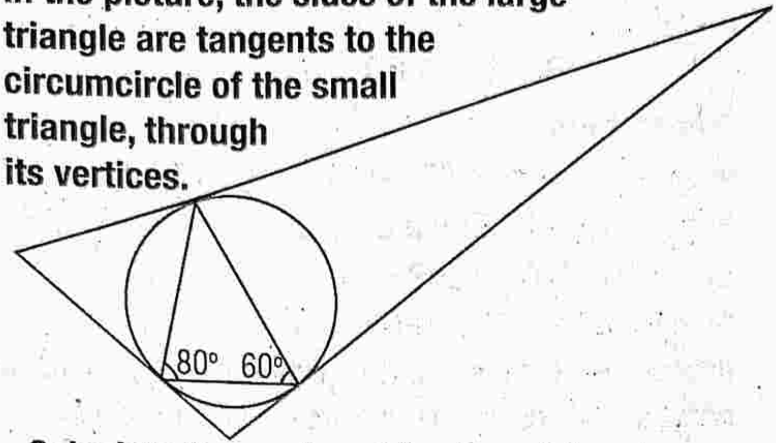
1. In the picture, the sides of the large triangle are tangents to the circumcircle of the small triangle, through its vertices.



Calculate the angles of the large triangle.

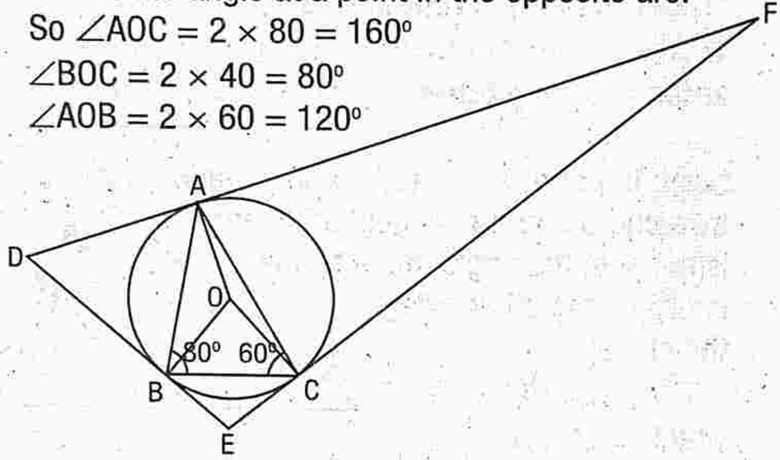
In ΔABC, $\angle A = 180 - (80 + 60) = 180 - 140 = 40°$

The angle made by an arc at the centre of a circle is double the angle at a point in the opposite arc.

So $\angle AOC = 2 \times 80 = 160°$

$\angle BOC = 2 \times 40 = 80°$

$\angle AOB = 2 \times 60 = 120°$



Since the angle between the radii of a circle through two points and the angle between the tangents at these points are supplementary,

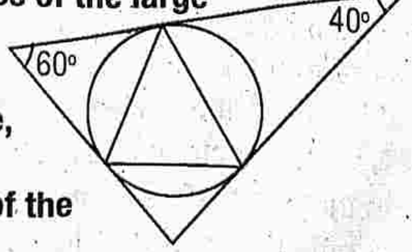
$\angle F = 180 - \angle AOC = 180 - 160 = 20°$

Similarly, $\angle E = 180 - 80 = 100°$

$\angle D = 180 - 120 = 60°$

Also we can find the angles of ΔDEF using the principle that the angles which a chord of a circle makes with the tangents at its ends on any side are equal to the angle which it makes on the part of the circle on the other side.

2. In the picture, the sides of the large triangle are tangents of the circumcircle of the smaller triangle, through its vertices. Calculate the angles of the smaller triangle.



In ΔXYZ, $\angle Y = 180° - (60 + 40) = 80°$

Since $\angle X = 60°$,

$\angle POQ = 180 - 60$

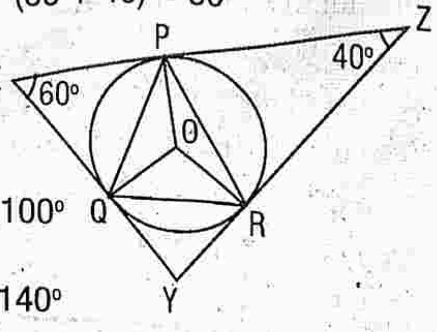
$= 120°$

Since $\angle Y = 80°$,

$\angle QOR = 180 - 80 = 100°$

Since $\angle Z = 40°$,

$\angle POR = 180 - 40 = 140°$



In ΔPQR, $\angle P = \frac{1}{2}\angle QOR = 50^\circ$

$\angle Q = \frac{1}{2}\angle POR = 70^\circ$

$\angle R = \frac{1}{2}\angle POQ = 60^\circ$

Another method

Since $\angle Z = 40^\circ$, $\angle ZPR + \angle ZRP = 180 - 40 = 140^\circ$

But $\angle ZPR$ and $\angle ZRP$ are equal.

