

Augustin-Louis Cauchy French mathematician, engineer,
 and physicist who made ploneering contributions to several branches of mathematics, including mathematical analysis and continuum mechanics. He was one of the first to state and rigorously prove theorems of calculus, rejecting the heuristic principle of the generality of algebra of earller authors. He almost singlehandedly founded complex analysis and the study of permutation aroups in abstract algebra.

## NOTES

1) In two similar triangles, the opposite sides of equal angles are proportional.

2) In any isosceles triangle the perpendicular from the point joining equal sides to the opposite side bisects the opposite side.

3) The length of the line joining the midpoints of two sides of a triangle is half the length of the third side. Also, this line will be always parallel to the third side.


To Remember: -
In questions, to prove

1) If the figure is not given, draw a rough picture related to the statement.
2) If the figure is given, the lines required for the proof should be drawn.
3) Find out two triangles with the sides mentioned and then prove that these triangles are equal or similar.
4) If the diameter of a circle is given, draw lines where angle in a semicircle is 90 .

To recall....
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1) In figure, $A B$ and $A C$ are the diameters and $A E$ and $A D$ are the chords of the larger and smaller circles respectively. Then,
i) $\angle \mathrm{ADC}=$ $\qquad$ $\left[30^{\circ}, \mathbf{6 0}^{\circ}, \mathbf{9 0}^{\circ}\right]$
ii) $\mathbf{A E B}=$ $\qquad$ $\left[60^{\circ}, 90^{\circ}, 30^{\circ}\right]$

$$
\therefore \angle \mathrm{ADC}=\angle \mathrm{AEB}
$$


iii) Name the common angle in the triangles

## $\triangle A D C$ and $\triangle A E B$.

If two angles of a triangle are equal to any two angles of another triangle, then the triangles are $\qquad$ .
iv) Name the centre of larger circle.
v) $\frac{\mathrm{AC}}{\mathrm{AB}}=\frac{--}{\mathrm{AE}}=\frac{1}{2}$
vi) $\mathrm{AE}=2 \times$ $\qquad$
$\therefore$ The smaller circle bisects the chord.

Another method:
i) Name the centre of larger circle.
ii) The perpendicular from the centre of a circle to a chord
$\qquad$ the chord.
iii) $\angle \mathrm{ADC}=$ $\qquad$
iv) $\mathrm{AD}=$
v) $\mathrm{AE}=2 \times$ $\qquad$


$$
\begin{array}{l|l}
\hline \text { Learn through } & \text { CLICK HERE } \\
\hline \text { Dynamics of Geometry } & \text { CLICK HERE } \\
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For widened thoughts...

1) In figure, $A B$ is a diameter and $D$ is a point on the circle.
If $\mathrm{AC}=\mathrm{BC}$ and $\angle \mathrm{ABC}=70^{\circ}$ then,
a) $\mathrm{ADB}=$ $\qquad$
b) $\mathbf{B A C}=$ $\qquad$
c) $\mathrm{BAD}=$ $\qquad$
d) $\mathrm{DAC}=$ $\qquad$
e) $\mathrm{ACD}=$ $\qquad$

2) In figure, $M$ is the midpoint of AB and $\mathrm{AM}=\mathrm{MC}$.
a) Find another side which is equal to AM and MC.
b) What type of triangles are $\triangle A M C$ and $\triangle B M C$ ?

c) If a circle is drawn with $A B$ as
diameter, check whether the point C is inside, on or outside the circle.
d) What is the measure of $\angle A C B$ ?



MATHEMATICS STANDARD 10


Exterior Angle theorem:
The measurement of an outer angle at any vertex of a triangle is the sum of inner angles at the other two vertices.


In the figure,
p is the external angle of $\triangle \mathrm{ABC}$
$y$ and $z$ are two internal angles which are away from $p$.
$x+(y+z)=180^{\circ}$ (sum of angles in a triangle)
and $\mathrm{x}+\mathrm{p}=180^{\circ}$ (linear pairs)
So, $\mathrm{p}=\mathrm{y}+\mathrm{z}$

## Q 1

In figure, $\angle \mathrm{C}=90^{\circ}, \angle \mathrm{ABC}=30^{\circ}$,
i) $\angle \mathrm{BAC}=$ $\qquad$
ii) $\angle \mathrm{BAD}=$
iii) $\angle \mathrm{ABC}+\angle \mathrm{ACB}$


## $\angle \mathrm{ABC}+\angle \mathrm{ACB}=\angle \mathrm{BAD}$

Is it true even when the measure of $\angle B$ is changed?

