Class - 6

Application of Derivatives

① Find equation of tangent and normal to the curve $y = x^3$ at $(1, 1)$.

$$\frac{dy}{dx} = 3x^2$$

$$m = \left(\frac{dy}{dx}\right)_{(1,1)} = 3$$

Eq: of the tangent is

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 3(x - 1)$$

$$3x - y - 2 = 0$$

Eq: of the normal is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - 1 = -\frac{1}{3}(x - 1)$$

$$3y - 3 = -x + 1$$

$$x + 3y - 4 = 0$$

② Find equation of tangent and normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(0, 5)$

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

$$m = \left(\frac{dy}{dx}\right)_{(0,5)} = -10$$

Eq: of the tangent is

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -10(x - 0)$$

$$10x + y - 5 = 0$$

Eq: of the normal is

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - 5 = \frac{-1}{-10}(x - 0)$$

$$10y - 50 = x$$

$$x - 10y + 50 = 0$$

(3) Find eq: of tangent and normal to the curve $x^{2/3} + y^{2/3} = 2$ at $(1,1)$.

$$x^{2/3} + y^{2/3} = 2$$

diff: w.r.t x

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

$$y^{-1/3}\frac{dy}{dx} = -x^{-1/3}$$

$$\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$$

$$m = \left(\frac{dy}{dx}\right)_{(1,1)} = -1$$

Eq: of the tangent is

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -1(x-1)$$

$$x + y - 2 = 0$$

Eq: of the normal is

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - 1 = \frac{-1}{-1}(x-1)$$

$$x - y = 0$$

(4) Find equation of tangent and normal to the curve $x = \cos\theta, y = \sin\theta$ at $\theta = \frac{\pi}{4}$

$$\frac{dx}{d\theta} = -\sin\theta \qquad \frac{dy}{d\theta} = \cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta}{-\sin\theta} = -\cot\theta$$

$$m = \left(\frac{dy}{dx}\right)_{\theta = \pi/4} = -\cot \pi/4 = -1$$

when $\theta = \pi/4$, $x = \cos \pi/4 = 1/\sqrt{2}$

$y = \sin \pi/4 = 1/\sqrt{2}$

Eq: of the tangent is

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{\sqrt{2}} = -1\left(x - \frac{1}{\sqrt{2}}\right)$$

$$\underline{\underline{x + y = \sqrt{2}}}$$

Eq: of the normal is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - \frac{1}{\sqrt{2}} = \frac{-1}{-1}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$\underline{\underline{x - y = 0}}$$

(5) Find equation of all lines having slope -1 that are tangents to the curve $y = \frac{1}{x-1}$, $x \neq 1$

$$y = \frac{1}{x-1}$$

$$\frac{dy}{dx} = \frac{-1}{(x-1)^2}$$

Given slope $= -1$

$$\therefore \frac{dy}{dx} = -1$$

$$\frac{-1}{(x-1)^2} = -1$$

$$(x-1)^2 = 1$$

$$x-1 = \pm 1$$

$x = 2$ or $x = 0$

when $x=2$, $y = 1$. Point is $(2,1)$

when $x=0$, $y = -1$. Point is $(0,-1)$

Eqⁿ of the tangent at $(2,1)$

$$y-1 = -1(x-2)$$

$$x+y-3 = 0$$

Eqⁿ of the tangent at $(0,-1)$

$$y+1 = -1(x-0)$$

$$\underline{\underline{x+y+1 = 0}}$$

H.W

(6) Find eqⁿ of all lines having slope 0 which are tangents to the curve

$$y = \frac{1}{x^2-2x+3}$$

$$\frac{dy}{dx} = \frac{-1}{(x^2-2x+3)^2} \times (2x-2)$$

$$\text{slope} = 0 \implies \frac{dy}{dx} = 0$$

$$\implies \frac{-(2x-2)}{(x^2-2x+3)^2} = 0$$

$$2x-2=0$$

$$x=1$$

When $x=1$, $y = 1/2$. Point is $(1, 1/2)$

Eq. of the tangent at $(1, 1/2)$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{2} = 0(x-1)$$

$$\underline{\underline{y - \frac{1}{2} = 0}}$$