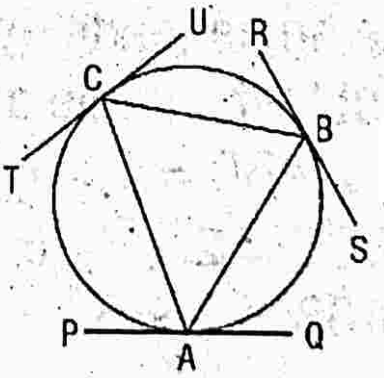
So $\angle ZPR = 70^\circ$, $\angle ZRP = 70^\circ$

Since $\angle ZPR = 70^\circ$, $\angle PQR = 70^\circ$

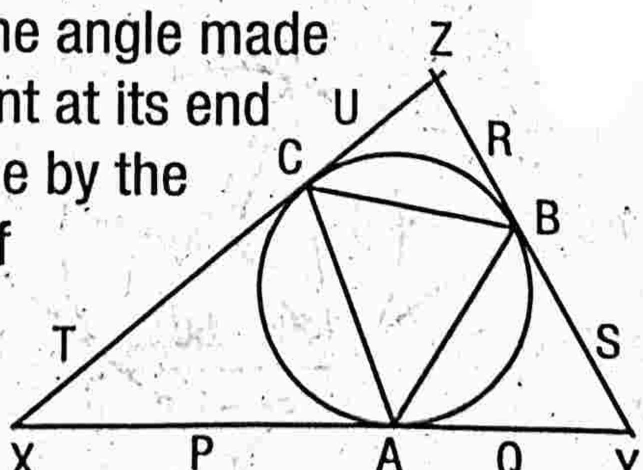
(In a circle the angles which a chord makes with the tangents at its ends on any side are equal to the angle which it makes on the part of the circle on the other side.)

In the same way we can find $\angle PRQ$ and $\angle QPR$.

3. In the picture, PQ, RS, TU are tangents to the circumcircle of ΔABC. Sort out the equal angles in the picture.



Using the principle that the angle made by a chord and the tangent at its end is equal to the angle made by the chord on the other side of the circle,



$\angle TCA = \angle CBA$

$\angle PAC = \angle CBA$

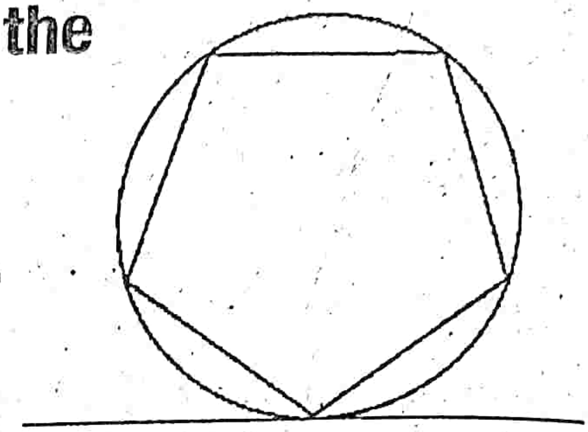
$\angle UCB = \angle CAB$, $\angle RBC = \angle CAB$

$\angle SBA = \angle ACB$, $\angle QAB = \angle ACB$

Since two angles of the isosceles triangle got by extending the tangents are equal,

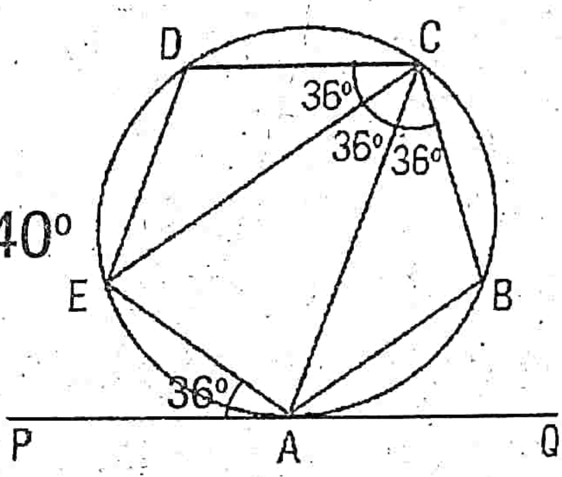
$\angle TCA = \angle PAC$, $\angle UCB = \angle RBC$, $\angle SBA = \angle QAB$

4. In the picture, the tangent to the circumcircle of a regular pentagon through a vertex is shown. Calculate the angle which the tangent makes with the two sides of the pentagon through the point of contact.



