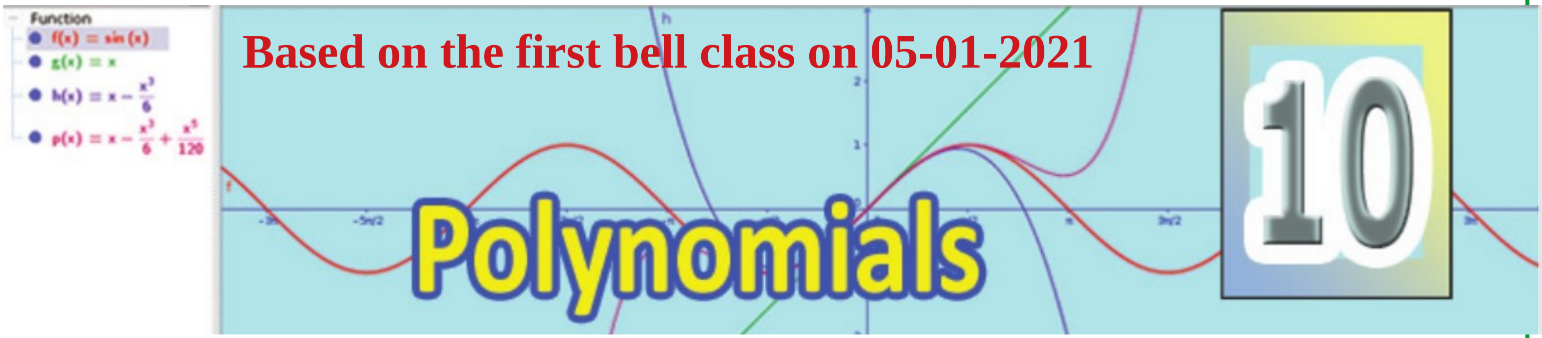

Polynomials

An algebraic expression is a polynomial if all the terms contains the same variable and the powers are non negative integers.

Eg: $x^2 - 3x + 2$ Here the variable is x and powers of x are non negative integers.

A polynomial in the variable x is represented by p(x).

That is, $p(x) = x^2 - 3x + 2$

A polynomial in the variable y is represented by p(y).

Using the identity $x^2 - y^2 = (x + y)(x - y)$

$x^2 - 1 = (x + 1)(x - 1)$

$x^2 - 4 = x^2 - 2^2 = (x + 2)(x - 2)$

$x^2 - 1$, $x^2 - 4$, $x^2 - 3x + 2$ are second degree polynomials.

Degree of a polynomial

Degree of a polynomial is the degree of its highest degree term.

$x^2 - 1 = (x + 1)(x - 1)$

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$x^2 - 1$ is a second degree polynomial.

$x + 1$, $x - 1$ are first degree polynomials.

We can write a second degree polynomial as the product of two first degree polynomials.

Factors and solutions

When we write a number as the product of two numbers, those multiplied numbers are called factors.

Eg: $10 = 5 \times 2$

Here 5 and 2 are factors of 10

If p(x) is the product of q(x) and r(x), then q(x) and r(x) are factors of p(x).

Using the identity $(x + a)(x + b) = x^2 + (a+b)x + ab$ we can write

$(x + 3)(x + 1) = x^2 + (3+1)x + 3\times1 = x^2 + 4x + 3$

$(x - 1)(x - 2) = x^2 + (-1+-2)x + -1\times -2 = x^2 - 3x + 2$

That is, $x^2 - 3x + 2 = (x - 1)(x - 2)$

Here the first degree polynomials $x - 1$, $x - 2$ are the factors of the second degree polynomial $x^2 - 3x + 2$

In the polynomial $p(x) = x^2 - 3x + 2$, find p(1) and p(2).

$p(x) = x^2 - 3x + 2$

$p(1) = 1^2 - 3\times1 + 2 = 1 - 3 + 2 = 3 - 3 = 0$

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That is, p(1) = 0

We can find p(1) in another way.

