

$y - y_1 = m(x - x_1)$

$y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right)$

$36y - 217 = -12x + 10$

$36y + 12x - 227 = 0$

(7) Find eq: of the normals to the curve $y = x^3 + 2x + 6$ which are Parallel to the line $x + 14y + 4 = 0$

Slope of the tangent $= \frac{dy}{dx} = 3x^2 + 2$

Slope of the normal $= -1 / \frac{dy}{dx}$

$= \frac{-1}{3x^2 + 2}$

Slope of the given line is $\frac{-1}{14}$

$$\therefore \frac{-1}{3x^2+2} = \frac{-1}{14}$$

$$3x^2+2 = 14$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

When $x=2$, $y=18$ Point is $(2,18)$

When $x=-2$, $y=-6$ Point is $(-2,-6)$

Eq: of the normal at $(2,18)$ is

$$y-y_1 = -\frac{1}{m}(x-x_1)$$

$$y-18 = \frac{-1}{14}(x-2)$$

$$14y-252 = -x+2$$

$$x+14y-254=0$$

Eq: of the normal at $(-2,-6)$

$$y+6 = -\frac{1}{14}(x+2)$$

$$14y+84 = -x-2$$

$$x+14y+86=0$$