Sum of the angles of a pentagon $= (n - 2) \times 180^\circ$

$= (5 - 2) \times 180^\circ = 540^\circ$



One angle of a regular pentagon $= 540 \div 5 = 108^\circ$

Since it is a regular pentagon, the three angles at C in the picture are equal. Each of them is equal to $108 \div 3 = 36^\circ$.

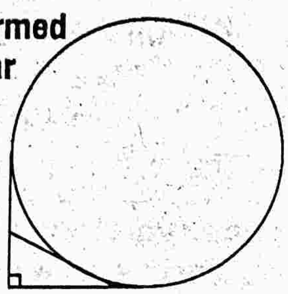
Since $\angle ECA = 36^\circ$, $\angle EAP = 36^\circ$. (The angle made by a chord and the tangent at its end is equal to the angle made by the chord on the other side of the circle.)

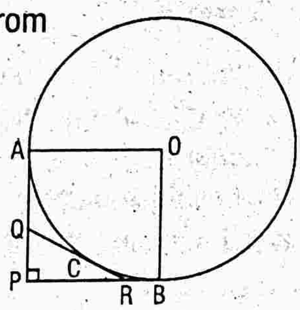
By the same reason, $\angle BAQ = \angle BCA = 36^\circ$

$\therefore$ The angle which the tangent makes with the two sides of the pentagon through the point of contact are 36° each.

Activities (Pages 179, 180, 181)

1. In the picture, a triangle is formed by two mutually perpendicular tangents to a circle and a third tangent. Prove that the perimeter of the triangle is equal to the diameter of the circle.





Since the tangents to a circle from a point are of the same length,

PA = PB ..... (1)

RB = RC ..... (2)

QA = QC ..... (3)

From the picture,

PR = PB − RB = PB − RC ... (4) (reason (2))

PQ = PA − QA = PA − QC ..... (5) (reason (3))

Perimeter of ΔPQR = PQ + QR + PR

= PA − QC + QR + PB − RC (reason (4), (5))

= PA + PB + QR − (QC + RC)

= PA + PB + QR − QR

= PA + PB ...... (6)

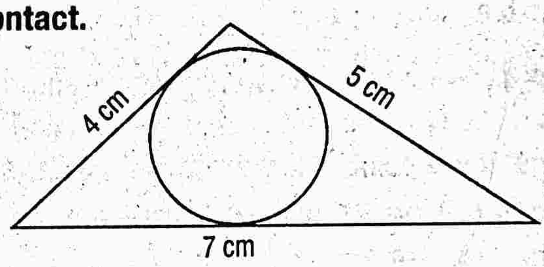
Since all the angles of quadrilateral PAOB are 90° each and its sides are equal, it is a square.

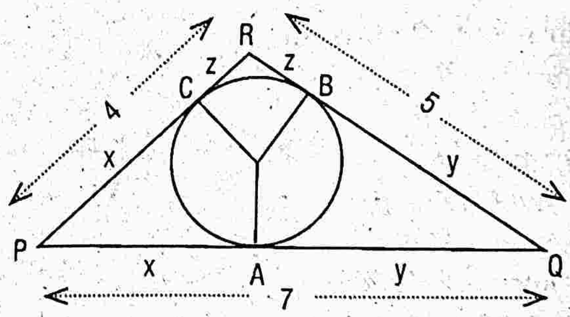
∴ PA = PB = OA = OB = radius of the circle ... (7)

From (6), perimeter of ΔPQR = PA + PB

= PA + PA = 2PA = 2 OA = diameter

2. The picture shows a triangle formed by three tangents to a circle. Calculate the length of each tangent from the corner of the triangle to the point of contact.