$p(x) = x^2 - 3x + 2 = (x - 1)(x - 2)$

$p(1) = (1 - 1)(1 - 2) = 0 \times -1 = 0$

Therefore, (x-1) is a factor of $p(x) = x^2 - 3x + 2$

$p(2) = 2^2 - 3\times2 + 2 = 4 – 6 + 2 = 6 – 6 = 0$

or

$p(2) = (2 - 1)(2 - 2) = 1 \times 0 = 0$

(x-1) is a factor 0f p(x), then p(1) = 0

(x-2) is a factor 0f p(x), then p(2) = 0

$p(x) = x^2 - 3x + 2$

If p(x) = 0, then

$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$(x – 1) = 0$ or $(x - 2) = 0$

x = 1 or x = 2

That is,

if x = 1, 2 then the equation $x^2 - 3x + 2 = 0$ becomes true.

These numbers are the solutions of the equation.

If the first degree polynomial (x-a) is a factor of the polynomial p(x); then p(a) = 0; that is, a is a solution of the equation p(x)=0

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In general,

If the polynomial p(x) can be split into first degree factors as $p(x) = (x-a_1)(x-a_1) \dots (x-a_n)$ then the numbers $a_1, a_2, \dots, a_n$ are the solutions of the equation p(x)=0

Activity

Write the second degree polynomial as the product of two first degree polynomials.

1. $p(x) = x^2 - 7x + 12$

$x^2 - 7x + 12 = x^2 + (a+b)x + ab$

Here $ab = 12$

$a + b = -7$

That is, product of two numbers = 12

sum of two numbers = -7

Therefore, the numbers are -3 and -4

That is, a = -3, b = -4

That is, $x^2 - 7x + 12 = (x - 3)(x - 4)$

$p(x) = x^2 - 7x + 12$

If p(x) = 0, then $x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$(x – 3) = 0$ or $(x - 4) = 0$

$x = 3$ or $x = 4$

These are the solutions of the equation $x^2 - 7x + 12 = 0$

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2. $p(x) = x^2 - 12x - 13$

$x^2 - 12x - 13 = x^2 + (a+b)x + ab$

Here $ab = -13$

$a + b = -12$

That is, product of two numbers = -13

sum of two numbers = -12

Therefore, the numbers are -13 and 1

Therefore, $a = -13$, $b = 1$

Therefore, $x^2 - 12x - 13 = (x-13)(x+1)$

If $p(x) = 0$, then $x^2 - 12x - 13 = 0$

$(x - 13)(x + 1) = 0$

$(x - 13) = 0$ or $(x + 1) = 0$

$x = 13$ or $x = -1$

These are the solutions of the equation $x^2 - 12x - 13 = 0$

Assignment

Write the second degree polynomials given below as the product of two first degree polynomials. Find also the solutions of the equation $p(x) = 0$ in each.

1. $p(x) = x^2 + 7x + 12$

2. $p(x) = x^2 - 8x + 12$

3. $p(x) = x^2 + 13x + 12$

4. $p(x) = x^2 + 12x - 13$

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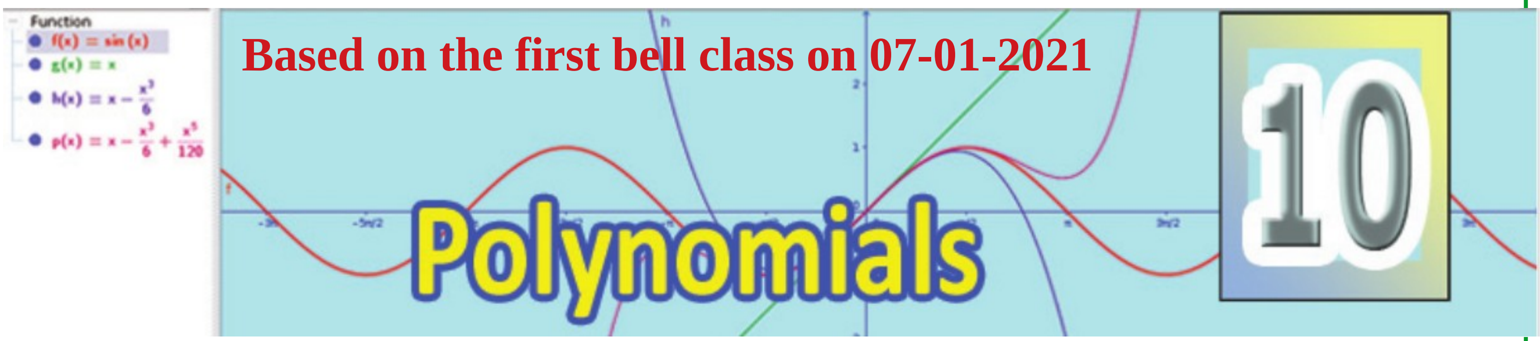

