WORK SHEET BASED ON THE FOCUS AREA
CHAPTER 3
MATRICES

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1. (a) Find values of x, y, z, w satisfying the matrix equation

$$2\begin{bmatrix} x & z \\ y & w \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

(b) If the matrices A and B are of orders $m \times n$ and $n \times m$ respectively,

Then find the order of AB and BA

(c) The diagonal elements of a skew symmetric matrix are …………

2. Consider the matrices $\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$

(a) Find $A+B$ (b) Find $A-B$ (c) Find $(A+B)(A-B)$

3. Consider a 3×2 matrix $A = \left[a_{ij}\right]_{3\times 2}$ where $a_{ij} = |i-2j|$

(a) Write A (b) Find A'

4. Let $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

(a) Find AB (b) Is BA defined ? Justify your answer

5. Consider the matrices $A = \begin{bmatrix} 2 & -6 \\ 1 & 2 \end{bmatrix}$ and $2A + B = \begin{bmatrix} 1 & -10 \\ -2 & -1 \end{bmatrix}$

(a) Find matrix B (b) Find AB (c) Find B'

6. If $X+Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}, X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, then

(a) Find X and Y (b) Find $2X+Y$

7. Consider the matrix $A=\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$

(a) Find A^2 (b) Find k so that $A^2=kA-2I$

8. (a) Construct a 2×2 matrix $A=\left[a_{ij}\right]_{2\times2}$ where $a_{ij}=i+j$

(i) Find A (ii) Find A^2

(b) Find x and y if $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

9. If $A=\begin{bmatrix}1 & 3\\-2 & 4\end{bmatrix}$, then

(a) Find A^2 (b) Show that $A^2-5A+10I=0$ (c) Find A^{-1}

10. $A=\begin{bmatrix}2 & 1 & 3\\2 & 3 & 1\\1 & 1 & 1\end{bmatrix}$,then

(a) Find A' (b) Find $A+A'$ and $A-A'$

(c) Express A as sum of a symmetric and skew – symmetric matrix.

11. If $A=\begin{bmatrix}\cos x & -\sin x\\\sin x & \cos x\end{bmatrix}$

(a) Find A' (b) Verify that $AA'=I$

12. Let $A=\begin{bmatrix}1 & 2 & -3\\2 & 1 & -1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\5 & 4\\1 & 6\end{bmatrix}$

(a) What is the order of AB

(b) Find A' and B'

(c) Verify that $(AB)'=B'A'$

13. (a) If A is a skew – symmetric matrix, then $A' = ...$

(b) If $A = \begin{bmatrix} 0 & 3 & a \\ b & 0 & -2 \\ 5 & 2 & 0 \end{bmatrix}$ is a skew – symmetric matrix , then find the values of a and b.

14. (a) If the matrix A is both symmetric and skew – symmetric, then A is a

(i) Diagonal matrix (ii) Zero matrix (iii) Square matrix (iv) Scalar matrix

(b) If $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix}$

(i) Find $A + B$

(ii) Find the matrix C such that $A + B + C = O$

15. (a) Which of the following values of x and y make the following matrices equal $\begin{bmatrix} 2x+4 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ y+1 & 4 \end{bmatrix}$

(i) $x = 1, y = 2$ (ii) $x = 2, y = 1$ (iii) $x = -2, y = 1$ (iv) $x = 1, y = -2$

HINTS AND ANSWERS

(WORKSHEET BASED ON THE FOCUS AREA)

CHAPTER 3

MATRICES

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1. (a) Given that $2\begin{bmatrix} x & z \\ y & w \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$$\begin{bmatrix} 2x & 2z \\ 2y & 2w \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2w \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Therefore $2x+3=9 \Rightarrow 2x=6 \Rightarrow x=3$

$2z-3=15 \Rightarrow 2z=18 \Rightarrow z=9$

$2y=12 \Rightarrow y=6$ and $2w=18 \Rightarrow w=9$

(b) Order of $AB = m \times m$ and Order of $BA = n \times n$

(c) The diagonal elements of a skew symmetric matrix are 0

2. (a) $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$

$B = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$

$A + B = \begin{bmatrix} 1 & 3 \\ 0 & 6 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 3 & -1 \\ 4 & 0 \end{bmatrix}$

