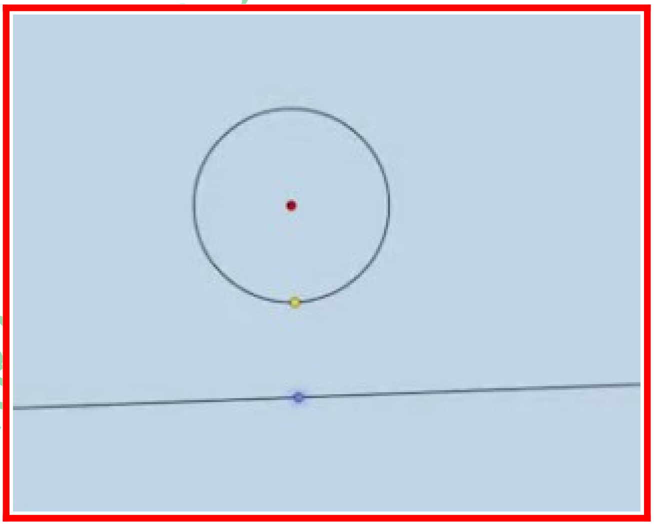
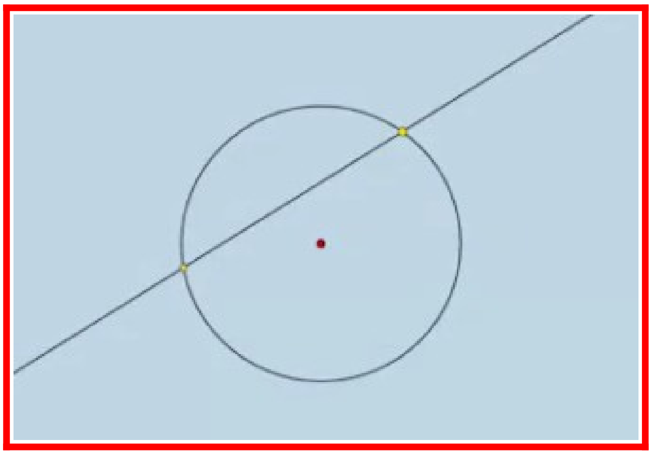
(Based on the online class on 07-12-2020)

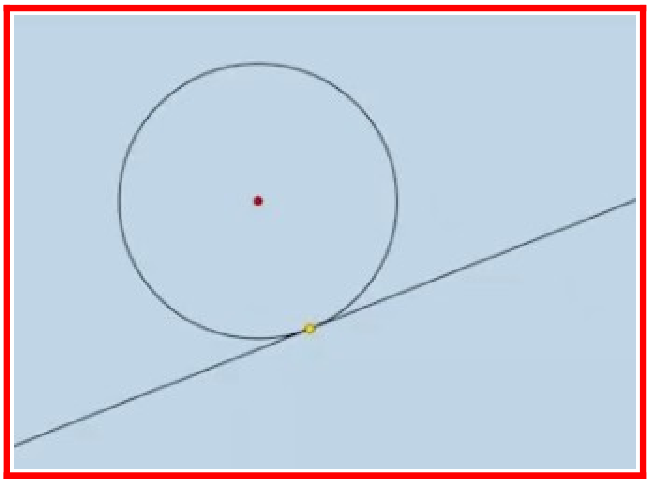
LINES AND CIRCLES



In the figure, a circle and a line are drawn. Here the line is not touching the circle. Therefore, there is no common point.

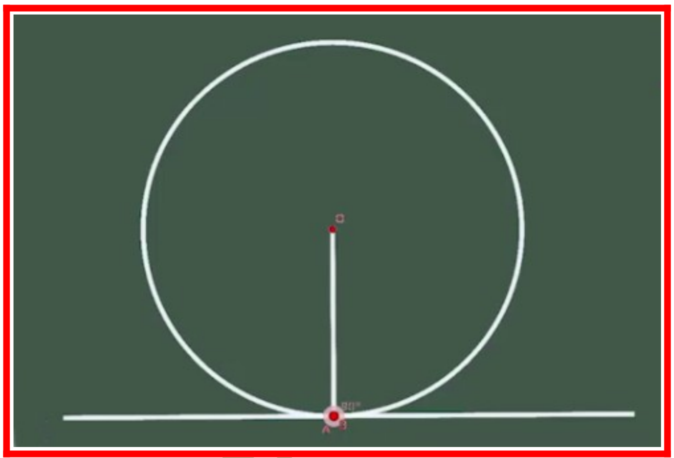


In the figure, a circle and a line are drawn. The line is cutting the circle. There is two common points.

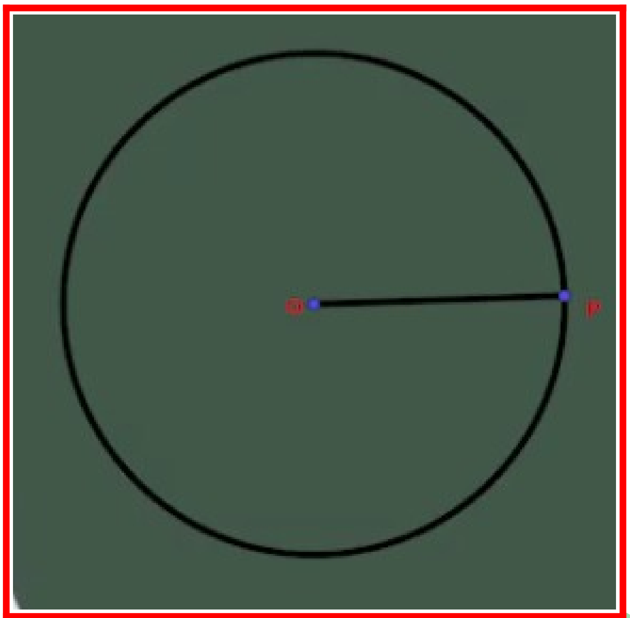


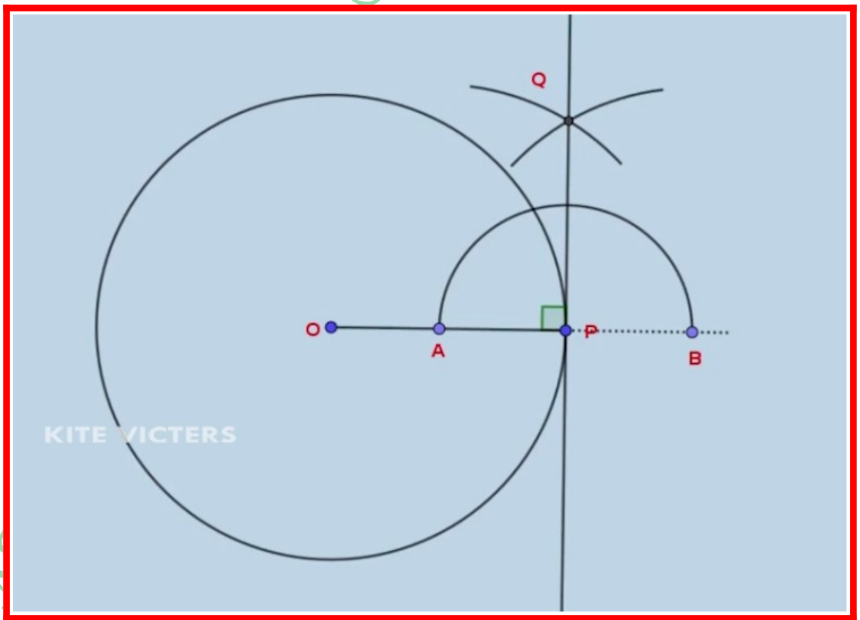
In the figure, a circle and a line are drawn. The line is just touching the circle. There is only one common point. Such lines are tangent of the circle through that point.

The tangent at a point on a circle is perpendicular to the diameter through that point.



How can we draw a tangent of a circle?





Steps

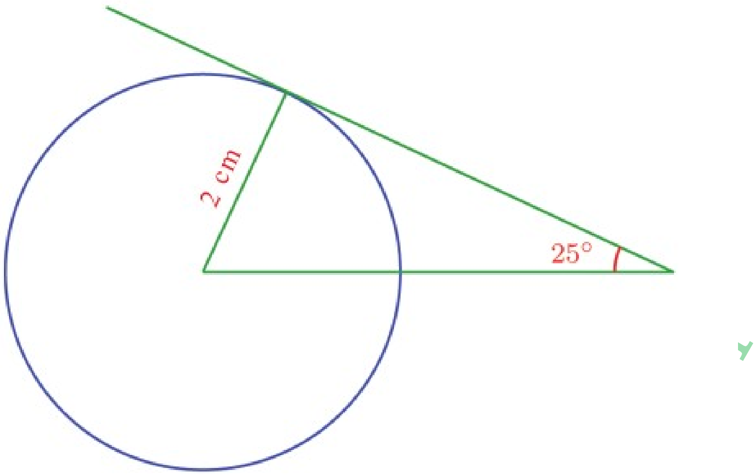
Draw a circle with centre O. Mark a point P on the circle.

Draw a perpendicular PQ to the radius through P.

(For this, extend OP to outside of the circle . Draw a small semi circle with centre at P and diameter AB. Then draw the perpendicular bisector(PQ) of AB.) PQ is the tangent of the circle.

Activity-1

Draw the picture in your notebook with the given measure.

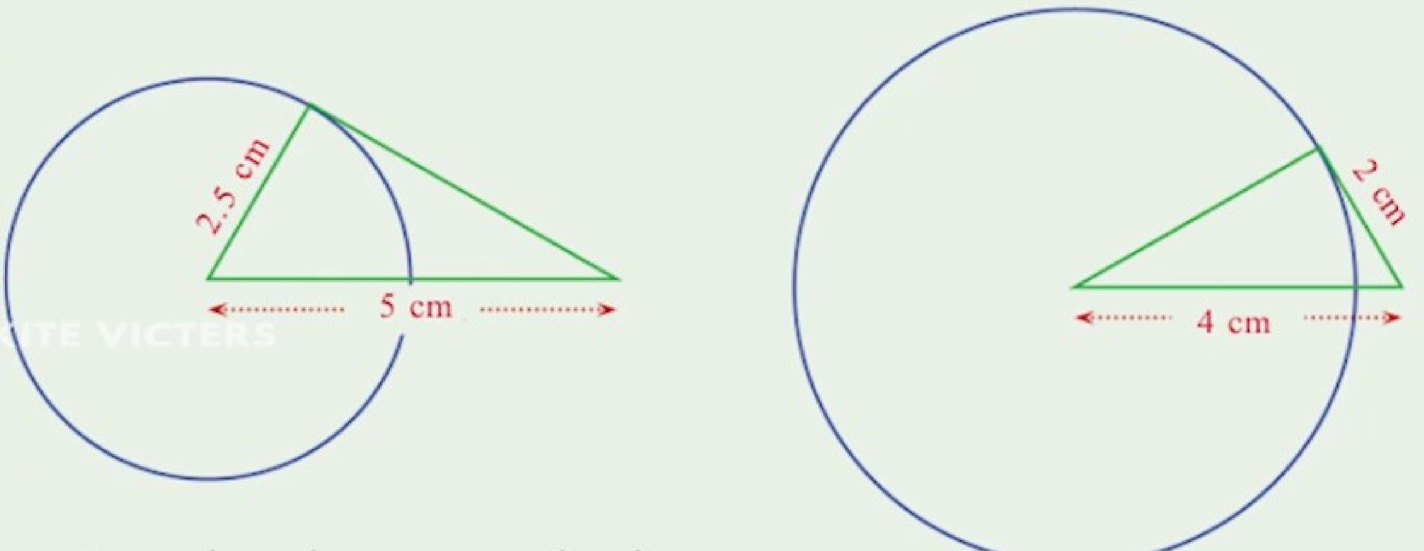


Steps

1. Draw a circle of radius 2 cm
2. Draw a radius.
3. Draw another radius with 65° between the two radii and extend this to outside of the circle.
4. Draw a tangent through the end of first radius. Now the angle between the tangent and the line will be 25°.

Activity-2 (Do yourself)

In each of the two pictures below, a triangle is formed by a tangent to a circle, the radius through the point of contact and a line through the centre:

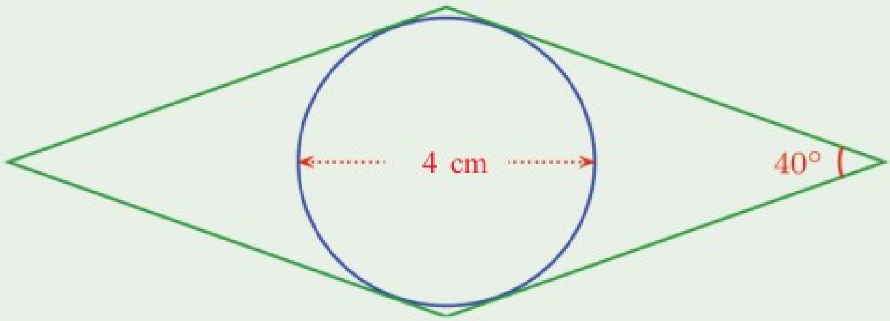


Draw these in your note book.

Activity-3 (Do yourself)

In the picture, all sides of a rhombus are tangents to a circle.

Draw this picture in your notebook.



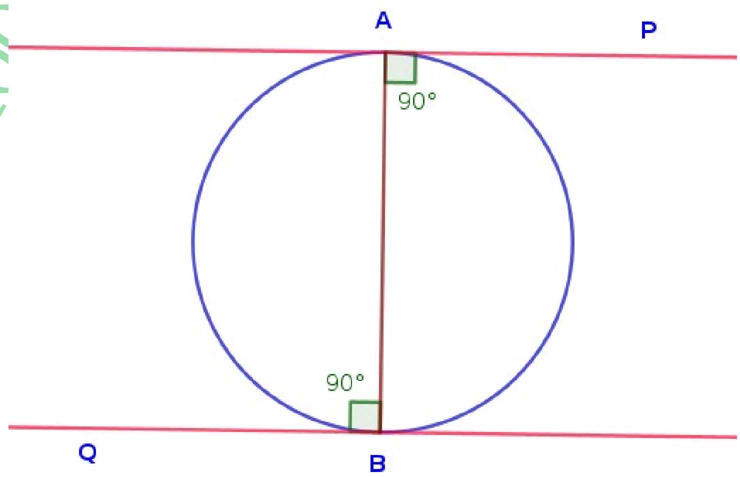
Activity-4

Prove that the tangents drawn to a circle at the two ends of a diameter are parallel.

Proof

In the figure, a circle is drawn with AB as diameter. AP and BQ are tangents of circle.

The tangent at a point on a circle is perpendicular to the diameter through that point.