Q 2
In figure, if $\angle \mathrm{PBC}=40^{\circ}$ and AB is a diameter of the circle,
i) $\angle \mathrm{ACB}=$ $\qquad$
ii) $\angle \mathrm{ACB}+\angle \mathrm{CBP}=$ $\qquad$
III) $\angle \mathrm{APB}=$ $\qquad$


The angle subtended by the diameter at a point inside the circle is greater than $90^{\circ}$

## Q 3

In figure, AC and BC are perpendicular to each other and $\angle \mathrm{ABC}=50^{\circ}$. Then,
i) $\angle \mathrm{CAB}=$ $\qquad$
ii) $\angle \mathrm{PCB}=$ $\qquad$

iii) If $\angle \mathrm{PBC}=30^{\circ}, \angle \mathrm{CPB}=$
iv) $\angle \mathrm{ACB}-\mathrm{PBC}=$ $\qquad$

## $\angle \mathrm{CPB}=\angle \mathrm{ACB}-\angle \mathrm{PBC}$

Is it true even when the measure of $\angle \mathrm{ABC}$ is changed?

Q 4
In figure, O is the centre, $\angle \mathrm{CAB}=35^{\circ}$ and $\angle \mathrm{CBP}=25^{\circ}$.
Then,
i) $\angle \mathrm{ACB}=$ $\qquad$
ii) $\angle \mathrm{ABC}=$ $\qquad$
iii) $\angle \mathrm{APB}=$ $\qquad$

$P$ is a point outside the circle. The angle subtended by the diameter at a point outside the circle is less than $90^{\circ}$


## More questions

## Q 1

In figure, PQ is a diameter. If the ratio between $\angle \mathrm{PAQ}, \angle \mathrm{PBQ}$ and $\angle \mathrm{PCQ}$ are

1: 2: 3 then,
a) $\angle \mathrm{PBQ}=$ $\qquad$
b) $\angle \mathrm{PAQ}=$ $\qquad$
c) $\angle \mathrm{PCQ}=$ $\qquad$


Q 2
In figure, $\angle \mathrm{P}=90^{\circ}, \angle \mathrm{Q}=70^{\circ}$ and $\angle \mathrm{S}=125^{\circ}$ then,
i) Check whether the points $Q$ and $S$ are inside, on or outside the circle whose diameter is PR.
ii) Check whether the points $P$ and $R$ are inside, on or outside the circle whose diameter is QS.


## Q 3

A circle is drawn with AB as diameter . A point C marked inside the circle. On drawing triangle ABC and measuring $\angle \mathrm{C}$ Remya got $70^{\circ}$ while Reema got $100^{\circ}$. Which is correct measure of $\angle \mathrm{C}$ ? Why?

## Do yourself and learn yourself......

## CLICK HERE

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CLICK HERE
Test yourself....


MATHEMATICS STANDARD 10


LET'S REVISE
MATCH THE FOLLOWING

| Line joining centre and any point on a circle | Diameter |
| :--- | :---: |
| The part of a circle | Chord |
| Double of the radius | Arc |
| Line joining any two points on a circle | $2 \pi r$ |
| Area of circle | Radius |
| Perimeter of a circle | $\pi r^{2}$ |

1) What is the measure of $\angle B$
in the figure?
(The sum of the angles of a triangle is 180 degree).
2) in the figure, what is the measure of $\angle A+\angle B ?$

3) $O A$ and $O B$ are the radii.

We know that the radii are equal.
What are the measures of $\angle A$ and $\angle B$ ?
(Hint 1: The sum of the angles of a triangle is $180^{\circ}$ Hint 2 : The angles opposite to equal sides are also equal).

4) $\angle A P B$ is the angle formed by the diameter $A B$ at the point $P$ on the circle.
(i.e. the angle in a semicircle)

Then, $\angle \mathrm{A}=$ $\qquad$ , $\angle B=$ $\qquad$ ,
$\angle O P A=$ $\qquad$ , $\angle \mathrm{OPB}=$ $\qquad$ and $\angle \mathrm{APB}=$ $\qquad$
5) In the given figure, $O$ is the centre of the circle and $A B$ is a diameter. C is a point on the circle.
i) Compute $\angle \mathrm{ACB}$
ii) Draw another figure like this by changing the size of $\angle C O B$ and calculate $\angle A C B$.