Assignment on 05-01-2021

Write the second degree polynomials given below as the product of two first degree polynomials. Find also the solutions of the equation p(x) = 0 in each.

1. $p(x) = x^2 + 7x + 12$
2. $p(x) = x^2 - 8x + 12$
3. $p(x) = x^2 + 13x + 12$
4. $p(x) = x^2 + 12x - 13$

Answers

1. $p(x) = x^2 + 7x + 12$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$x^2 + 7x + 12 = x^2 + (a+b)x + ab$

Therefore, $a+b = 7$

$ab = 12$

That is, sum of two numbers = 7

Their product = 12

Therefore, $a = 3 \quad b = 4$

Therefore, $x^2 + 7x + 12 = (x+3)(x+4)$

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If $p(x) = 0$, then $x^2 + 7x + 12 = 0$

Therefore, $(x+3)(x+4) = 0$

That is, $(x+3) = 0$ or $(x+4) = 0$

That is, $x = -3$ or $x = -4$

Therefore,

$x = -3, x = -4$ are the solutions of $p(x) = x^2 + 7x + 12$.

2. $p(x) = x^2 - 8x + 12$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$x^2 - 8x + 12 = x^2 + (a+b)x + ab$

Therefore, $a+b = -8$

$ab = 12$

That is, sum of two numbers = -8

Their product = 12

Therefore, $a = -6$ $b = -2$

Therefore, $x^2 - 8x + 12 = (x-6)(x-2)$

If $p(x) = 0$, then $x^2 - 8x + 12 = 0$

Therefore, $(x-6)(x-2) = 0$

That is, $(x-6) = 0$ or $(x-2) = 0$

That is, $x = 6$ or $x = 2$

Therefore,

$x = 6, x = 2$ are the solutions of $p(x) = x^2 - 8x + 12$.

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3. $p(x) = x^2 + 13x + 12$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$x^2 + 13x + 12 = x^2 + (a+b)x + ab$

Therefore, $a+b = 13$

$ab = 12$

That is, sum of two numbers = 13

Their product = 12

Therefore, $a = 12 \quad b = 1$

Therefore, $x^2 + 13x + 12 = (x+12)(x+1)$

If $p(x) = 0$, then $x^2 + 13x + 12 = 0$

Therefore, $(x+12)(x+1) = 0$

That is, $(x+12) = 0$ or $(x+1) = 0$

That is, $x = -12$ or $x = -1$

Therefore,

$x = -12$, $x = -1$ are the solutions of $p(x) = x^2 + 13x + 12$.

4. $p(x) = x^2 + 12x - 13$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$x^2 + 12x - 13 = x^2 + (a+b)x + ab$

Therefore, $a+b = 12$

$ab = -13$

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That is, sum of two numbers = 12

Their product = -13

Therefore, a = -1 b = 13

Therefore, $x^2 + 12x - 13 = (x-1)(x+13)$

If p(x) = 0, then $x^2 + 12x - 13 = 0$

Therefore, $(x-1)(x+13) = 0$

That is, $(x-1) = 0$ or $(x+13) = 0$

That is, $x = 1$ or $x = -13$

Therefore,

$x = 1$, $x = -13$ are the solutions of $p(x) = x^2 + 12x - 13$.

Note:

We know that $x^2 - a^2 = (x+a)(x-a)$

That is, $(x-a)$ is a factor of $x^2 - a^2$

Let $p(x) = x^2$

$p(a) = a^2$

$p(x) - p(a) = x^2 - a^2$

$= (x+a)(x-a)$

That is, $(x-a)$ is a factor of $p(x) - p(a)$.