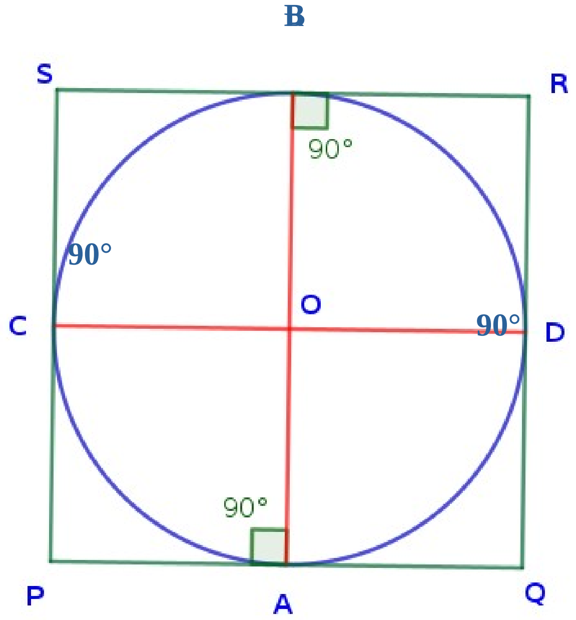
Therefore, $\angle BAP = \angle ABQ = 90^\circ$

Since the alternate angles are equal, AP and BQ are parallel.

Activity-5

What sort of a quadrilateral is formed by the tangents at the ends of two perpendicular diameters of a circle?

Answer



AB and CD are two perpendicular diameters. PQ and SR are tangents at the end points of diameter AB. QR and PS are tangents at the end points of diameter CD.

We know that the tangents at the end point of a diameter are parallel.

Therefore, PQ ∥ SR and PS ∥ QR

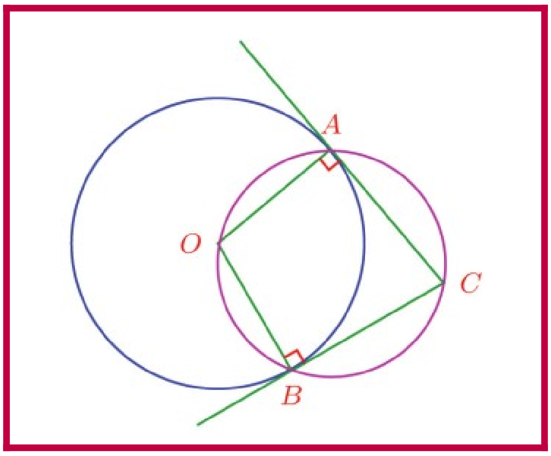
PQ = SR = PS = QR = diameter.

In the squarePAOC, $\angle$PAO = $\angle$PCO = $\angle$AOC = 90°

Therefore, $\angle$P = 90°. Similarly, $\angle$Q = $\angle$R = $\angle$S = 90°

Therefore, PQRS is a square.

Tangents and Angles



In the picture, A and B are two points on the circle. OA, OB are radii. AC and BC are tangents.

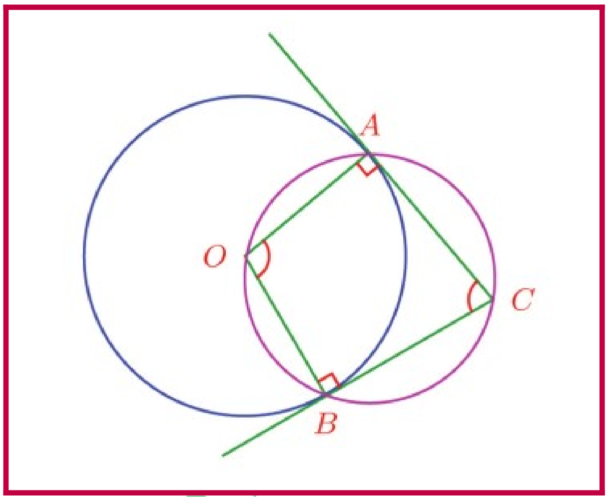
The tangents at the points A, B meet at C.

In the quadrilateral OACB, the angles at the opposite corners A, B are right; so their sum is 180°. Thus the quadrilateral is cyclic.

The quadrilateral with vertices at the centre of a circle, two points on it and the point where the tangents at these points meet, is cyclic.

In such a quadrilateral the sum of the other two angles is also 180°.

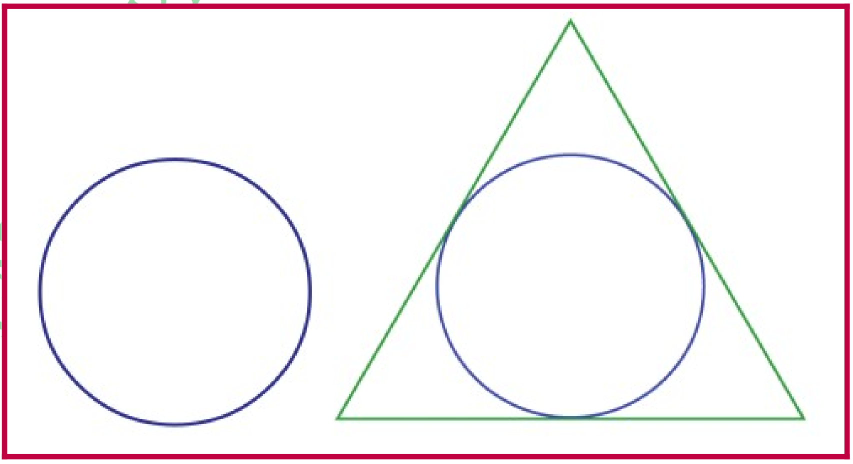
That is, ∟AOB + ∟ACB = 180°

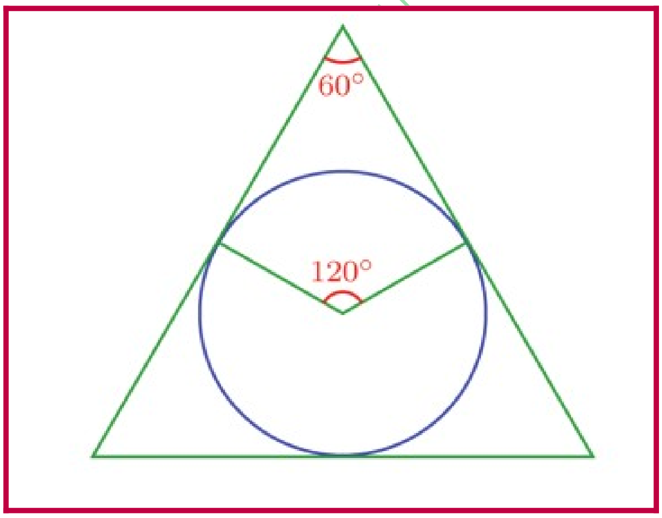


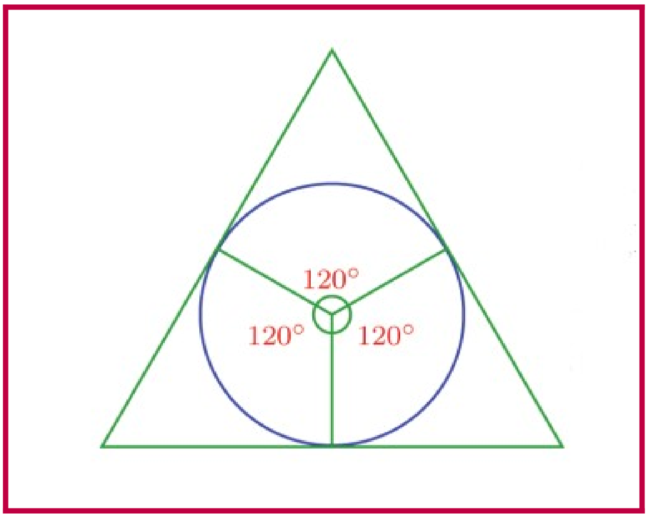
> **In a circle, the angles between the radii through two points and the angle between the tangents at these points are supplementary.**

How can we draw an equilateral triangle, exactly covering a circle?

The sides of the triangle must be the tangents of the circle. Since the triangle is to be equilateral, the angle between them must be 60°. Therefore, the angle between the radii and point of contact of tangents is 180° – 60° = 120°.







Steps to draw the picture in measure

1. Draw a circle.

2. Draw 3 radius with angle 120° apart.

3. Draw tangents at the ends of these radii.

Draw a circle of radius 3 centimetres and draw an equilateral triangle exactly covering the circle.

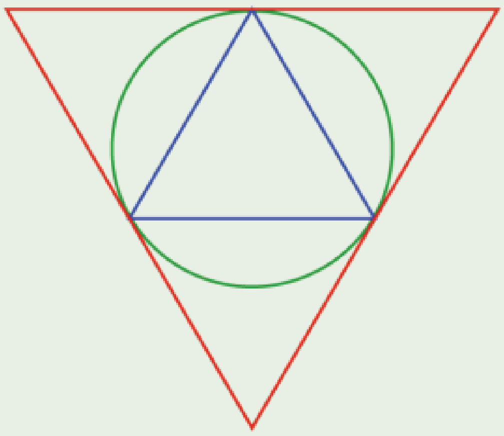
Activity-6 (Draw yourself)

Draw a circle of radius 2.5 centimetres. Draw a triangle of angles 40°, 60°, 80° with all its sides touching the circle.

Hint : Angle between the radii are 180° – 40° = 140°, 180° – 60° = 120°, 180° – 80° = 100°

Activity-7

In the picture, the small (blue) triangle is equilateral. The sides of the large (red) triangle are tangents to the circumcircle of the small triangle at its vertices.



i) Prove that the large triangle is also equilateral and its sides are double those of the small triangle.

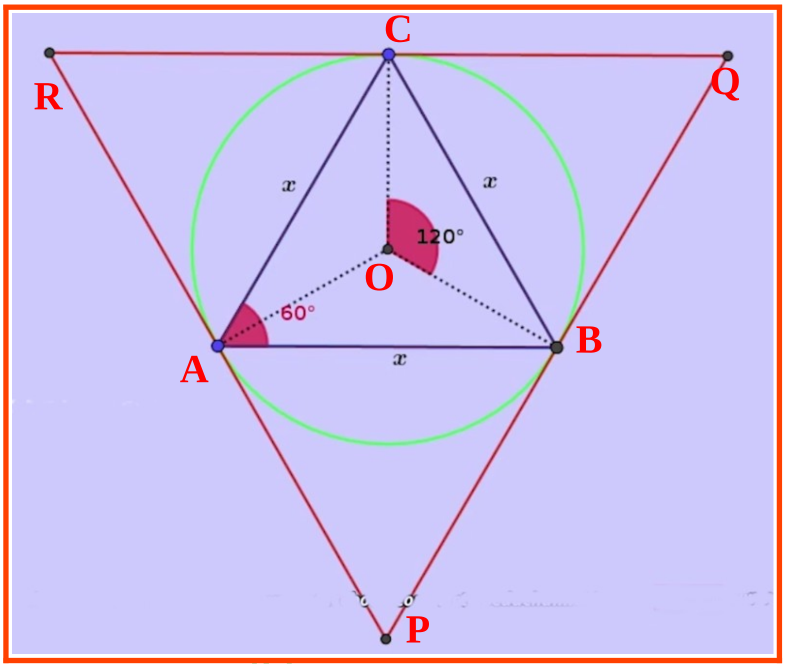
ii) Draw this picture, with sides of the smaller triangle 3 centimetres.

Answer

(i) In the figure, ΔABC is an equilateral triangle. Each angle is 60°.

$\angle$BAC = $\angle$ABC = $\angle$ACB = 60°

Therefore, $\angle$BOC = $\angle$AOC = $\angle$AOB = 2 × 60° = 120°



PAOB, QBOC, RAOC are cyclic quadrilaterals.
Therefore,
$\angle P = \angle Q = \angle R = 180° – 120°$
$= 60°$

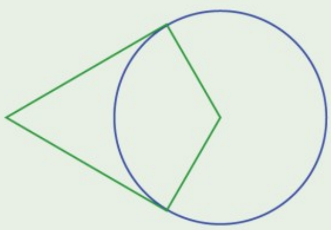
Therefore, PQR is an equilateral triangle.

Let the side of small triangle = x
$\angle A = \angle Q = 60°$, $\angle B = \angle R = 60°$, $\angle P = \angle C = 60°$,

Since the opposite angles are equal; ABQC, ABCR, PBCA are rhombus of sides x.
Therefore, PB = BQ = QC = CR = RA = AP = x
Therefore, PQ = 2x, QR = 2x, PR = 2x
That is, the sides of large triangle is double the side of small triangle.
(ii) Draw yourself.

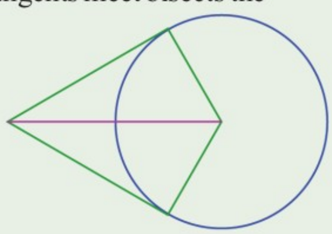
Assignment

The picture shows the tangents at two points on a circle and the radii through the points of contact.



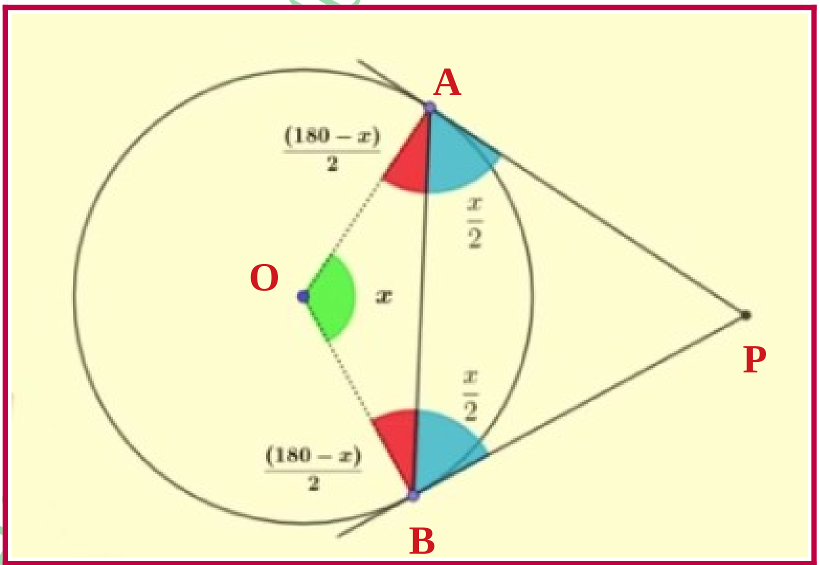
i) Prove that the tangents have the same length.

ii) Prove that the line joining the centre and the point where the tangents meet bisects the angle between the radii.