In any circle, if the end points of a diameter are joined to another point on the circle, what would be the angle obtained?
6) $\angle A P B$ is the angle formed by the diameter $A B$ at the point $P$ on the circle. (l.e. the angle in a semicircle)

Then, $\angle \mathrm{A}=$ $\qquad$ $\angle A O P=$ $\qquad$
$\angle \mathrm{OPB}=$ $\qquad$ and $\angle \mathrm{APB}=$ $\qquad$

7) $\angle A C B$ is the angle formed by the diameter $A B$ at the point $C$ on the circle.(i.e. the angle in a semicircle) Then,
$\angle O C A=$ $\qquad$ , $\angle \mathrm{AOC}=$ $\qquad$
$\angle O C B=$ $\qquad$ , $\angle B O C=$ $\qquad$ and $\angle \mathrm{ACB}=$ $\qquad$

8) $\angle B P A$ and $\angle B F A$ are the angles formed by the diameter $A B$ at the points $P$ and $F$ respectively on the circle. Then,
$\angle B P A=$ $\qquad$ , $\angle B F A=$ $\qquad$
$\angle \mathrm{BPA}+\angle \mathrm{BFA}=$ $\qquad$ and $\angle P A B=$ $\qquad$ .

9) $A B$ is the diameter.
$P$ is the point on the circle.
Then $\angle \mathrm{APB}=$ $\qquad$ and $\angle P A B=$ $\qquad$

10) AB is the diameter. $\angle \mathrm{P}+\angle \mathrm{Q}=$ ?


## Note:- (Angle in a semicircle is $90^{\circ}$ )

 $C$ is the point on the circle of diameter AB

> A,B are the end points of the diameter.

The $\angle A C B$ is obtained by joining the points $A$ and $B$ to the point $C$ on the circle.

$$
\angle \mathbf{A C B}=90^{\circ}
$$

## That is

The angle obtained by joining the ends of a diameter of a circle with a point on the circle is $90^{\circ}$


Watch and learn.....

## CIRCLES AROUND US

Have you observed an umbrella? Are they of same size? yours and your grandpa's? Are the number of spokes equal? What about the angle between 2 spokes? What does the size 345 mm of an umbrella means relating to a circle? Send your findings to your teacher.



## Class - 18 (Worksheet -2)

1) For the arithmetic sequence $4,12,20,28$......
(a) What is the first term?
(b) What is the common difference?
(c) What is the relation between the first term and the common differnce?
(d) What is the sum of first two terms?
(e) What is the sum of first three terms?
(f) What is the sum of first four terms?
(g) What is the sum of first five terms?

| (a) | First term | 4 |
| :---: | :---: | :---: |
| (b) | Common difference | 8 |
| (c) | Relation between the first term and the common differnce | Common difference is two times the first term |
| (d) | Sum of first two terms | $4+12=16=4^{2}$ |
| (e) | Sum of first three terms | $4+12+20=36=6^{2}$ |
| (f) | Sum of first four terms | $4+12+20+28=64=8{ }^{2}$ |
| (g) | Sum of first five terms | $4+12+20+28+36=100=10^{2}$ |

2) For the arithmetic sequence $\mathbf{1 6}, \mathbf{4 8}, \mathbf{8 0}, . . . .$.
(a) What is the first term?
(b) What is the common difference?
(c) What is the relation between the first term and the common differnce?
(d) What is the sum of first two terms?
(e) What is the sum of first three terms?
(f) What is the sum of first four terms?
(g) What is the sum of first five terms?

| (a) | First term | 16 |
| :---: | :---: | :---: |
| (b) | Common difference | 48-16=32 |
| (c) | Relation between the first term and the common differnce | Common difference is two times the first term |
| (d) | Sum of first two terms | $16+48=64=8^{2}$ |
| (e) | Sum of first three terms | $16+48+80=144=12^{2}$ |
| (f) | Sum of first four terms | $16+48+80+112=256=16^{2}$ |
| (g) | Sum of first five terms | $16+48+80+112+144=400=20^{2}$ |

## Conclusion:

In an arithmetic sequence, if the first term is a perfect sqaure and the common difference is twice the first term then the sum of any number of terms of the sequence will be a perfect square.

Now, we know that for the arithmetic sequence, $1,3,5,7,9, \ldots$. the sum of any number of terms is a perfect square.

Example:

| One term | Two terms | Three terms |
| :---: | :---: | :---: |
| 1 | $1+3=4$ | $1+3+5=9$ |
| $\mathbf{1}^{2}$ | $2^{2}$ | $3^{2}$ |

What about in the sequence $4,12,20,28,36$, $\qquad$ ?

| One term | Two terms | Three terms |
| :---: | :---: | :---: |
| 4 | $4+12=16$ | $4+12+20=36$ |
| $2^{2}$ | $4^{2}$ | $6^{2}$ |
| $(1 \times 2)^{2}$ | $(2 \times 2)^{2}$ | $(3 \times 2)^{2}$ |

And what do you think in this sequence? 9, 27, 45, 63, $\qquad$

| One term | Two terms | Three terms |
| :---: | :---: | :---: |
| 9 | $9+27=36$ | $9+27+45=81$ |
| $3^{2}$ | $6^{2}$ | $9^{2}$ |
| $(1 \times 3)^{2}$ | $(2 \times 3)^{2}$ | $(3 \times 3)^{2}$ |

Similarly, we can find any number of sequences and sum of any number of terms of each sequence.