Example:1

Let $p(x) = 3x^2 + 2x - 1$

$p(a) = 3a^2 + 2a - 1$

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$p(x) - p(a) = 3x^2 + 2x - 1 - (3a^2 + 2a - 1)$

$= 3x^2 + 2x - 1 - 3a^2 - 2a + 1$

$= 3x^2 + 2x - 3a^2 - 2a$

$= 3x^2 - 3a^2 + 2x - 2a$

$= 3(x^2 - a^2) + 2(x-a)$

$= 3(x+a)(x-a) + 2(x-a)$

$= (x-a)[3(x+a) + 2]$

That is, (x-a) is a factor of p(x) – p(a).

Example:2

Let $p(x) = lx^2 + mx + n$

$p(a) = la^2 + ma + n$

$p(x) - p(a) = lx^2 + mx + n - (la^2 + ma + n)$

$= lx^2 + mx + n - la^2 - ma - n$

$= lx^2 + mx - la^2 - ma$

$= lx^2 - la^2 + mx - ma$

$= l(x^2 - a^2) + m(x-a)$

$= l(x+a)(x-a) + m(x-a)$

$= (x-a)[l(x+a) + m]$

$= (x-a)[lx + la + m]$

$= (x-a)[lx + (la+m)]$

That is, (x-a) is a factor of p(x) – p(a).

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For any second degree polynomial p(x) and for any number a, the polynomial x−a is a factor of the polynomial p(x)−p(a).

For any second degree polynomial $p(x)$ and for any number a, if $p(a) = 0$, then the first degree polynomial $x - a$ is a factor of the polynomial $p(x)$

In each pair of polynomials given below, find the number to be subtracted from the first to get a polynomial for which the second is a factor. Find also the second factor of the polynomial got on subtracting the number.

(i) $x^2 - 3x + 5$, $x - 4$

(ii) $x^2 - 3x + 5$, $x + 4$

(iii) $x^2 + 4x + 6$, $x + 1$

Activity

Answer(i) $x^2 - 3x + 5$; $x-4$

Let $p(x) = x^2 - 3x + 5$

(x-a) is a factor of p(x) – p(a)

Therefore, (x-4) is a factor of p(x) – p(4)

$p(4) = 4^2 - 3\times4 + 5$

$= 16 - 12 + 5 = 4 + 5 = 9$

That is, 9 is to be subtracted from $x^2 - 3x + 5$ to get a polynomial in which x-4 is a factor.

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$p(x) - 9 = x^2 - 3x + 5 - 9 = x^2 - 3x - 4$

That is, x-4 is a factor of $x^2 - 3x - 4$

We have to find the second factor of $x^2 - 3x - 4$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$x^2 - 3x - 4 = x^2 + (a+b)x + ab$

Therefore, $a + b = -3$ $ab = -4$

$-4 + 1 = -3$ $-4\times1 = -4$

Therefore, $a = -4$ $b = 1$

Therefore, $x^2 - 3x - 4 = (x-4)(x+1)$

(ii) $x^2 - 3x + 5$; $x+4$

$x+4 = x-(-4)$

Let $p(x) = x^2 - 3x + 5$

$(x-a)$ is a factor of $p(x) - p(a)$

Therefore, $x-(-4)$ is a factor of $p(x) - p(-4)$

$p(-4) = (-4)^2 - 3\times-4 + 5$

$= 16 + 12 + 5 = 28 + 5 = 33$

That is, 33 is to be subtracted from $x^2 - 3x + 5$ to get a polynomial in which x+4 is a factor.

$p(x) - 33 = x^2 - 3x + 5 - 33 = x^2 - 3x - 28$

That is, x+4 is a factor of $x^2 - 3x - 28$

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We have to find the second factor of $x^2 - 3x - 28$