(Based on the online class on 08-12-2020 -2)

Chord and tangent



In the figure, OA, OB are radii; PA, PB are tangents and AB is a chord.

Let angle between the radii = x

That is, $\angle AOB = x°$

OA = OB (radii)

ΔAOB is isosceles.

Therefore, $\angle OAB = \angle OBA = \frac{180-x}{2}$

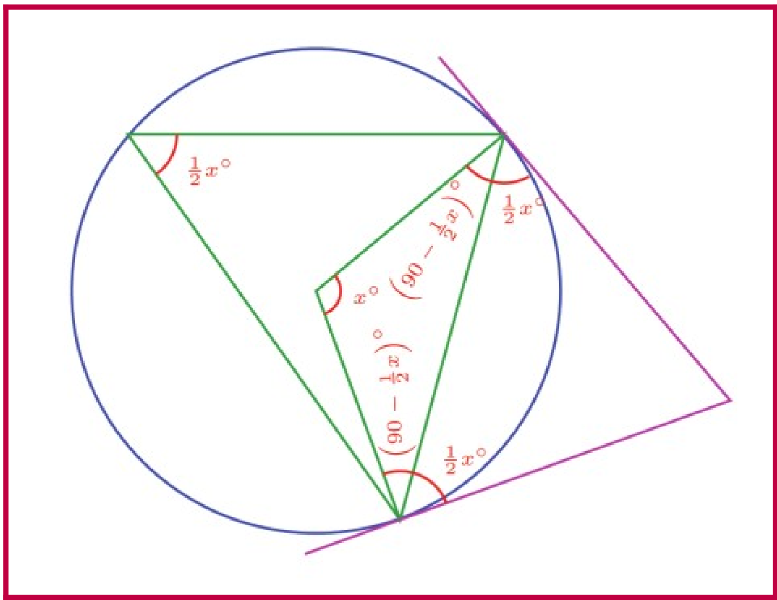
$\angle OAP = \angle OBP = 90°$ (tangent and radius through the point of contact are perpendicular)

$\angle PAB$ and $\angle PBA$ are the angle between tangent and chord.

$$\angle PAB = \angle PBA = 90-\frac{180-x}{2} = \frac{180}{2}-\frac{180-x}{2}$$

$$= \frac{180-(180-x)}{2} = \frac{180-180+x}{2} = \frac{x}{2}$$

In a circle, the angle between a chord and tangent at either end is half the central angle of the chord.

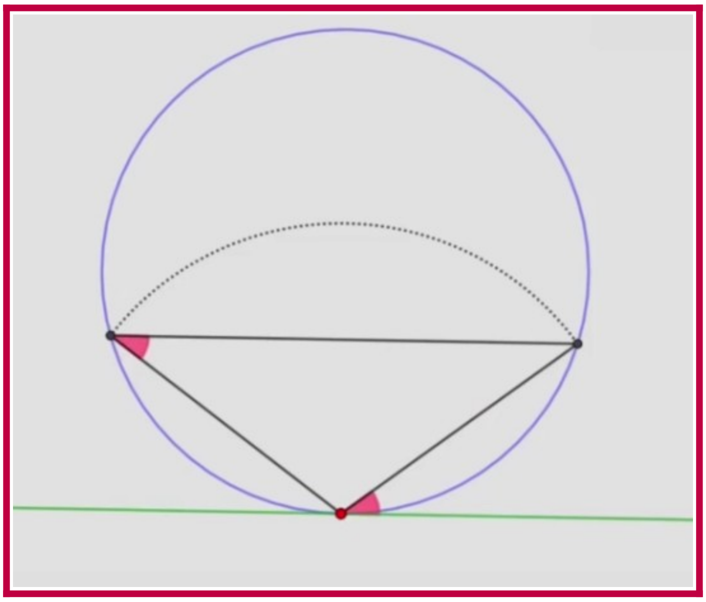


In other words,

In a circle, the angle which a chord makes with the tangent at one end on any side is equal to the angle which it makes on the part of the circle on the other side.

How can we draw a tangent of a circle without knowing the centre?

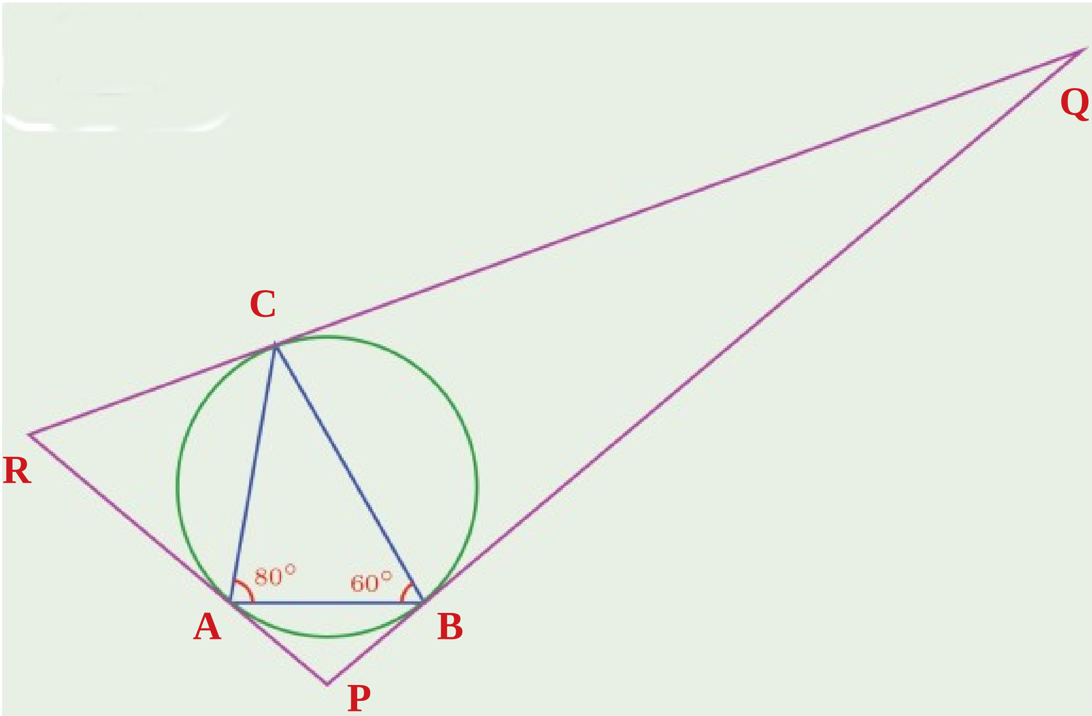
Steps



1. Draw a circle.
2. Mark a point on it.
3. Draw two equal chords on either side of the circle.
4. Draw a chord joining the ends of these equal chords.
5. Draw a line parallel to this new chord through the point marked on the circle . This line will be the tangent of the circle.

Activity

In the picture, the sides of the large triangle are tangents to the circumcircle of the small triangle, through its vertices. Calculate the angles of the large triangle.



Answer

The angle between a tangent and chord of a circle is equal to the angle made by the chord at the opposite opposite part of the circle.

Therefore,

∟QBC = ∟QCB = 80°

Therefore, ∟Q = 180 – (80° + 80°) = 180 – 160° = 20°

∟RAC = ∟RCA = 60°

Therefore, ∟R = 180 – (60° + 60°) = 180 – 120° = 60°

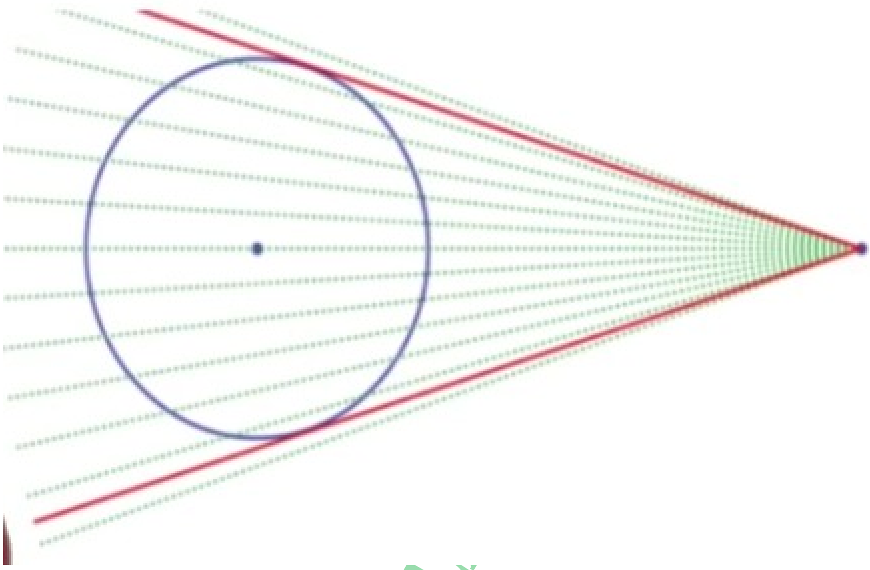
∟P = 180 – (20° + 60°)

= 180 – 80° = 100°

Therefore, The angles of large triangle are 20°, 60°, 100°

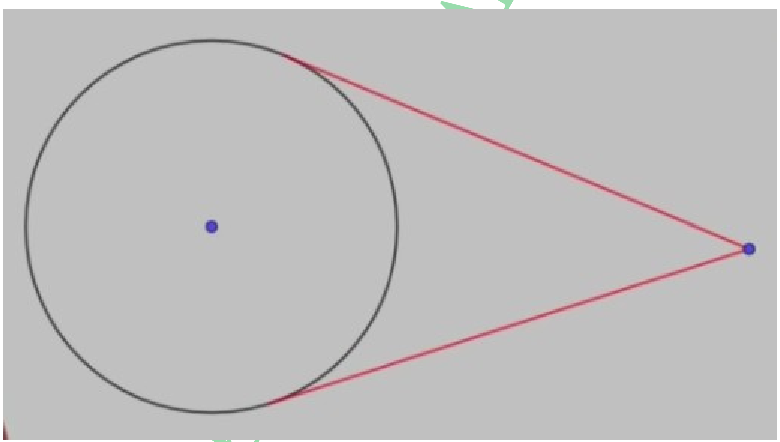
A tangent from outside

From a point outside a circle, we can draw many lines towards the circle. Two of them are tangents of the circle.



That is, **From a point outside a circle, two tangents can be drawn.**

How can we draw tangents from a point outside the circle



Steps

1. Draw a circle of given radius.

2. Mark a point outside the circle.

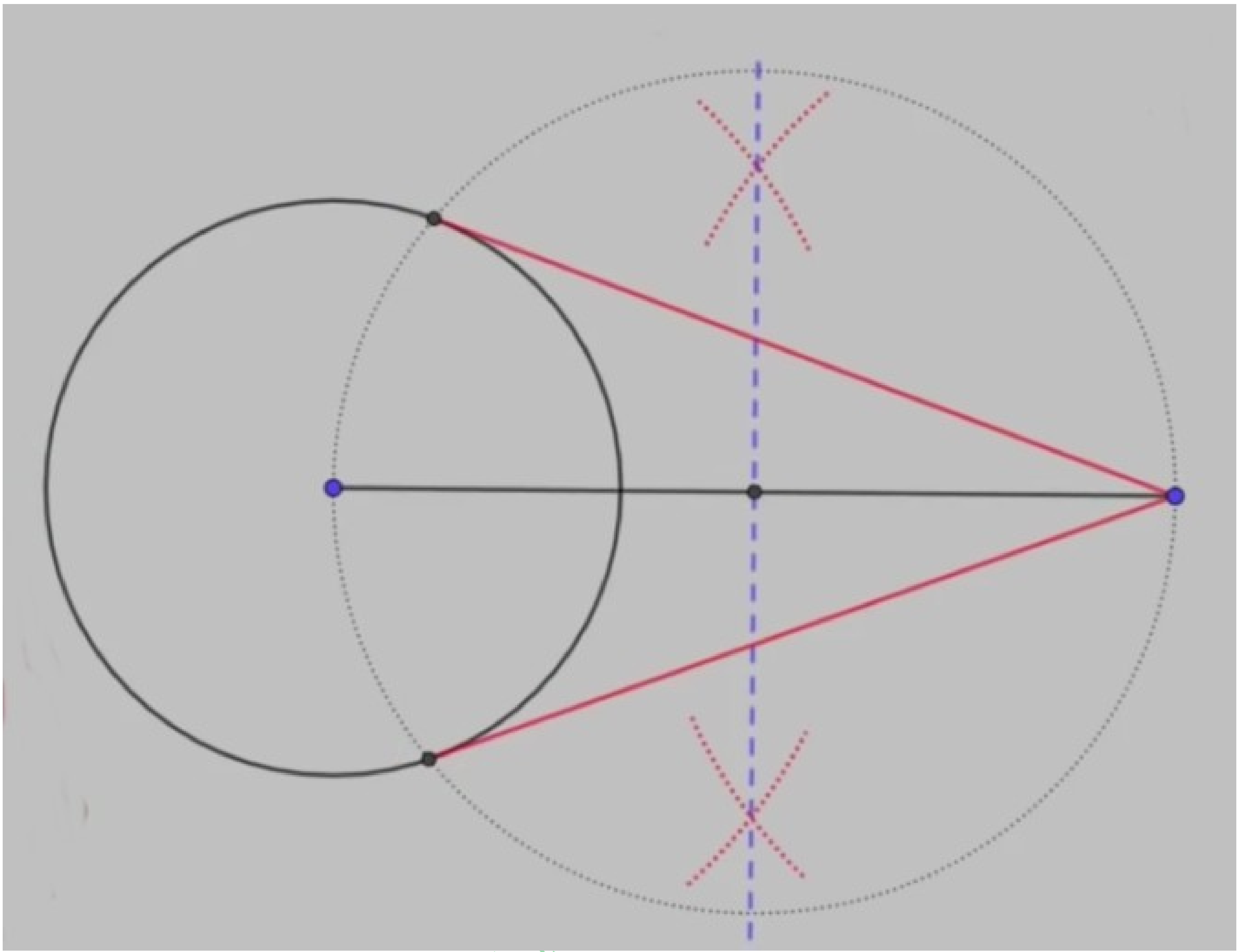
4. Draw a line joining the centre and the point.

