ARITHMETIC SEQUENCE

1

A sequence of numbers in which each term is obtained by adding a constant number to the previous term is called an arithmetic sequence.The difference of any two consecutive terms of an AS will be same. It is called its common difference.

The common difference of an AS $x_1, x_2, x_3, x_4, x_5 . . . . .$is $d = x_2 - x_1$

Eg: The common difference of the AS 8 , 13 , 18 . . . . . is d = 13 – 8 = 5

2

If the first term is f ,common difference is d , then the AS = f , f+d , f+2d ,f+3d , f+4d , . , .,

or AS = x - d , x , x + d

Its n^{th} term is given by = dn + f – d

Eg : Find the n^{th}term and 18^{th} term of the AS 8 , 13 , 18 . . . ..

n^{th} term (x_n) = dn + f – d

= 5n + 8 – 5 = 5n + 3

18^{th} term (x_{18}) = f + 17d

= 8 + 17 × 5 = 8 + 85 = 93

3

To get m^{th} term from n^{th} term ,we add (m-n) common difference to n^{th} term

ie, $x_m = x_n + (m-n)d$

Eg : Find 26^{th}term of A.S, if its 15^{th} term is 95 and common difference is 7

$x_{26} = x_{15} + 11d = 95 + 11 \times 7 = 172$

4

The difference between m^{th} term and n^{th} term is (m-n)d ie , $x_m - x_n = (m-n)d$

Eg : In an AS, 8,13,18,......Find the difference between 12^{th} term and 25^{th} term.

$x_{25} - x_{12} = 13d = 13 \times 5 = 65$

Eg: Is 100 a term of the AS 8 ,13,18,......... ?

Ans: No , 100 – 8 = 92 is not a multiple of common difference.

5

In an AS , the difference of any two terms is always a multiple of its common difference.

$$d = \frac{x_m - x_n}{m - n}$$

Eg: In an AS,first term is 8 , 15^{th}term is 78. Find common difference ?

Ans: $d = \frac{78 - 8}{15 - 1} = \frac{70}{14} = 5$

6 In an AS, the remainders on dividing any term by the common difference are same.

Eg: Is 100 a term of the AS 8, 13, 18, ...?

Ans: When we divide 100 by common difference 5, we get remainder 0.

When we divide each term by common difference, we get remainder 3.

100 is not a term of this sequence.

7 Number of terms in the AS, $x_1, x_2, x_3, x_4, x_5, \ldots, x_n$ is given by

$$n = \frac{x_n - x_1}{d} + 1$$

Eg: Find the number of terms in the AS 8, 13, 18, ..., 158

$$n = \frac{158 - 8}{5} + 1 = \frac{150}{5} + 1 = 30 + 1 = 31$$

8 If a, b, c are any three consecutive terms of an AS, then $2b = a + c$

Eg: Find the value of x if 8, x, 18 are three consecutive terms of an AS.

$2 \times x = 8 + 18$

$2x = 26$

$x = 13$

9 In an AS of odd number of terms, Midterm = Average of first and last term.

Eg: In an A.S, first term is 8 and 15th term is 78. Find its 8th term.

$$8^{th}\text{ term } (x_8) = \text{Mid term} = \frac{x_1 + x_{15}}{2} = \frac{8 + 78}{2} = 43$$

Eg: In an A.S, 9th term is 43 and 17th term is 83. Find its 13th term.

$$13^{th}\text{ term } (x_{13}) = \frac{x_9 + x_{17}}{2} = \frac{43 + 83}{2} = 63$$

10 In an AS of odd number of terms, their sum is the product of middle term and number of terms. ie, Sum of the terms = Midterm × Number of terms.

Eg: In an AS, the 8th term is 43. Find the sum of first 15 terms.

Here 8th term is the middle term

Sum of the first 15 terms $= 43 \times 15 = 645$.

Eg: In an AS, the sum of first 15 terms is 645, find its 8th term.

Here 8th term is the middle term

$$8^{th}\text{ term} = \text{Middle term} = \frac{645}{15} = 43$$

11

In an AS the pairs of terms equidistant from each end will have the same sum.

If there are 10 terms, then, $x_1 + x_{10} = x_2 + x_9 = x_3 + x_8 = x_4 + x_7 = x_5 + x_6$

If there are 9 terms, then, $x_1 + x_9 = x_2 + x_8 = x_3 + x_7 = x_4 + x_{76} = 2 \times x_5$

In an AS the sum of terms at the 4^{th} and 11^{th} positions is 100, find the sum of 7^{th} and 8^{th} terms ?