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

$$x^2 - 3x - 28 = x^2 + (a+b)x + ab$$

Therefore, $a + b = -3$ $\qquad ab = -28$

$-7 + 4 = -3$ $\qquad -7\times4 = -28$

Therefore, $a = -7 \quad b = 4$

Therefore, $x^2 - 3x - 28 = (x+4)(x-7)$

(iii) $x^2 + 4x + 6$; $x+1$

$$x+1 = x-(-1)$$

Let $p(x) = x^2 + 4x + 6$

$(x-a)$ **is a factor of** $p(x) - p(a)$

Therefore, $x-(-1)$ **is a factor of** $p(x) - p(-1)$

$$p(-1) = (-1)^2 + 4\times-1 + 6$$

$$= 1 - 4 + 6 = 7 - 3 = 3$$

That is, 3 is to be subtracted from $x^2 + 4x + 6$ **to get a polynomial in which** $x+1$ **is a factor.**

$$p(x) - 3 = x^2 + 4x + 6 - 3 = x^2 + 4x + 3$$

That is, $x+1$ **is a factor of** $x^2 + 4x + 3$

We have to find the second factor of $x^2 + 4x + 3$

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

$$x^2 + 4x + 3 = x^2 + (a+b)x + ab$$

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Therefore, $a + b = 4$ $ab = 3$

$1 + 3 = 4$ $1\times3 = 3$

Therefore, $a = 1$ $b = 3$

Therefore, $x^2 + 4x + 3 = (x+1)(x+3)$

Assignment

In each pair of polynomials given below, find the number to be subtracted from the first to get a polynomial for which the second is a factor. Find also the second factor of the polynomial got on subtracting the number.

(i) $x^2 + 5x - 7$, $x-1$

(ii) $x^2 - 4x - 3$, $x-1$

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Assignment on 07-01-2021

In each pair of polynomials given below, find the number to be subtracted from the first to get a polynomial for which the second is a factor. Find also the second factor of the polynomial got on subtracting the number.

(i) $x^2 + 5x - 7$, $x-1$

(ii) $x^2 - 4x - 3$, $x-1$

Answer

(i) $x^2 + 5x - 7$, $x-1$

Let $p(x) = x^2 + 5x - 7$

$(x-a)$ is a factor of $p(x) - p(a)$

Therefore, $(x-1)$ is a factor of $p(x) - p(1)$

$p(1) = 1^2 + 5\times1 - 7$

$= 1 + 5 - 7 = 6 - 7 = -1$

That is, -1 is to be subtracted from $x^2 + 5x - 7$ to get a polynomial in which $x-1$ is a factor.

$p(x) - -1 = x^2 + 5x - 7 - -1 = x^2 + 5x - 7 + 1$

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$= x^2 + 5x - 6$

That is, x-1 is a factor of $x^2 + 5x - 6$

We have to find the second factor of $x^2 + 5x - 6$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$x^2 + 5x - 6 = x^2 + (a+b)x + ab$

Therefore, $a + b = 5$ $ab = -6$

$6 + -1 = 5$ $6\times-1 = -6$

Therefore, $a = 6$ $b = -1$

Therefore, $x^2 + 5x - 6 = (x+6)(x-1)$

(ii) $x^2 - 4x - 3$, x-1

Let $p(x) = x^2 - 4x - 3$

(x-a) is a factor of p(x) – p(a)

Therefore, x-1 is a factor of p(x) – p(1)

$p(1) = 1^2 - 4\times1 - 3$

$= 1 - 4 - 3 = -3 -3 = -6$

That is, -6 is to be subtracted from $x^2 - 4x - 3$ to get a polynomial in which x-1 is a factor.

$p(x) - -6 = x^2 - 4x - 3 - -6 = x^2 - 4x - 3 + 6$

$= x^2 - 4x + 3$

That is, x-1 is a factor of $x^2 - 4x + 3$

We have to find the second factor of $x^2 - 4x + 3$

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$(x+a)(x+b) = x^2 + (a+b)x + ab$

$x^2 - 4x + 3 = x^2 + (a+b)x + ab$

Therefore, $a + b = -4$ $ab = 3$

$-3 + -1 = -4$ $-3 \times -1 = 3$

Therefore, $a = -3$ $b = -1$

Therefore, $x^2 - 4x + 3 = (x-3)(x-1)$

Note:

(x-a) is a factor of p(x) – p(a).

Therefore, The polynomial p(x) – p(a) can be written as the product of (x-a) and a polynomial q(x)

That is, $p(x) - p(a) = (x-a)\,q(x)$

Therefore, $p(x) = (x-a)\,q(x) + p(a)$

That is, p(a) is the remainder when p(x) is divided by (x-a).