5. Find the midpoint of line by drawing its perpendicular bisector.

6. Draw a circle with the length of the line as diameter.

7. The meeting points of the circles are the point of contacts of the tangents

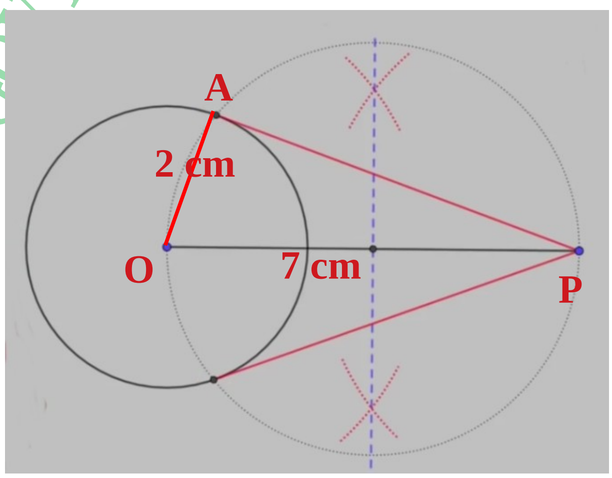
8. Draw the tangents by joining the point outside the circle and meeting points of the circles.



Activity

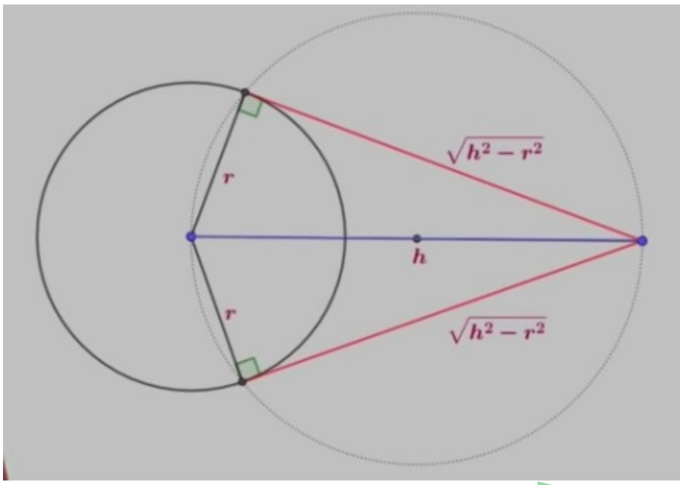
Draw a circle of radius 2 cm. Mark a point P at a distance 7 cm from the centre. Draw tangents from P.

Draw the figure yourself in measure.



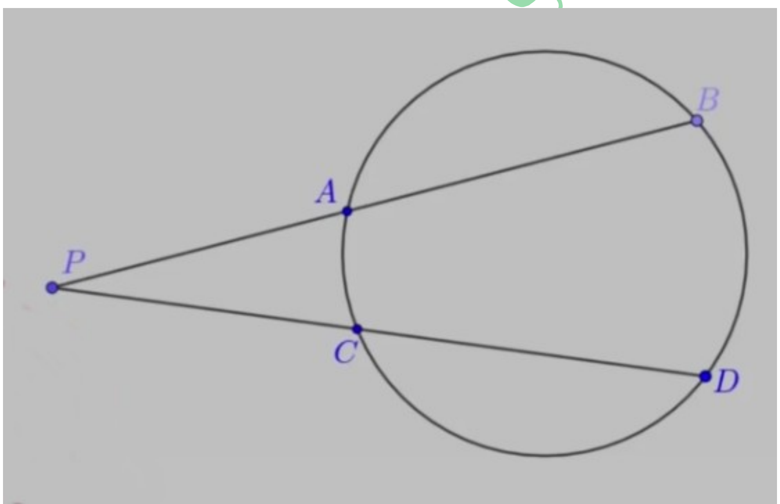
Note:

The tangents to a circle from a point are of the same length

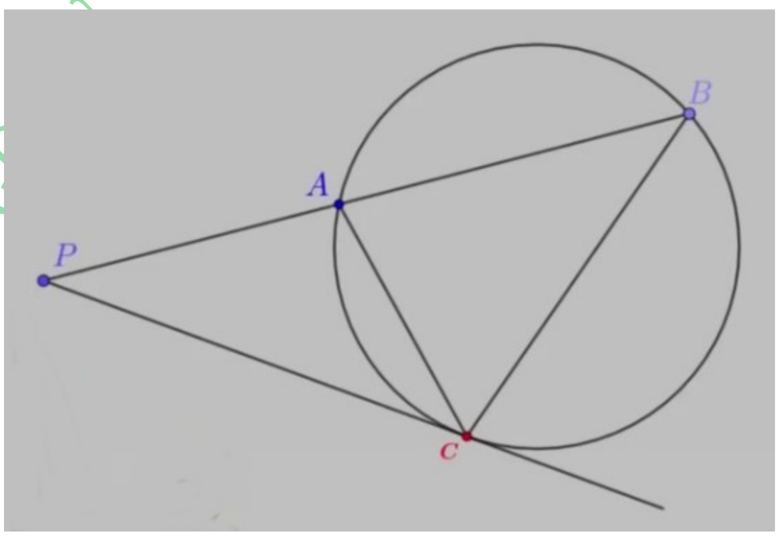


Note:

In the figure, two chords of a circle AB and CD meet at P.



We know that PA × PB = PC × PD. When the point C and D comes closer we get a picture like this:



Consider ΔPAC and ΔPCB.

∟P is common for both triangles.

∟PCA = ∟PBC (The angle between a tangent and chord of a circle is equal to the angle made by the chord at the opposite opposite part of the circle.)

∟PAC = ∟PCB (Sum of angles of a triangle is 180°)

That is, Angles of ΔPAC and ΔPCB are equal.

Therefore, ΔPAC and ΔPCB are similar.

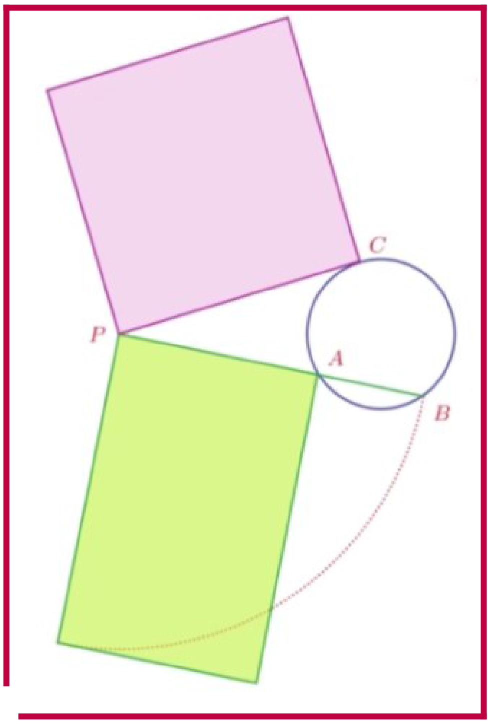
In similar triangles, sides opposite to equal angles are proportional.

Therefore, $\frac{PA}{PC} = \frac{PC}{PB}$

By cross multiplication, we get $PA \times PB = PC \times PC = PC^2$

The product of an intersecting line and the part of it outside the circle is equal to the square of the tangent.

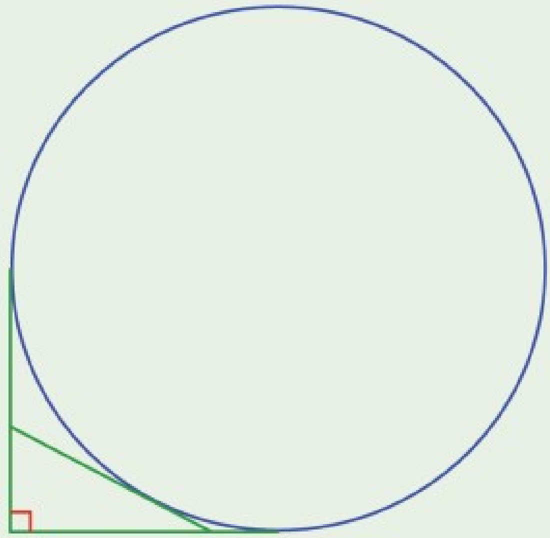
The rectangle with the intersecting line and its part outside the circle as sides and the square with sides equal to the tangent have the same area.



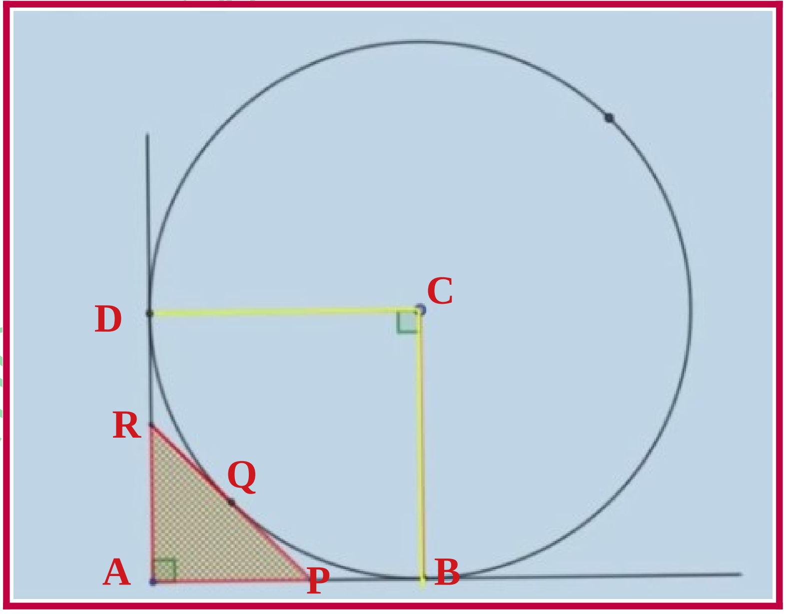
Activity

In the picture, a triangle is formed by two mutually perpendicular tangents to a circle and a third tangent.

Prove that the perimeter of the triangle is equal to the diameter of the circle.



Answer



In the figure, AB and AD are two mutually perpendicular tangents. PR is another tangent touching the circle at Q.

$\angle$ADC = $\angle$ABC = 90° (The tangent and radius through the point of contact are perpendicular)

$\angle$A = 90° (given)

Therefore, $\angle$C = 90°

BC = DC (radii of circle)

All angles of ABCD are 90° and two adjacent sides are equal.

Therefore, ABCD is a square.

Then, AB = BC = CD = AD = radius

The tangents to a circle from a point are of the same length

Therefore, PB = PQ, RD= RQ

AD = AR + RD

AD= AR + RQ 1

AB = AP + PB

AB= AP + PQ 2

Adding two equations

AD + AB = AR + RQ + AP + PQ

= AR + AP + PQ + RQ

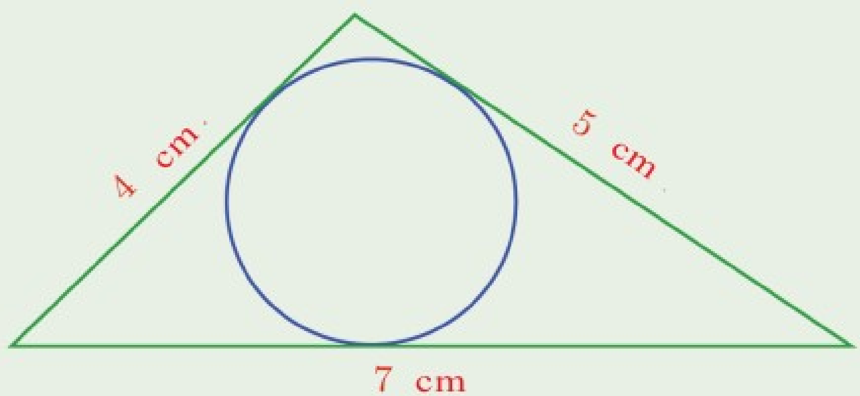
= AR + AP + PR = perimeter of ΔAPR

Also AD + AB = radius + radius = diameter of the circle.

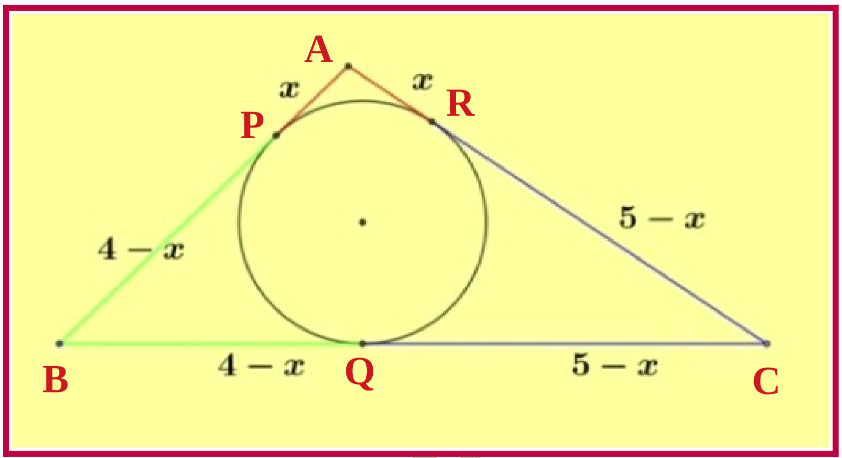
Therefore, perimeter of triangle is equal to the diameter of the circle.

Activity

The picture shows a triangle formed by three tangents to a circle.



Calculate the length of each tangent from the corner of the triangle to the point of contact.



Answer

In ΔABC,

AB = 4 cm, BC = 7 cm

AC = 5 cm

Sides of the triangle are tangents of circle. P, Q, R are the points of contacts.

Let AP = x

The tangents to a circle from a point are of the same length

Therefore, AP = AR = x

BP = BQ = 4 – x, CR = CQ = 5 – x

BC = 7 cm

That is, 4 – x + 5 – x = 7

9 – 2x = 7

2x = 9 – 7 = 2

Therefore, x = 1

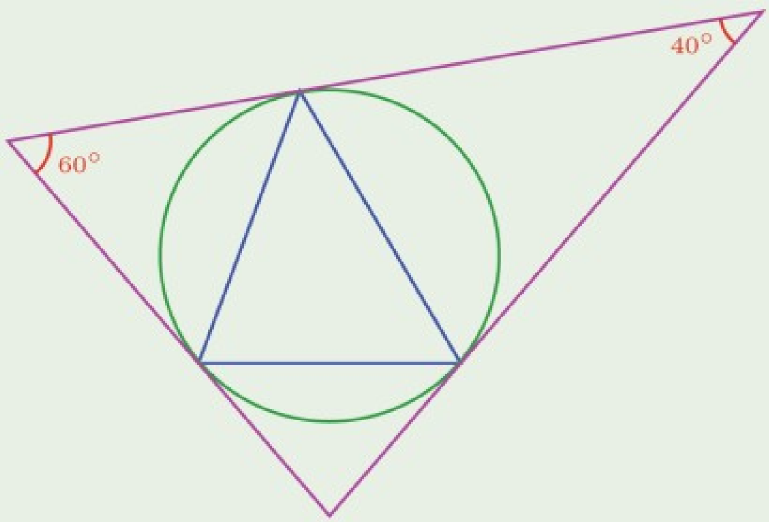
BP = 4 – 1 = 3 cm

CR = 5 – 1 = 4 cm

Assignment-1

In the picture, the sides of the large triangle are tangents of the circumcircle of the smaller triangle, through its vertices.