(c) $(A+B)(A-B)=\begin{bmatrix}1 & 3\\ 0 & 6\end{bmatrix}\begin{bmatrix}3 & -1\\ 4 & 0\end{bmatrix}=\begin{bmatrix}1\times 3+3\times 4 & 1\times -1+3\times 0\\ 0\times 3+6\times 4 & 0\times -1+6\times 0\end{bmatrix}$

$$=\begin{bmatrix}3+12 & -1+0\\ 0+24 & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}15 & -1\\ 24 & 0\end{bmatrix}$$

3. (a) Given that $A=\left[a_{ij}\right]_{3\times 2}$ where $a_{ij}=|i-2j|$

$$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\\ a_{31} & a_{32}\end{bmatrix}=\begin{bmatrix}1 & 3\\ 0 & 2\\ 1 & 1\end{bmatrix}$$

(b) $A'=\begin{bmatrix}1 & 0 & 1\\ 3 & 2 & 1\end{bmatrix}$

4. (a) Given that $A=\begin{bmatrix}2 & 4\\ 1 & 3\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 0\\ 0 & 2 & 1\end{bmatrix}$

$$AB=\begin{bmatrix}2 & 4\\ 1 & 3\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ 0 & 2 & 1\end{bmatrix}=\begin{bmatrix}2+0 & -2+8 & 0+4\\ 1+0 & -1+6 & 0+3\end{bmatrix}=\begin{bmatrix}2 & 6 & 4\\ 1 & 5 & 3\end{bmatrix}$$

(b) no. of columns of $B=3$ and no of rows of $A=2$

$\therefore$ no. of columns of $B\neq$ no. of rows of B

$\therefore$ BA is not defined

5. (a) Given that $A=\begin{bmatrix}2 & -6\\ 1 & 2\end{bmatrix}$ and $2A+B=\begin{bmatrix}1 & -10\\ -2 & -1\end{bmatrix}$

$$2A+B=\begin{bmatrix}1 & -10\\ -2 & -1\end{bmatrix}\Rightarrow B=\begin{bmatrix}1 & -10\\ -2 & -1\end{bmatrix}-2A$$

$$=\begin{bmatrix}1 & -10\\ -2 & -1\end{bmatrix}-2\begin{bmatrix}2 & -6\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1 & -10\\ -2 & -1\end{bmatrix}-\begin{bmatrix}4 & -12\\ 2 & 4\end{bmatrix}=\begin{bmatrix}-3 & 2\\ -4 & -5\end{bmatrix}$$

(b) $AB=\begin{bmatrix}2 & -6\\ 1 & 2\end{bmatrix}\begin{bmatrix}-3 & 2\\ -4 & -5\end{bmatrix}=\begin{bmatrix}-6+24 & 4+30\\ -3+-8 & 2+-10\end{bmatrix}=\begin{bmatrix}18 & 34\\ -11 & -8\end{bmatrix}$

(c) $B'=\begin{bmatrix}-3 & -4\\ 2 & -5\end{bmatrix}$

6. (a) $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}\rightarrow(1)$

$X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\rightarrow(2)$

$(1)+(2)\Rightarrow 2X=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}\Rightarrow X=\frac{1}{2}\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$

$(1)-(2)\Rightarrow 2Y=\begin{bmatrix}4 & 0\\ 2 & 2\end{bmatrix}\Rightarrow Y=\frac{1}{2}\begin{bmatrix}4 & 0\\ 2 & 2\end{bmatrix}=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$

(b) $2X+Y=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}+\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}=\begin{bmatrix}12 & 0\\ 3 & 9\end{bmatrix}$

7. (a) $A^2=AA=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}=\begin{bmatrix}9+-8 & -6+4\\ 12+-8 & -8+4\end{bmatrix}=\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}$

(b) $A^2=kA-2I \Rightarrow \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}=k\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}=\begin{bmatrix}3k & -2k\\ 4k & -2k\end{bmatrix}-\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}=\begin{bmatrix}3k-2 & -2k\\ 4k & -2k-2\end{bmatrix}\Rightarrow 4k=4$

$\Rightarrow \quad k=1$

8. (a) Given that $A=\left[a_{ij}\right]_{2\times 2}$ where $a_{ij}=i+j$

(i) $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}=\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