Note:

p(-a) is the remainder when p(x) is divided by (x+a).

Activity

In the polynomial $x^2 + kx + 6$, what number must be taken as k to get a polynomial for which $x - 1$ is a factor? Find also the other factor of that polynomial.

Answer

$p(x) = x^2 + kx + 6$

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If (x-1) is a factor of p(x), then p(1) = 0

p(1) = 0

That is, $1^2 + k\times1 + 6 = 0$

That is, $1 + k + 6 = 0$

That is, $k + 7 = 0$

Therefore, $k = -7$

Therefore, $p(x) = x^2 - 7x + 6$

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$x^2 - 7x + 6 = x^2 + (a+b)x + ab$

Therefore, $a + b = -7$ $ab = 6$

$-1 + -6 = -7$ $-1 \times -6 = 6$

Therefore, $a = -1$ $b = -6$

Therefore, $x^2 - 7x + 6 = (x-1)(x-6)$

Activity

In the polynomial $kx^2 + 2x - 5$, what number must be taken as k to get a polynomial for which x − 1 is a factor?

Answer

$p(x) = kx^2 + 2x - 5$

If (x-1) is a factor of p(x), then p(1) = 0

Therefore, $p(1) = 0$

That is, $k\times1^2 + 2\times1 - 5 = 0$

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That is, $k + 2 - 5 = 0$

That is, $k - 3 = 0$

Therefore, $k = 3$

Activity

Find the remainder when $p(x) = x^{75} + 2x^{50} + x^2 + 1$ is divided by $(x-1)$

Answer

$$p(x) = x^{75} + 2x^{50} + x^2 + 1$$

when p(x) is divided by (x-a), the remainder is p(a).

Therefore,

when p(x) is divided by (x-1), the remainder is p(1).

$$P(1) = 1^{75} + 2\times1^{50} + 1^2 + 1 = 1+2+1+1 = 5$$

Therefore, the remainder = 5

Activity

Find a second degree polynomial p(x) such that p(2) = 0 and p(-2) = 0.

Answer

If p(a) = 0, then (x-a) is a factor of p(x).

Given that, p(2) = 0

Therefore, (x-2) is a factor of p(x).

Also given that, p(-2) = 0

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Therefore, (x+2) is a factor of p(x).

Therefore, the polynomial p(x) = (x-2)(x+2)

$= x^2 - 2^2 = x^2 - 4$

Activity

Which first degree polynomial is added to $3x^3 - 2x^2$ gives a polynomial p(x) for which $x^2 - 1$ is a factor?

Answer

Let the first degree polynomial added = ax + b

Therefore, $p(x) = 3x^3 - 2x^2 + ax + b$

$x^2 - 1$ is a factor of p(x)

$x^2 - 1 = (x + 1)(x - 1)$

That is, (x + 1) and (x – 1) are factors of p(x).

If (x – 1) is a factor of p(x), then p(1) = 0

That is, $3\times1^3 - 2\times1^2 + a\times1 + b = 0$

$3\times1 - 2\times1 + a\times1 + b = 0$

$3 - 2 + a + b = 0$

$1 + a + b = 0$

$a + b = -1$ .... (1)

If (x + 1) is a factor of p(x), then p(-1) = 0

That is, $3\times(-1)^3 - 2\times(-1)^2 + a\times-1 + b = 0$

$3\times-1 - 2\times1 - a + b = 0$

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-3 – 2 - a + b = 0

-5 - a + b = 0

-a + b = 5 2

a + b = -1 1

Adding these two equations, we get

2b = 4

b =4 ÷ 2 = 2

a + b = -1

That is, a + 2 = -1

a = -1 – 2 = -3

Therefore,

the first degree polynomial is to be added = -3x + 2

Assignment

1. Find the value of k, if x-2 is a factor of p(x) = $3x^2 – 5x + k$.

2. The solutions of $x^2 + ax + b = 0$ are 3 and -4.

(a) Write $x^2 + ax + b$ as the product of two first degree polynomials

(b) Find the value of a and b.

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