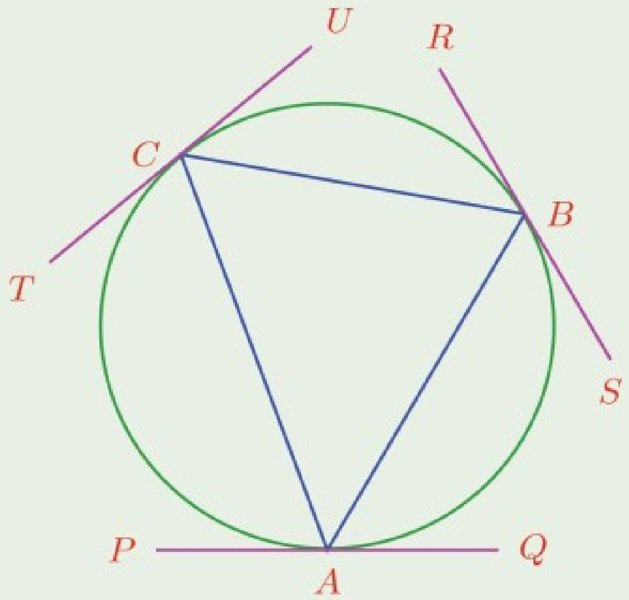
Calculate the angles of the smaller triangle.



Assignment-2

In the picture, PQ, RS, TU are tangents to the circumcircle of ΔABC.

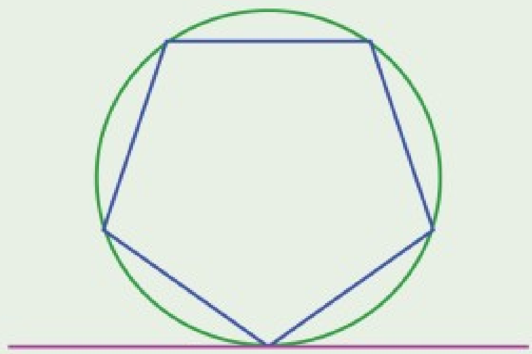
Sort out the equal angles in the picture.



Assignment-3

In the picture, the tangent to the circumcircle of a regular pentagon through a vertex is shown.

Calculate the angle which the tangent makes with the two sides of the pentagon through the point of contact.

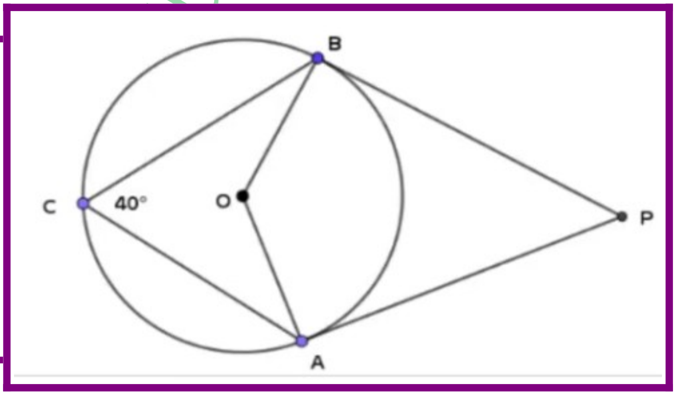


Prepared by Jaisingh G R ;HST(Maths) Govt.V&HSS Kulathoor

(Based on the online class on 09-12-2020)

Activity

In the figure, O is the centre of the circle, A, B, C are points on the circle. PA and PB are tangents.
(a) What is the measure of ∠AOB?
(b) Find the angles of triangle APB

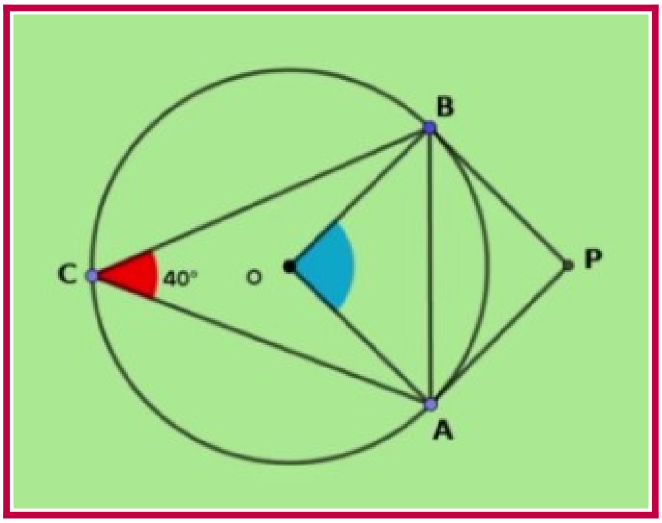


Answer

(a) ∟C = 40°

∟AOB = 2 × 40° = 80° (Angle made by an arc at the centre is double the angle made at its opposite arc)

(b) Join AB.



OAPB is a cyclic quadrilateral.

Therefore, ∟P = 180 – 80° = 100°

PA = PB (tangents from a point to a circle are same)

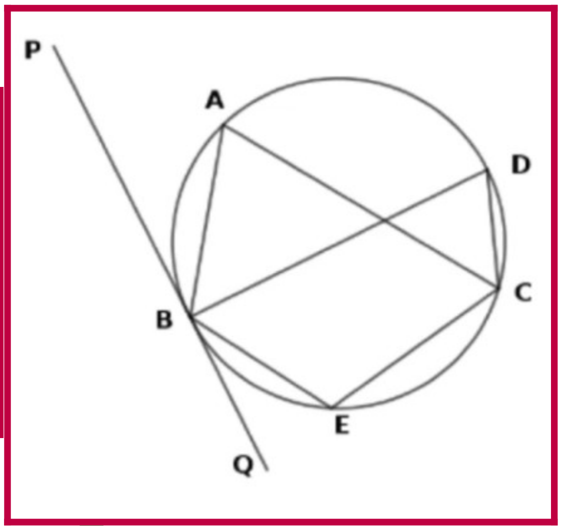
Therefore, ΔAPB is isosceles.

Therefore, ∟PAB = ∟PBA = $\frac{180-100}{2} = \frac{80}{2}$ = 40°

Therefore angles of ΔAPB are 40°, 40°, 100°.

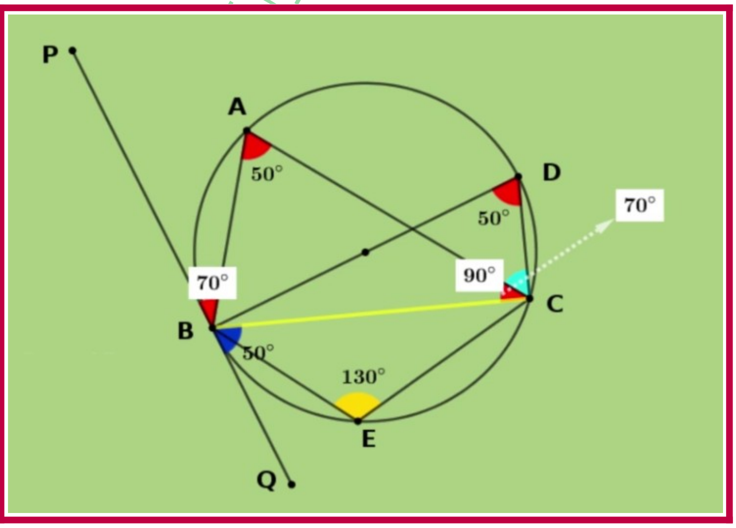
Activity

In the figure, A,B,C,D,E are points on the circle. PQ is the tangent through B and BD is the diameter. $\angle BDC = 50^\circ$, $\angle PBA = 70^\circ$
Find the following angles
$\angle BCD$, $\angle BAC$, $\angle BEC$, $\angle ACB$, $\angle QBC$



Answer

Join BC.



BD is the diameter of the circle.

Therefore, $\angle BCD = 90^\circ$ (Angle in a semicircle is right)

$\angle BDC = 50^\circ$

Therefore, $\angle BAC = 50^\circ$ (Angles in the same arc are equal)

$\angle BEC = 180 - 50^\circ = 130^\circ$ (ABEC is a cyclic quadrilateral)

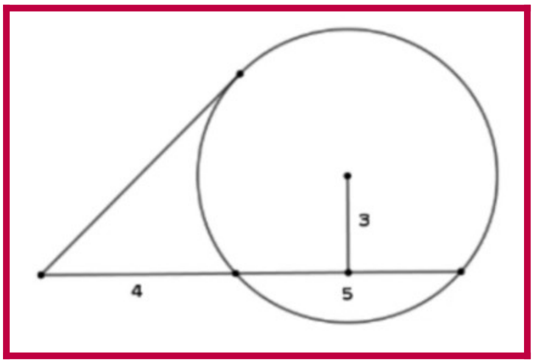
$\angle PBA = 70^\circ$

$\angle ACB = \angle PBA = 70^\circ$ (Angle between a tangent a chord through the point of contact is equal to the angle made by the chord at the opposite segment)

$\angle QBC = \angle BAC = 50^\circ$ (Angle between a tangent a chord through the point of contact is equal to the angle made by the chord at the opposite segment)

Activity

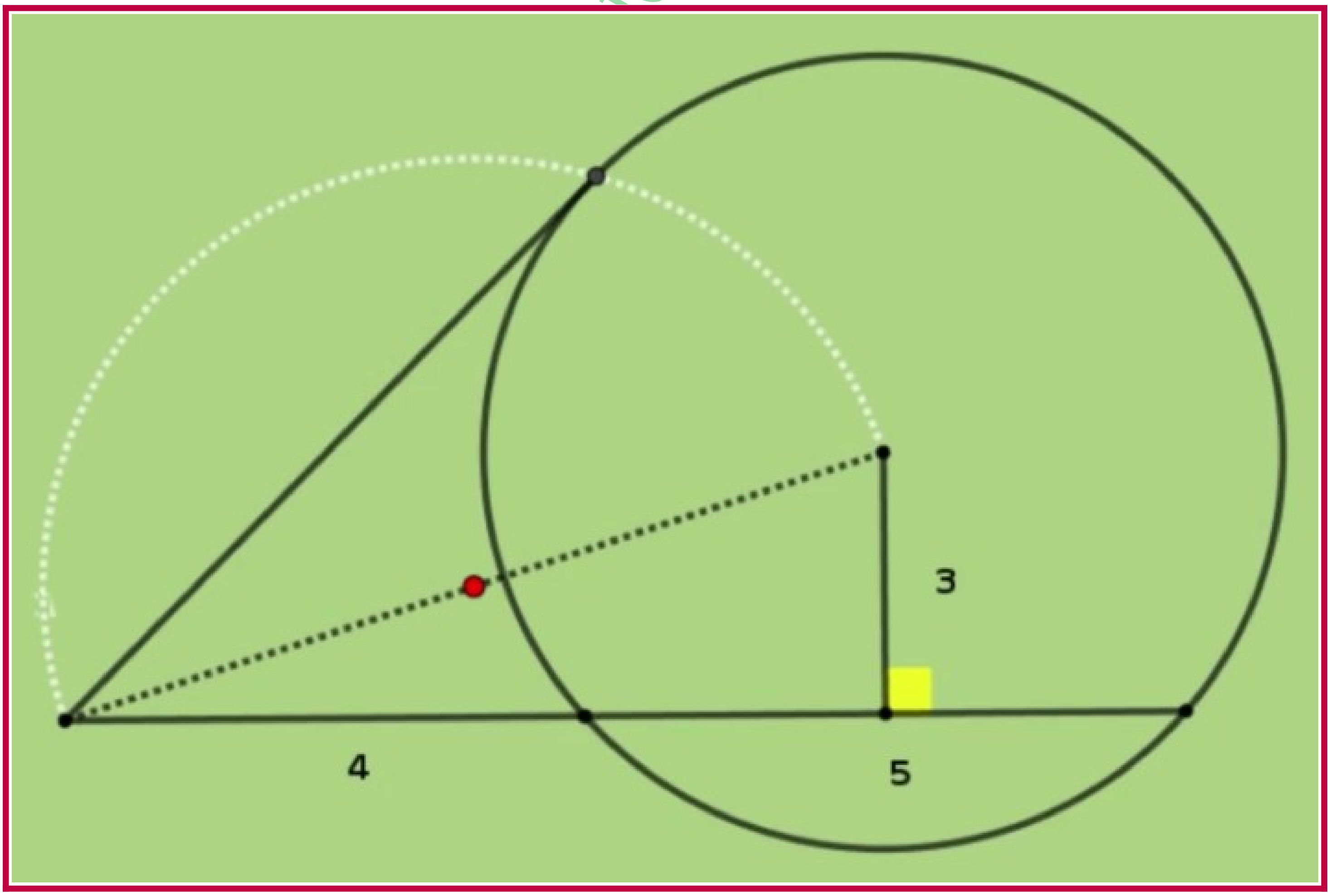
Draw the figure with the given measures.



Answer

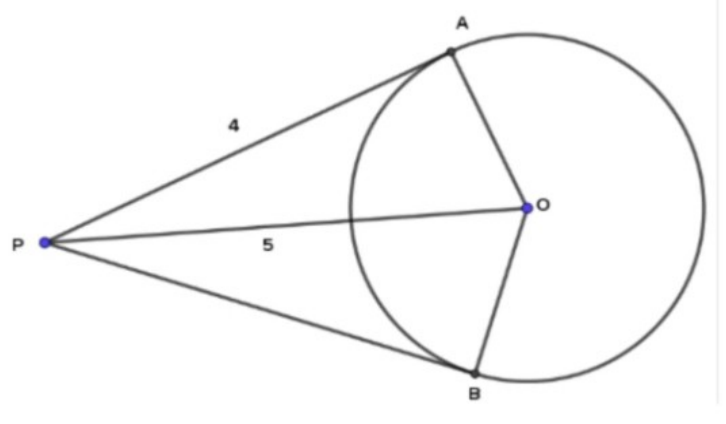
Steps

1. Draw a line of length 5 cm (This line is a chord of the circle)
2. Draw the perpendicular bisector of this line.
3. Mark a point on this perpendicular at 3 cm away from the midpoint.
4. Draw a circle passing through the ends of the chord.
5. Extend the chord by 4 cm.
6. Draw a line joining centre of the circle and outer point.
7. Find the midpoint of this line by drawing its perpendicular bisector.
8. Draw a semicircle with the line as diameter.
9. Mark the meeting point of the semicircle and the circle, which is the point of contact.
10. Draw the tangent.