(ii) $A^2=AA=\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}=\begin{bmatrix} 4+9 & 6+12 \\ 6+12 & 9+16 \end{bmatrix}=\begin{bmatrix} 13 & 18 \\ 18 & 25 \end{bmatrix}$

(b) $x\begin{bmatrix} 2 \\ 3 \end{bmatrix}+y\begin{bmatrix} -1 \\ 1 \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix}+\begin{bmatrix} -y \\ y \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$2x-y=10 \quad \to (1)$

$3x+y=5 \quad \to (2)$

$(1)+(2) \Rightarrow 5x=15 \quad \Rightarrow x=3$

$3x+y=5 \quad \Rightarrow 3\times 3+y=5 \quad \Rightarrow y=-4$

9. (a) $A^2=AA=\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}=\begin{bmatrix} 1+-6 & 3+12 \\ -2+-8 & -6+16 \end{bmatrix}=\begin{bmatrix} -5 & 15 \\ -10 & 10 \end{bmatrix}$

(b) $A^2-5A+10I=\begin{bmatrix} -5 & 15 \\ -10 & 10 \end{bmatrix}-5\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}+10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$=\begin{bmatrix} -5 & 15 \\ -10 & 10 \end{bmatrix}-\begin{bmatrix} 5 & 15 \\ -10 & 20 \end{bmatrix}+\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}=0$

(c) $A^2-5A+10I=0$

Multiplying by A^{-1}

$A^{-1}A^2-5A^{-1}A+10A^{-1}I=A^{-1}0$

$A-5I+10A^{-1}=0 \quad \Rightarrow \quad 10A^{-1}=5I-A$

$A^{-1}=\frac{1}{10}(5I-A)=\frac{1}{10}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}-\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}\right)=\frac{1}{10}\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$

10. (a) $A' = A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(b) $A + A' = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 4 \\ 3 & 6 & 2 \\ 4 & 2 & 2 \end{bmatrix}$

$$A - A' = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$$

11 (a) $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ and $A' = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

(b) $AA' = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \cos x \sin x - \sin x \cos x \\ \sin x \cos x - \cos x \sin x & \sin^2 x + \cos^2 x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

12. (a) Order of AB is 2×2

(b) $A' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -3 & -1 \end{bmatrix}$ and $B' = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 4 & 6 \end{bmatrix}$

(c) $AB = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 5 & 4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 2+10+-3 & 3+8+-18 \\ 4+5+-1 & 6+4+-6 \end{bmatrix} = \begin{bmatrix} 9 & -7 \\ 8 & 4 \end{bmatrix}$

$(AB)' = \begin{bmatrix} 9 & 8 \\ -7 & 4 \end{bmatrix} \rightarrow (1)$

$$B'A' = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} 2+10+-3 & 4+5+-1 \\ 3+8+-18 & 6+4+-6 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ -7 & 4 \end{bmatrix} \rightarrow (2)$$

From (1) and (2), we have $(AB)' = B'A'$

13. (a) If A is a skew – symmetric matrix, then $A' = -A$

(b) Since A is skew-symmetric, we have $A' = -A$

$$\therefore \begin{bmatrix} 0 & b & 5 \\ 3 & 0 & 2 \\ a & -2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 3 & a \\ b & 0 & -2 \\ 5 & 2 & 0 \end{bmatrix}$$

$$\text{i.e., } \begin{bmatrix} 0 & b & 5 \\ 3 & 0 & 2 \\ a & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -3 & -a \\ -b & 0 & 2 \\ -5 & -2 & 0 \end{bmatrix}$$

So $a = -5$ and $b = -3$

14. (a) Zero matrix

(b) $A + B = \begin{bmatrix} 2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$

(c) $A + B + C = O \Rightarrow C = -(A+B) = -\begin{bmatrix} 4 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -4 & -1 & 0 \\ -3 & -1 & -1 \end{bmatrix}$

15. (a) Given that $\begin{bmatrix} 2x+4 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ y+1 & 4 \end{bmatrix} \Rightarrow 2x + 4 = 0$ and $y + 1 = 2$

$\Rightarrow x = -2$ and $y = 1$