Relations and Functions

Focus area - Class 4

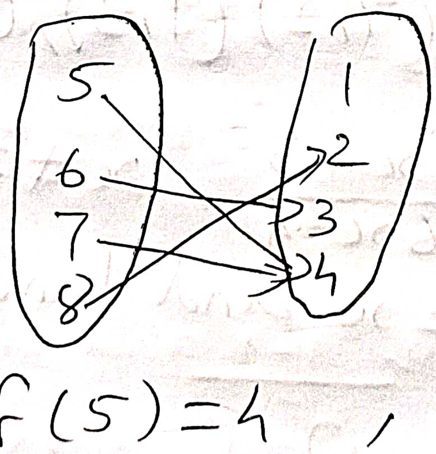
Invertibile function

A function $f : X \longrightarrow Y$ is said to be invertibile if there exist another function $g : Y \longrightarrow X$ such that $g \circ f = I_X$ and $f \circ g = I_Y$.

The function g is called inverse of f and it is denoted by $f^{-1} = g$.

A function f is invertibile iff f is bijective.

1. Let $f : \{5, 6, 7, 8\} \longrightarrow \{1, 2, 3, 4\}$ be a function defined as $f = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$. Is f is invertibile ? If so find f^{-1}.



$f(5) = 4$, $f(7) = 4$

$\therefore f$ is not one-one

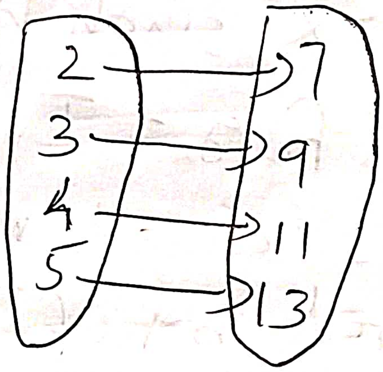
$\therefore f$ is not bijective

Hence f is not invertibile.

② Let $g:\{2,3,4,5\} \longrightarrow \{7,9,11,13\}$ be a function defined as

$g = \{(2,7),(3,9),(4,11),(5,13)\}$

Is g is invertibile? If so find g^{-1}.



Every element has distinct images.
Hence f is one-one.
Every element in the second set has pre-image. Hence f is onto.
since f is both one-one and onto it is bijective. Hence f is invertibile.

$\therefore g^{-1} = \{(7,2)\,(9,3),(11,4),(13,5)\}$

③ If $f(x) = \dfrac{4x+3}{6x-4}$, $x \neq 2/3$ then show that $f\circ f(x) = x$ and hence find f^{-1}.

$f\circ f(x) = f[f(x)]$

$= f\left[\dfrac{4x+3}{6x-4}\right]$

$$= \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$$

$$= \frac{16x+12+18x-12}{24x+18-24x+16}$$

$$= \frac{34x}{34} = x$$

$f \circ f(x) = x$

$$\therefore f^{-1} = f(x) = \frac{4x+3}{6x-4}$$

④ Show that $f: R \longrightarrow R$ given by $f(x) = 4x+3$ is invertibile and hence find f^{-1}.

Let $f(x) = f(y)$

$4x+3 = 4y+3$

$4x = 4y$

$x = y$

$\therefore f$ is one - one

Let $y \in R$, $f(x) = y$

$4x+3 = y$

$4x = y-3$

$\therefore x = \frac{y-3}{4} \in R$

$\therefore f$ is onto.

Since f is both one-one and onto f is bijective. Hence f is invertibile.

$\therefore f^{-1} = \frac{x-3}{4}$

(5) Let $f : R_+ \rightarrow [4, \infty)$ be a function defined by $f(x) = x^2+4$, Show that f is invertibile and hence find f^{-1}.

Let $f(x) = f(y)$	Let $y \in [4, \infty)$
$x^2+4 = y^2+4$	$f(x) = y$
$x^2 = y^2$	$x^2+4 = y$
$x^2-y^2 = 0$	$x^2 = y-4$
$(x+y)(x-y) = 0$	$x = \sqrt{y-4} \in R_+$
$x+y \neq 0$, $x-y = 0$	$\therefore f$ is onto
$\therefore x = y$	
$\therefore f$ is 1-1	

Since f is both 1-1 and onto, f is bijective, hence f is invertibile.

$\therefore f^{-1} = \sqrt{x-4}$

6, consider a function $f: R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$, show that f is invertibile and hence find f^{-1}.

$$f(x) = 9x^2 + 6x - 5 = (3x+1)^2 - 6$$

Let $f(x) = f(y)$

$$(3x+1)^2 - 6 = (3y+1)^2 - 6$$

$$(3x+1)^2 = (3y+1)^2$$

$$3x+1 = 3y+1$$

$$3x = 3y$$

$$\therefore x = y$$

Hence f is 1-1

Let $y \in [-5, \infty)$, $f(x) = y$

$$(3x+1)^2 - 6 = y$$

$$(3x+1)^2 = y+6$$

$$3x+1 = \sqrt{y+6}$$

$$3x = \sqrt{y+6} - 1$$

$$x = \frac{\sqrt{y+6} - 1}{3} \in R_+$$

$\therefore f$ is onto. Hence f is bijective

$\therefore f$ is invertibile.

$$\therefore f^{-1} = \frac{\sqrt{x+6} - 1}{3}$$