The tangents to a circle from a point are of the same length. So let PA = PC = x

QA = QB = y

and RC = RB = z

In the figure, x + y = 7, x + z = 4, y + z = 5

Perimeter of triangle = 7 + 5 + 4 = 16 cm .... (1)

PQ + QR + PR = x + y + y + z + x + z

= 2x + 2y + 2z = 2(x + y + z) .... (2)

From (1) and (2), 2(x + y + z) = 16

x + y + z = 8 .... (3)

x = (x + y + z) − (y + z) = 8 − 5 = 3

y = (x + y + z) − (x + z) = 8 − 4 = 4

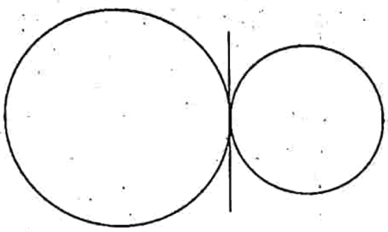
z = (x + y + z) − (x + y) = 8 − 7 = 1

Lengths of the tangents : PA = PC = 3 cm

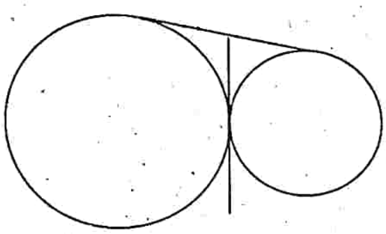
QA = QB = 4 cm

RB = RC = 1 cm

3. In the picture, two circles touch at a point and the common tangent at this point is drawn.



(i) Prove that this tangent bisects another common tangent of these circles.



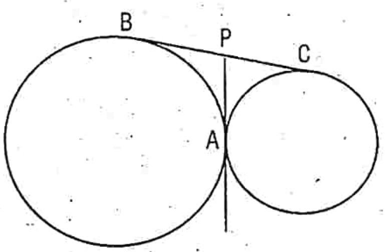
Tangents to a circle from a point are of the same length.

∴ PA = PB ... (1)

Also PA = PC ... (2)

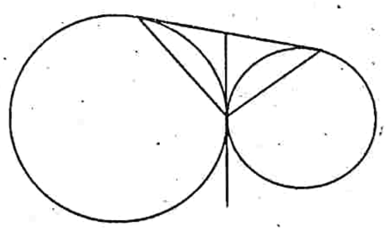
From (1) and (2),

PB = PC.



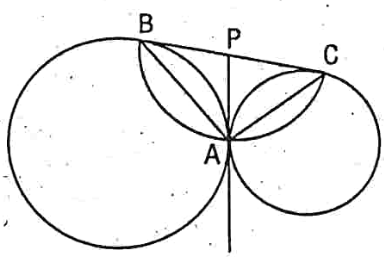
That is, the tangent through A bisects BC.

(ii) Prove that the points of contact of these two tangents form the vertices of a right triangle.



In (i) we have seen that PB = PC.

So P is the midpoint of BC.

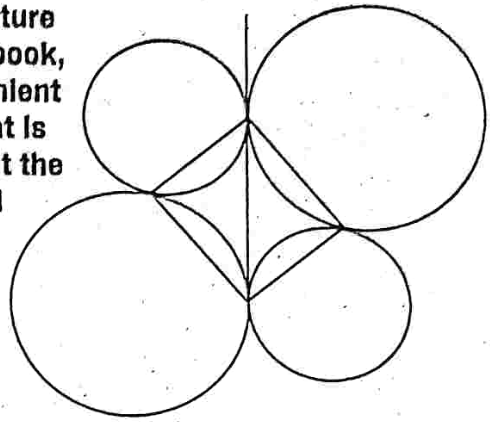


The circle drawn with P as centre passes through B and C. Since the angle in a semicircle is a right angle,

∠BAC = 90°

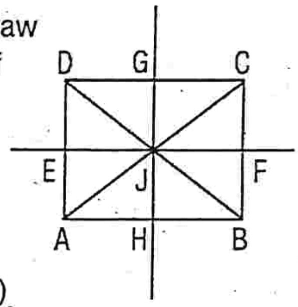
∴ ΔBAC is a right triangle.

(iii) Draw the picture in your notebook, using convenient lengths. What is special about the quadrilateral formed by joining the points of contact of the circles?

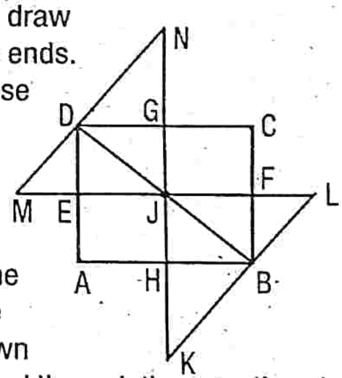


Since the angle in a semicircle is a right angle, then quadrilateral drawn by joining the meeting points of the circles is a rectangle. The picture can be drawn as follows.