Activity

In the figure O is the centre of the circle, PA and PB are tangents. PA= 4 cm, OP = 5 cm. Find the area of quadrilateral PAOB



Answer

In the figure; OA, OB are radii.

PA and PB are tangents

PA = PB = 4 cm

∟A = ∟B = 90°(Tangent and radius through the point of contact are mutually perpendicular)

Therefore, ΔPAO and ΔPBO are right triangles.

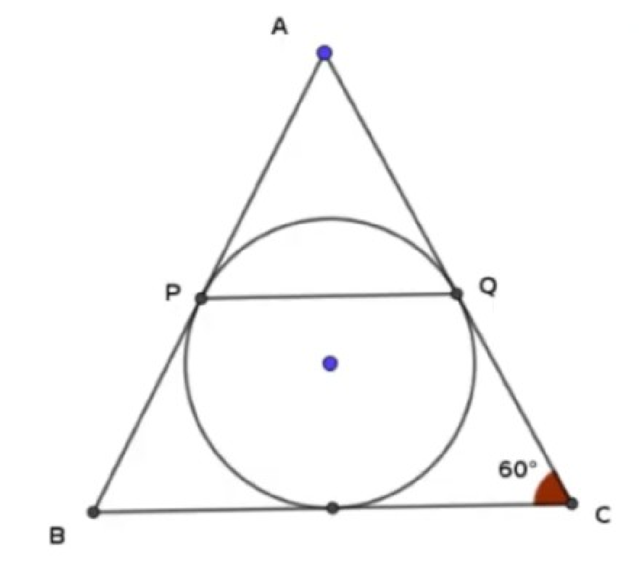
Therefore, OA = $\sqrt{5^2-4^2}$ = $\sqrt{25-16}$ = $\sqrt{9}$ = 3 cm

Area of ΔPAO = ½ × 4 × 3 = 6 cm^2

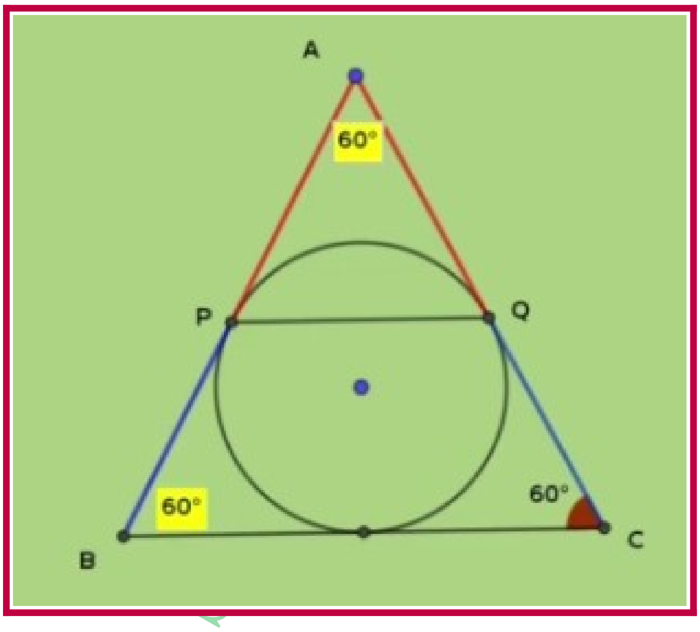
Therefore, Area of quadrilateral PAOB = 2 × 6 = 12 cm^2

Activity

In the figure sides of triangle ABC are the tangents of the circle. <C = 60°. PQ is parallel to BC. Find other angles of the triangle.



Answer



In the figure,

AP = AQ (tangents from a point to a circle are same)

PQ and BC are parallel.

Therefore, AP:PB = AQ:QC

Therefore, PB = QC

Therefore, AB = AC

Therefore, ΔABC is isosceles.

Therefore, ∟C = ∟B = 60°

∟A = 180 – (60° + 60°)

= 180 - 120° = 60°

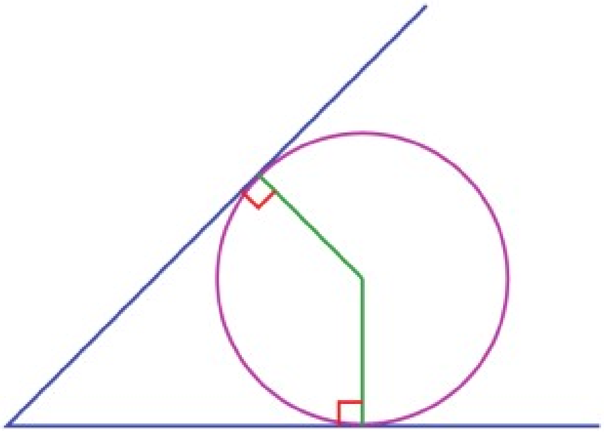
That is, ΔABC is an equilateral triangle.

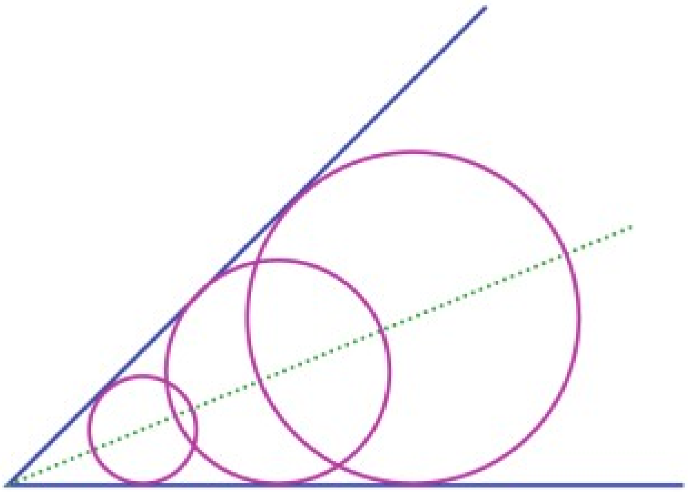
Prepared by Jaisingh G R ;HST(Maths) Govt.V&HSS Kulathoor

(Based on the online class on 11-12-2020)

Circles touching a line

How can we draw a circle touching two lines meeting at a point?





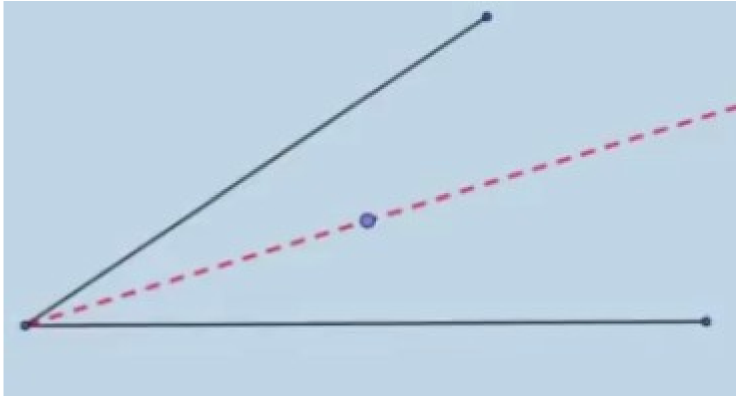
The radii are perpendicular to these lines. The centre of the circle must be at same distance from these lines. So it must be on the bisector of the angle formed by these lines.

> **The centre of a circle touching two lines meeting at a point is on the bisector of the angle formed by the lines.**

Steps for drawing

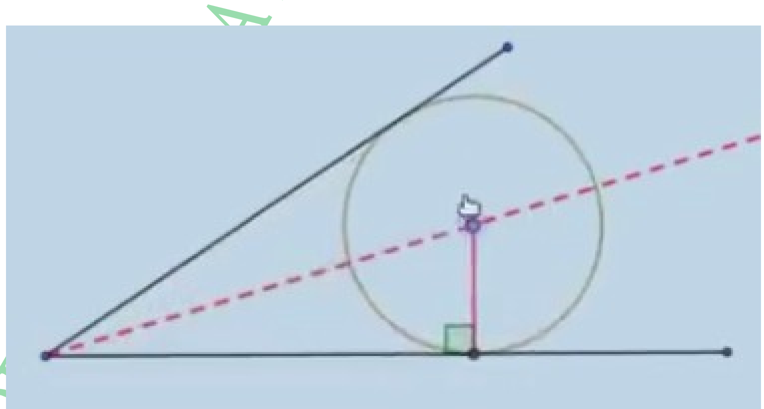
1. Draw two non parallel lines meeting at a point to form an angle.

2. Draw the bisector of this angle.



(For this, Mark two points equidistant from the vertex on either lines or arms. Draw two arcs meeting at point centred at these points. Draw a line joining this point and vertex, which is the bisector of the angle)

3. Draw a perpendicular from the meeting point of arc to any one arm.

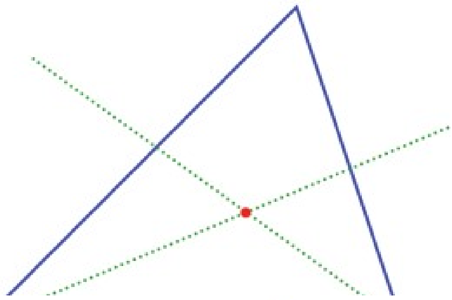


4. Draw a circle taking this perpendicular line as radius.

How can we draw a circle touching all sides of a triangle or

How can we draw the incircle of a triangle

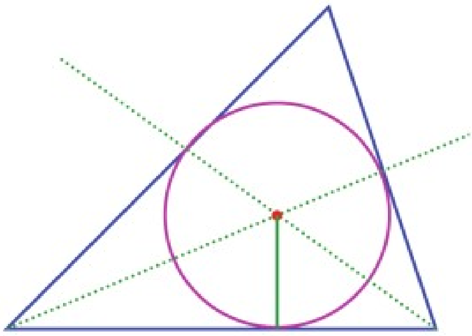
Steps



1. Draw a triangle

2. Draw the bisector of any two angles

(The meeting point of these two bisectors is the centre of the circle, which touches 3 sides of the triangle)



3. Draw a perpendicular from the centre to any one side which is the radius.

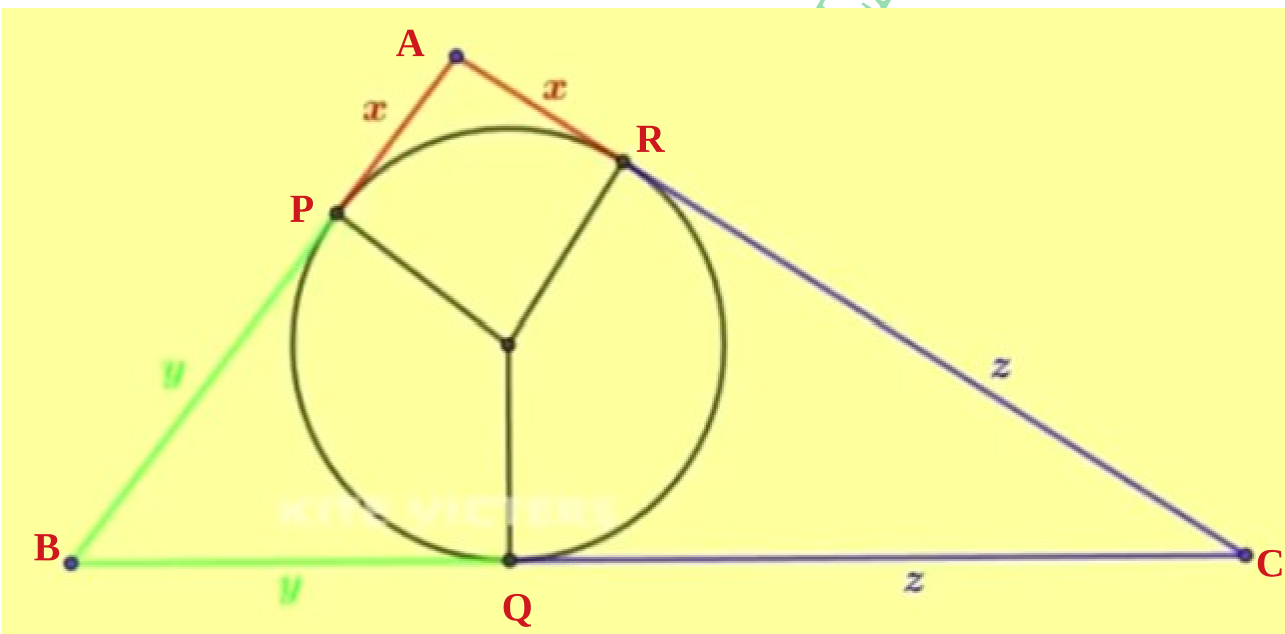
4. Draw the circle taking this perpendicular as radius.

This circle is called the incircle of the triangle. Centre is called the incentre of the triangle.

The bisectors of all three angles of a triangle meet at a point.

Note-1

The incircle of a triangle touche the sides of the triangle. Then, the sides of the triangle are tangents to the circle.



Tangents from a point outside the circle are same.

Therefore, AP = AR = x, BP = BQ = y, CQ = CR = z

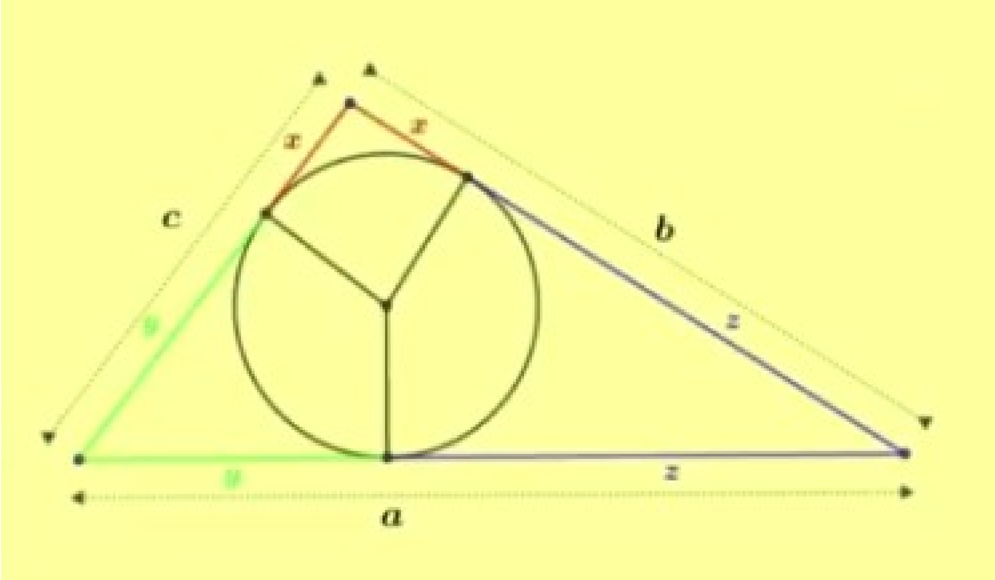
Perimeter of triangle, p $= x + x + y + y + z + z$

$= 2x + 2y + 2z$

$= 2(x + y + z)$

That is, x + y + z is half the perimeter of the triangle.