## Conclusion:

Here, each term in the sequence of sum is in the form $n^{2} \times f$. For an arithmetic sequence, if the common difference is twice the first term, the sum of first $n$ terms will always be in the form $n^{2} \times f$. It is understood that the first term ' $f$ ' of the sequence should be a perfect square in order to get the sum a perfect square.

## *LET'S ASSES

1. Can you write an arithmetic sequence, the sum of whose any number of terms, starting from the first, is 400 ?
2. Prove that the sum of any number of terms of the arithmetic sequence 16, 24, 32, starting from the first, added to 9 gives a perfect square.

## VIDEO $\rightarrow$ CLICK HERE

ONLINE TEST $\longrightarrow$ CLICK HERE


## $18^{\text {TH }}$ CLASS

Consider the following Arithmetic Sequences,
i) $\mathbf{1 , 2 , 3 , 4 , 5} . . .$. . Sequence of Natural Numbers. $1^{2}=1,2^{2}=4,2^{3}=8,2^{4}=16,3^{2}=9,3^{3}=27,4^{2}=16 \ldots$.
If we take any power of this sequence it is a Natural Number. Hence ' Every power of every term is again a term of the same Arithmetic Sequence.'
ii) 2,4,6,8,...........sequence of Even Numbers
$2^{2}=4,2^{3}=8,2^{4}=16,4^{2}=16,4^{3}=64,6^{2}=36$
If we take any power of this sequence it is an Even Number.
Hence ' Every power of every term is again a term of the same Arithmetic Sequence.'
iii) $\mathbf{3 , 6 , 9 , 1 2}$. sequence of Multiples of 3

We know the powers of 3 is again a multiple of 3,Hence ' Every power of every term is again a term of the same Arithmetic Sequence.'

Similarly we can say in the sequence of Multiples of 4 ,' Every power of every term is again a term of the same Arithmetic Sequence.'
iv) 3,5 7,9..........Arithmetic sequence of Odd Numbers excluding 1.
$3^{2}=9,3^{3}=27,3^{4}=81,5^{2}=25,5^{3}=125 \ldots .$. All the powers are Odd Numbers which belongs to the same sequence.
v) $\mathbf{5 , 9}, \mathbf{1 3}, 17$....... When we check we can say that ' Every power of every term is again a term of the same Arithmetic Sequence.'
vi) $4,7,10,13$....Here also When we check we can say that ' Every power of every term is again a term of the same Arithmetic Sequence.'

Let's write the Algebraic form of the above sequences.

| ARITHMETIC SEQUENCE | Xn |
| :---: | :---: |
| 1,2,3,4,5..... | n |
| 2,4,6,8,....... | 2n |
| 3,6,9,12...... | 3n |
| 3, 5 7,9...... | 2n+1 |
| 5,9,13,17..... | $4 \mathrm{n}+1$ |
| 4,7,10,13.. | $3 \mathrm{n}+1$ |

The algebraic form of all the above sequences are in the form of 'an' or in the form of 'an+1''

## CONCLUSION:

IF THE GENERAL FORM OF AN ARITHMETIC SEQUENCE IS IN THE FORM "an" or "an + 1", THEN EVERY POWER OF EVERY TERM IS AGAIN A TERM OF THE SAME SEQUENCE. WHERE "a" IS A NATURAL NUMBER.

## *LET'S ASSES

In the following Arithmetic Sequences check whether every power of every term is again a term of the same sequence. Write the reason.
a) $5,10,15,20, \ldots \ldots$.
b) $6,11,16,21 \ldots$.
c) $8,15,22,29 \ldots$
d) $10,20,30,40 \ldots$....

## Note:

When an is divided by the common difference $a$, the remainder is 0 .
We know that the remainder we get when $(a n)^{2}=a^{2} n^{2}$ is divided by $a$,
is also 0 .
But the remainder we get when $(a n+1)$ is divided by $a$ is 1 .
The remainder we get when $(a n+1)^{2}=a^{2} n^{2}+2 a n+1$ is divided by $a$ is also 1.

Hence, if any power of $(a n+1)$ is divided by the common difference $a$, the remainder is $\mathbf{1}$. Isn't it?