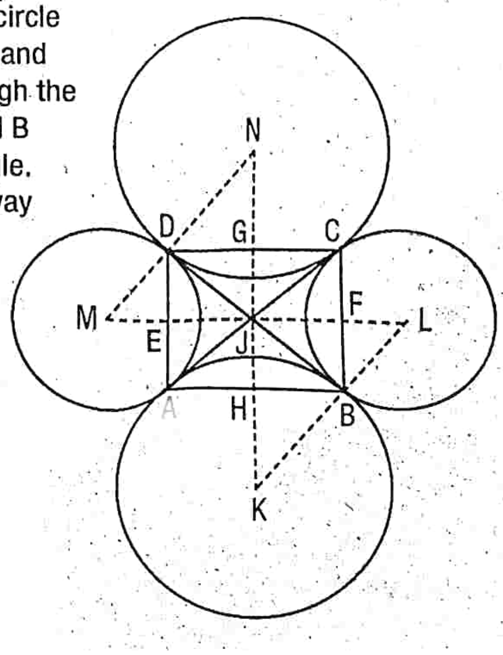
Step 1 : Draw a rectangle and draw the perpendicular bisectors of its sides. (Since each circle passes through two adjacent vertices of the rectangle, the centre of the circle is on the perpendicular bisector of the line joining these vertices.)



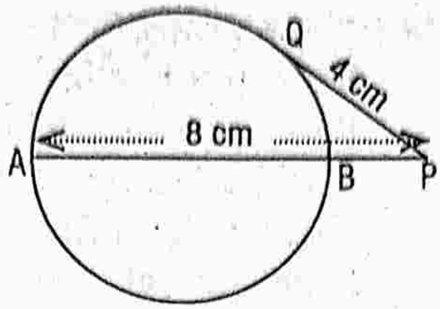
Step 2 : Draw a diagonal and draw perpendiculars through its ends. Mark the points where these perpendiculars meet the perpendicular bisectors of the sides. (Since each diagonal touches some circles at the vertices of the triangle, the centres of the circles are on the line drawn perpendicular to the diagonal through these vertices.)



Step 3 : Draw a circle with centre K and passing through the vertices A and B of the rectangle. In the same way draw circles with centres L, M and N.



4. In the picture below, AB is a diameter and P is a point on AB extended. A tangent from P touches the circle at Q. What is the radius of the circle?



Don't you remember the result $PA \times PB = PC^2$

Here $PA \times PB = PQ^2$

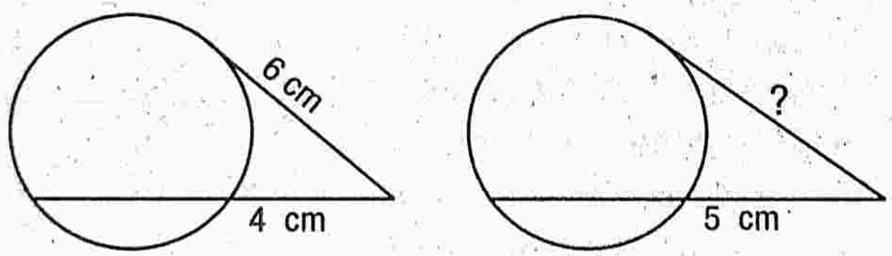
In the picture, let $PB = x$.

$8 \times x = 4^2$, $8x = 16$, $x = \frac{16}{8} = 2$

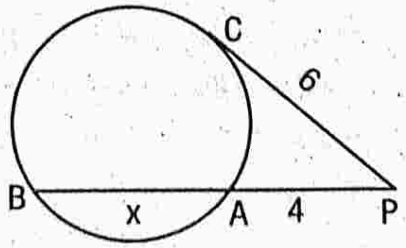
$\therefore PB = 2$ cm, $AB = AP - PB = 8 - 2 = 6$ cm

Diameter of the circle = 6 cm, radius = 3 cm.

5. In the first picture below, the line joining two points on a circle is extended by 4 cm and a tangent is drawn from this point. Its length is 6 cm, as shown.



The second picture shows the same line extended by 1 centimetre more and a tangent drawn from this point. What is the length of this tangent?



Since $AB = x$, $PB = x + 4$

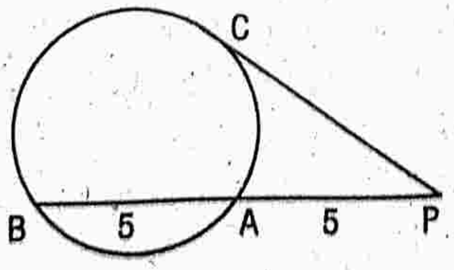
Since $PA \times PB = PC^2$

$4(x + 4) = 6^2$

$4x + 16 = 36$

$4x = 36 - 16 = 20$

$x = \frac{20}{4} = 5$



In this picture $PA = 5$, $PB = 5 + 5 = 10$

$PC^2 = PA \times PB = 5 \times 10 = 50$

$PC = \sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$ cm