That is, s = x + y + z, where s = semi perimeter or half of perimeter.



Let the sides of the triangle are a, b, c

That is, $a = y + z$, $b = x + z$, $c = x + y$

$$x + y + z = s$$

$$x = s - (y + z) = s - a$$

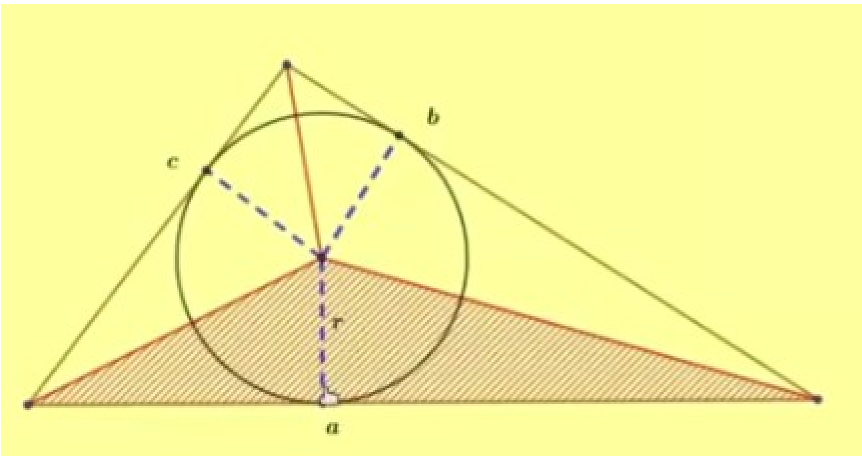
Similarly, $$y = s - (x + z) = s - b$$

$$z = s - (x + y) = s - c$$

That is, To get the length of tangents from one vertex, we have to subtract the length of its opposite side from the semi perimeter.

Note-2

The radius of the incircle has a relation with the area of the triangle. The lines joining the centre of the incircle to the vertices divide the triangle into three.



Let the sides of the triangle are a, b, c and the radius of the incircle is r.

Then, Area of the shaded triangle = ½ × a × r

Similarly, Area of other two triangles are ½ × b × r and ½ × c × r.

Therefore,

Area of large triangle, A = sum of area of these three triangles

= ½ × a × r + ½ × b × r + ½ × c × r

= ½ × r(a + b + c)

= ½ × r × s

Therefore, $r = \frac{A}{s}$ **or A = r × s**

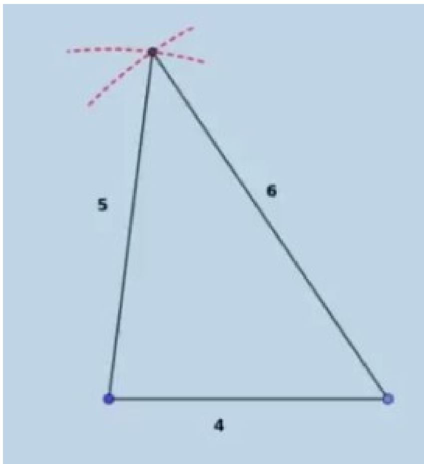
The radius of the incircle of a triangle is its area divided by half the perimeter.

Activity

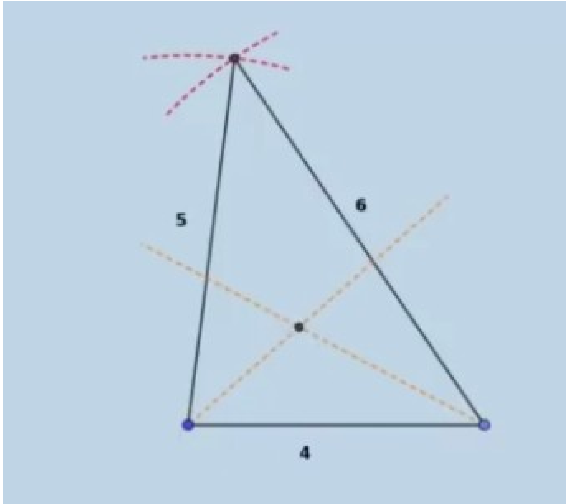
Draw a triangle of sides 4 centimetres, 5 centimetres, 6 centimetres and draw its incircle. Calculate its radius.

(or) Draw a circle touching the sides of a triangle having sides 4 cm, 5 cm, 6 cm

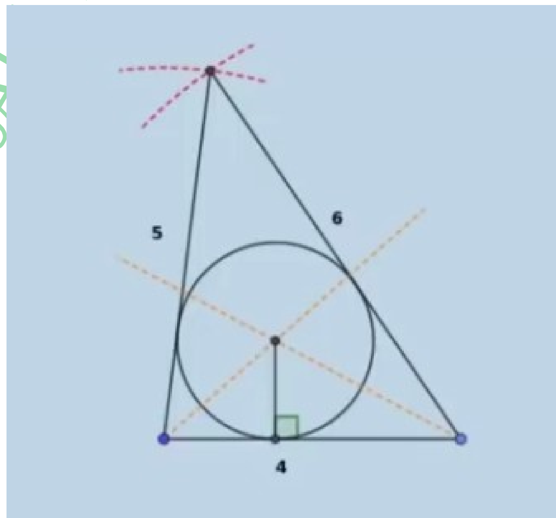
Steps:



1. Draw a triangle of sides 4 cm, 5 cm, 6 cm



2. Draw the bisector of any two angles

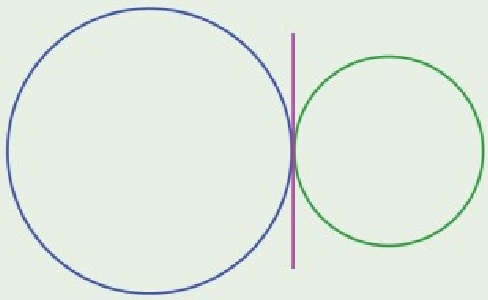


3. Draw a perpendicular from the centre to any one side which is the radius. Draw the circle taking this radius.

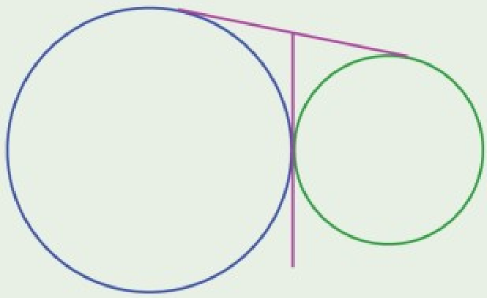
Measure the radius using scale and write.

(Based on the online class on 14-12-2020)

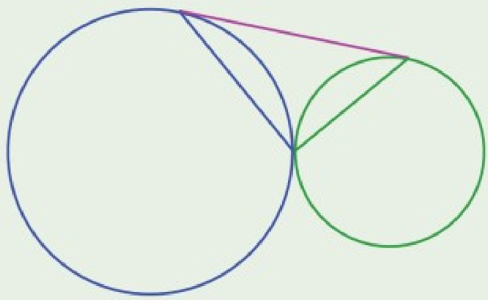
Activity



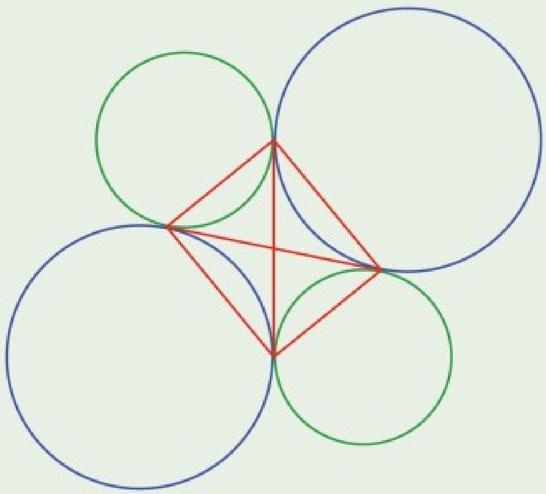
In the picture, two circles touch at a point and the common tangent at this point is drawn.



i) Prove that this tangent bisects another common tangent of these circles.



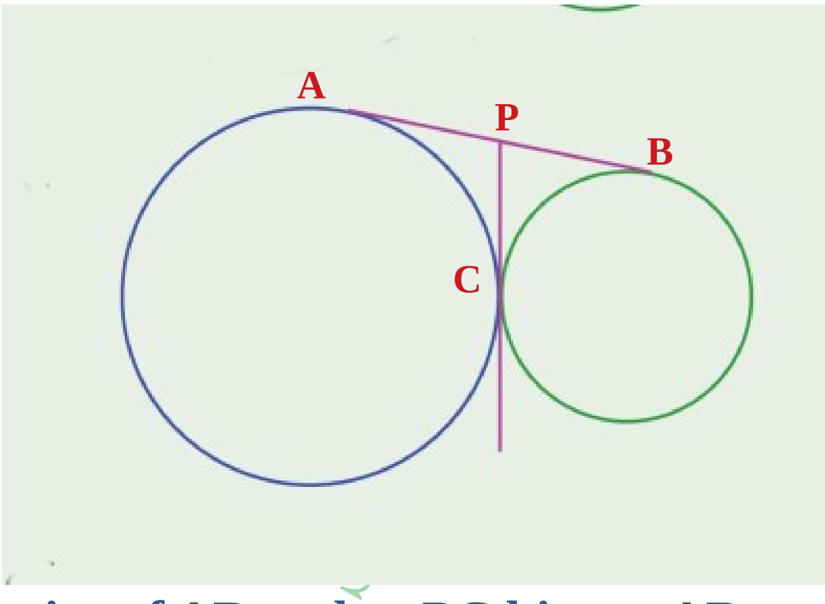
ii) Prove that the points of contact of these two tangents form the vertices of a right triangle.



Draw the picture on the right in your notebook, using convenient lengths.

What is special about the quadrilateral formed by joining the points of contact of the circles?

Answer

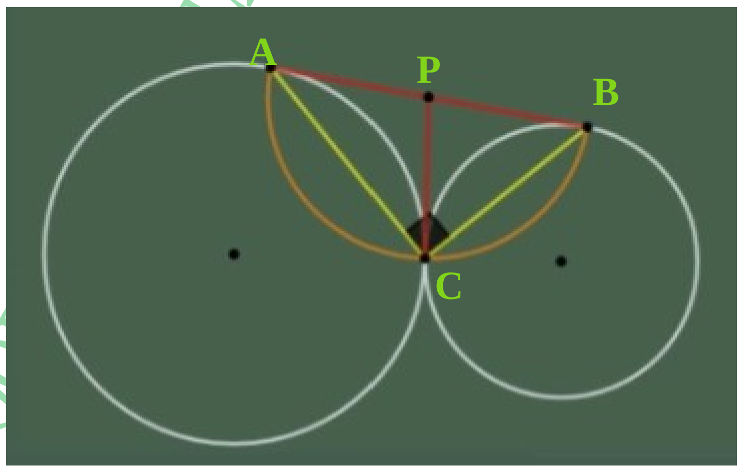


i) In the figure, PC and AB are common tangents.

Tangents from a point outside a circle are same.

Therefore, PA = PC, PB = PC

That is, PA, PB, PC are equal. Therefore, P is the midpoint of AB and so PC bisects AB.



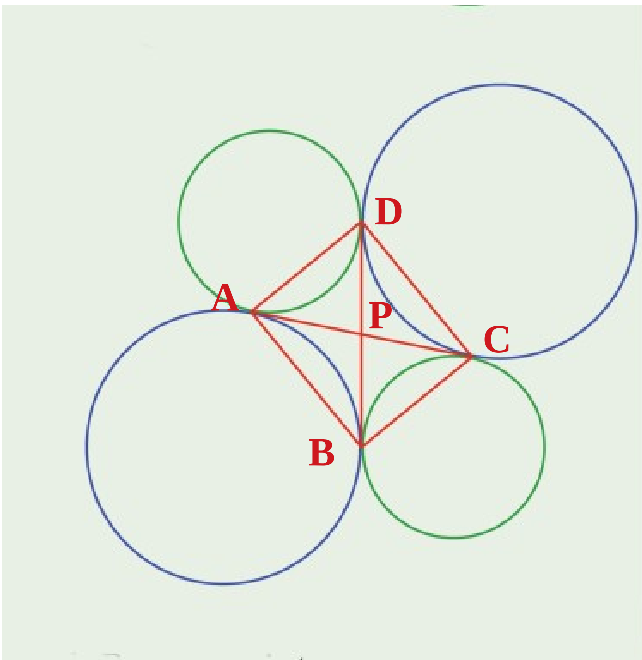
ii) In the picture, PA = PB = PC.

Therefore, if we draw a circle with centre P and radius PA, that circle passes through A, B and C.

Angle in a semicircle is 90°.

Therefore, ∟ACB = 90°

Therefore, ΔABC is a right triangle.



Iii) In the picture, PA, PB, PC, PD are tangents to the circles.

Tangents from a point outside a circle are same.

Therefore, PA = PB, PB = PC, PC = PD and PA = PD

That is, PA = PB = PC = PD

Therefore, AC and BD are equal. Also P is the midpoint of AC and BD.

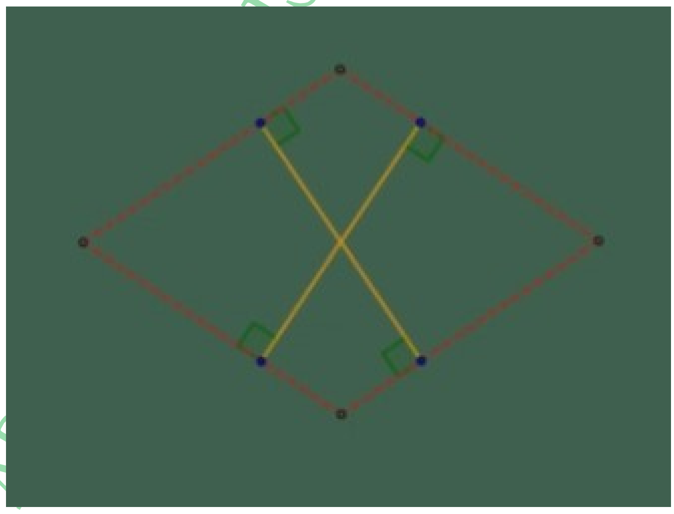
That is, Diagonals of ABCD are equal and they bisects each other.

Therefore, ABCD is a rectangle.