$x_7 + x_8 = x_4 + x_{11} = 100$

12

Sum of the first 'n' natural numbers is $\frac{n(n+1)}{2}$

$1 + 2 + 3 + \ldots\ldots\ldots\ldots + n = \frac{n(n+1)}{2}$

Eg : Find the sum of first 50 natural numbers

$1 + 2 + 3 + \ldots\ldots\ldots\ldots + 50 = \frac{50 \times 51}{2} = 1275$

Eg : Find 4 + 8 + 12 + ...+ 120

$4 + 8 + 12 + \ldots\ldots\ldots + 120 = 4\ (1 + 2 + 3 + \ldots\ldots + 30)$

$= 4 \times \frac{30 \times 31}{2} = = 1860$

13

Sum of the first 'n' odd numbers is n^2

$1 + 3 + 5 + \ldots\ldots\ldots + (2n-1) = n^2$

Eg: Find the sum of first 30 odd numbers

$1 + 3 + 5 + \ldots\ldots\ldots + (2 \times 30 - 1) = 30^2 = 900$

Eg : Find 1 + 3 + 5 + ...+ 99

Here,there are number of terms (n) = $\frac{99 - 1}{2} + 1 = 49 + 1 = 50$

$1 + 3 + 5 + \ldots\ldots\ldots + 99 = 50^2 = 2500$

14

Sum of first 'n' even numbers is n(n+1)

$2 + 4 + 6 + \ldots\ldots\ldots + 2n = n(n+1)$

Eg : Find the sum of first 40 even numbers .

$2 + 4 + 6 + \ldots\ldots\ldots + 2 \times 40 = 40 \times 41 = 1640$

Eg : Find , 2 + 4 + 6 + ...+ 40

Here , number of terms (n) = $\frac{40 - 2}{2} + 1 = 19 + 1 = 20$

$2 + 4 + 6 + \ldots\ldots\ldots + 40 = 20 \times 21 = 420$

15 The sum of terms of an AS with n terms equals half the product of number of terms with sum of the first and last terms.

$$x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots\ldots + x_n = \frac{n}{2}[x_1 + x_n]$$

Eg : Find the sum of the AS 8 ,13 ,18 . . . .. , 78

Here $x_1 = 8$, $x_n = 78$ and $n = \frac{78-8}{5} + 1 = 14 + 1 = 15$

sum of first 15 terms = $\frac{15}{2}[8 + 78] = 645$

16 The sum of first n terms of an AS with first term f and common difference d .

$$f + (f+d) + (f+2d) + \ldots\text{n terms} = \frac{n}{2}[2f + (n-1)d]$$

Eg : Find the sum of first 15 terms of 8,13,18,...

Here f = 8 ,d = 5

Sum of first 15 terms = $\frac{15}{2}[2 \times 8 + (15-1)5] = 645$

17 The algebraic form (n^{th} term) of an Arithmetic sequence is of the form $x_n = an + b$, where

Common differene (d) = Coefficient of n = a

First term (f) = Sum of the coefficient = a + b

Eg : Find the first term and common difference of an AS,$x_n = 5n + 3$

First term = 5 + 3 = 8 Common difference = 5

18 The difference between the sum of first 'n' terms and the sum of next 'n'terms is n^2d, where 'd' is the common difference.

Eg : Find the difference between the sum of first 25 terms and next 25 terms of an Arithmetic sequence ,8,13,18,23,.......

Here , d= 5 and n = 25

Difference = $n^2d = 25^2 \times 5 = 3125$

19 Sum of the first 'n' terms from its n^{th} term.

n^{th} term = an + b

Eg : The algebraic form of an Arithmetic sequence is 4n + 3.

Find the sum of first 20 terms

n^{th} term = 4n + 3

Sum of first 20 terms = $4 \times \frac{20 \times 21}{2} + 3 \times 20 = 900$

20

Consider the pattern

f
$f + d \quad f + 2d$
$f + 3d \quad f + 4d \quad f + 5d$
....
....

Algebraic form = $an + b$ Number of terms in n^{th} line = n

Last term of the n^{th} line = $a \dfrac{n(n+1)}{2} + b$,

Eg : Consider the pattern

5
8 11
14 17 20
....
....

Find the last term and first term in 20^{th} row.

Here , Algebraic form = $3n + 2$

Last term in 20^{th} row = $3 \times \dfrac{20 \times 21}{2} + 2 = 630 + 2 = 632$

First term in 20^{th} row = $632 – 19d = 632 – 19 \times 3 = 575$

21

The pattern

f
$f + d \quad f + 2d \quad f + 3d$
$f + 4d \quad f + 5d \quad f + 6d \quad f + 7d \quad f + 8d$
....
....

Algebraic form = $an + b$ Number of terms in n^{th} line = $2n - 1$

Last term of the n^{th} line = $an^2 + b$

Eg : Consider the pattern

5
8 11 14
17 20 23 26 29
....
....

Find the last term and first term in 20^{th} row.

Here , Algebraic form = $3n + 2$

Number of terms in 20^{th} row = $2 \times 20 – 1 = 39$

Last term in 20^{th} row = $3 \times 20^2 + 2 = 1200 + 2 = 1202$

First term in 20^{th} row = $1202 – 38d = 1202 – 38 \times 3 = 1088$