How can we draw this picture?

Tangents and radius through the point of contact are mutually perpendicular.

Therefore, centres of the circles are the intersecting points of perpendiculars to AC and BD through the end points.



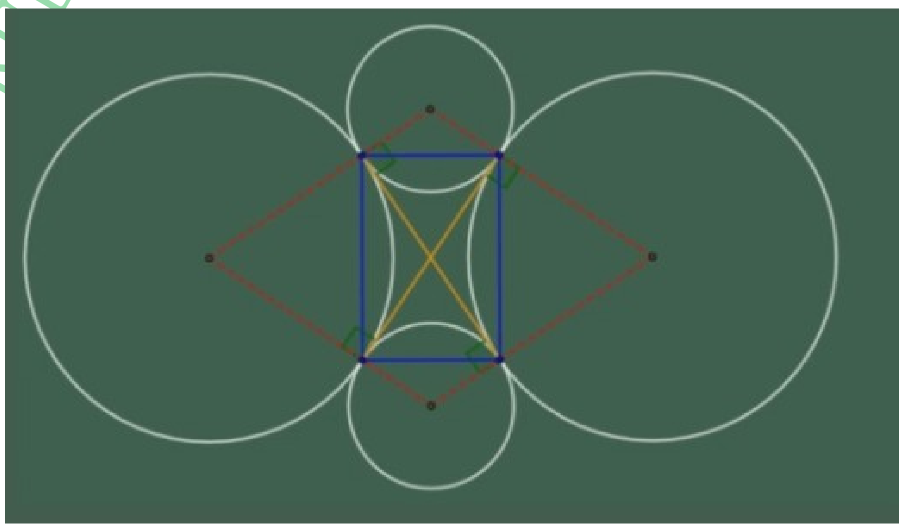
Steps

1. Draw a rectangle.

2. Draw its diagonals

3. Draw perpendicular to diagonals through the end points.

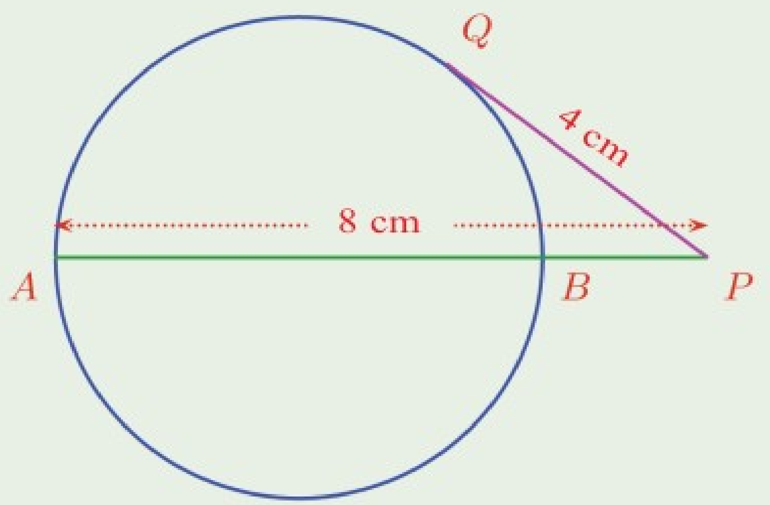
4. Mark the intersecting point of these perpendiculars.



5. Draw circles with centre as these points and passing through the corresponding vertices of the rectangle.

Activity

In the picture below, AB is a diameter and P is a point on AB extended. A tangent from P touches the circle at Q. What is the radius of the circle?



Answer

In the picture, PQ = 4 cm, PA = 8 cm

Let, PB = x

$PA \times PB = PQ^2$

$8 \times x = 4^2$

$8x = 16$

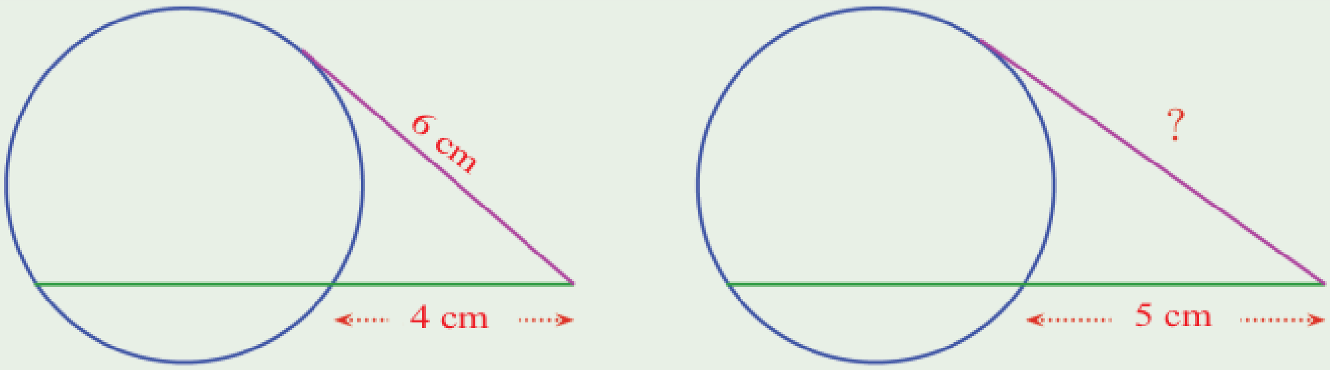
$x = 2$

Therefore, Diameter = AB = 8 – 2 = 6 cm

Therefore, radius = 3 cm

Activity

In the first picture below, the line joining two points on a circle is extended by 4 centimetres and a tangent is drawn from this point. Its length is 6 centimetres, as shown:



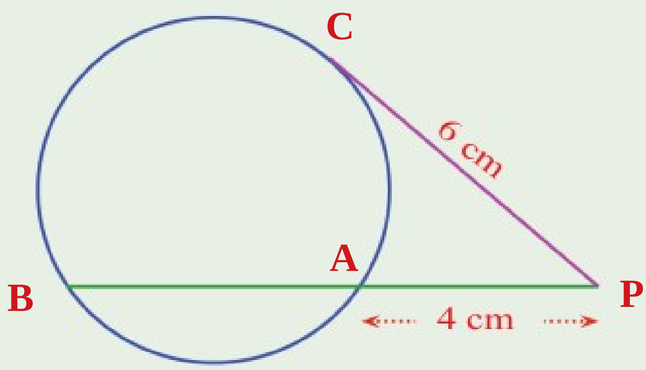
The second picture shows the same line extended by 1 centimetre more and a tangent drawn from this point. What is the length of this tangent?

Answer

In the picture,

PA = 4 cm, PC = 6 cm

Let AB = x, then PB = x + 4



$$PA \times PB = PC^2$$

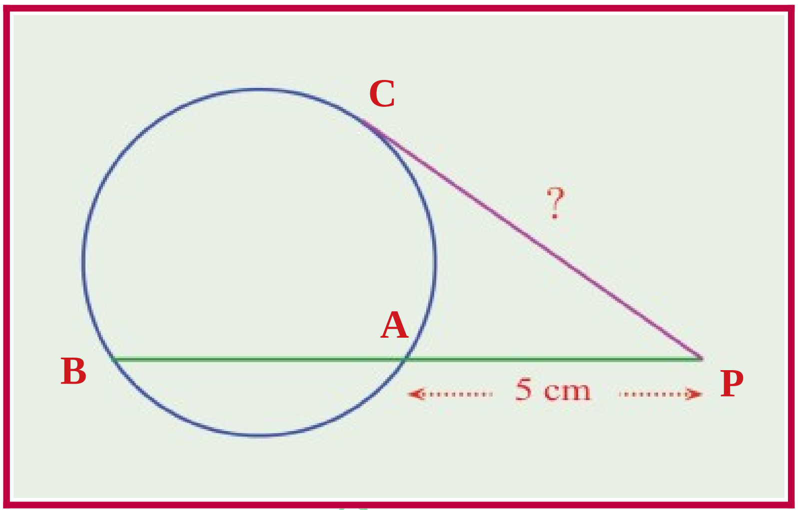
$$4 \times (x + 4) = 6^2$$

$$4x + 16 = 36$$

$$4x = 36 - 16 = 20$$

$$x = \frac{20}{4} = 5$$

Therefore, AB = 5 cm



In this picture,

PA =5, AB = 5

Therefore, PB = 5 + 5 = 10 cm

$PA \times PB = PC^2$

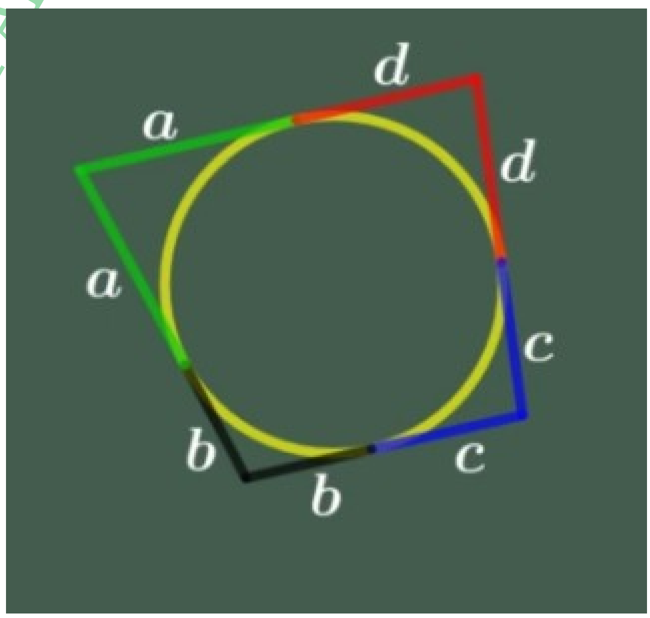
$5 \times 10 = PC^2$

$PC^2 = 50$

$PC = \sqrt{50} = 5\sqrt{2}$ cm

Note:

In a quadrilateral formed by the tangents at four points on a circle, the sum of the opposite sides are equal.



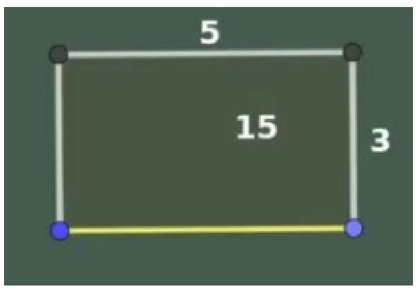
If the sum of opposite sides of a quadrilateral are equal, then we can draw incircle of that quadrilateral.

Activity

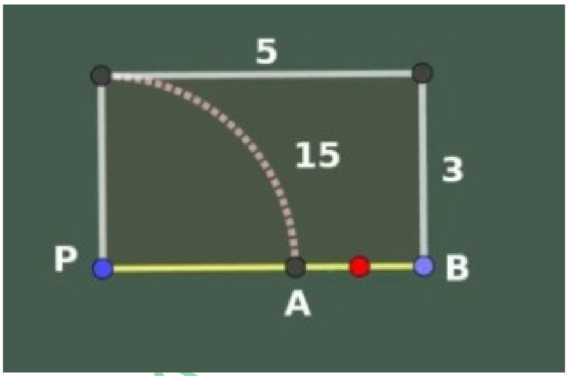
Draw a rectangle of sides 5 cm, 3 cm, then draw a square of the same area.

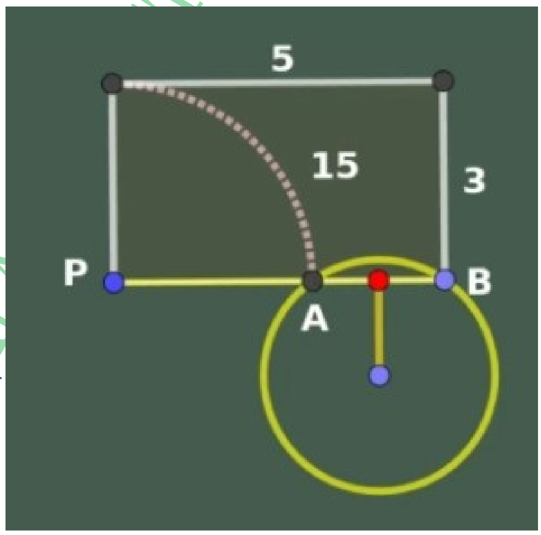
Steps ($PA \times PB = PC^2$)

1. Draw a rectangle of sides 5 cm and 3 cm



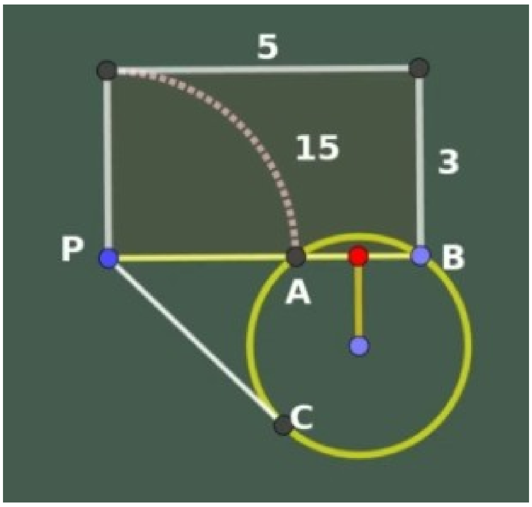
2. Mark a point A such that PA = 3 cm (breadth of the rectangle), where P is the corner of side 5cm



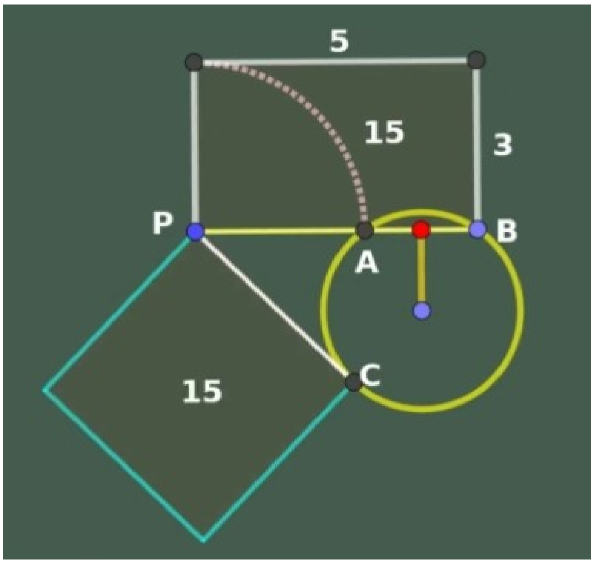


3. Draw the perpendicular bisector of AB.

4. Mark a point on this perpendicular and draw a circle passing through A and B with centre as the marked point.



5. Draw tangent from P to this circle (PC) which is the side of the required square.



6. Draw a square of side PC, which has the area equal to the area of the rectangle.