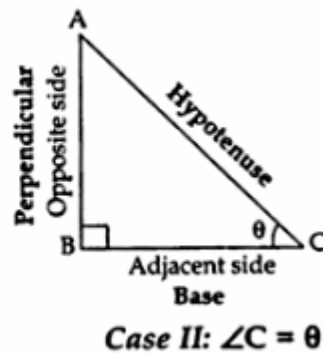
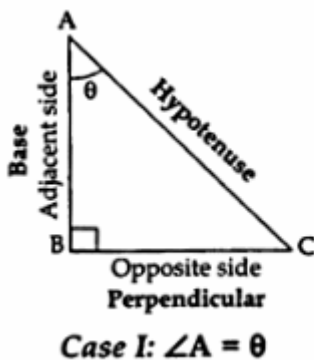


- Position of a point P in the Cartesian plane with respect to co-ordinate axes is represented by the ordered pair (x, y).
- Trigonometry is the science of relationships between the sides and angles of a right-angled triangle.
- **Trigonometric Ratios:** Ratios of sides of right triangle are called trigonometric ratios. Consider triangle ABC right-angled at B. These ratios are always defined with respect to acute angle 'A' or angle 'C'.
- If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of an angle can be easily determined.
- How to identify sides: Identify the angle with respect to which the t-ratios have to be calculated. Sides are always labelled with respect to the 'θ' being considered.

Let us look at both cases:



In a right triangle ABC, right-angled at B. Once we have identified the sides, we can define six t-Ratios with respect to the sides.

case I	case II
(i) sine A = $\frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC}$	(i) sine C = $\frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{AC}$
(ii) cosine A = $\frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC}$	(ii) cosine C = $\frac{\text{base}}{\text{hypotenuse}} = \frac{BC}{AC}$
(iii) tangent A = $\frac{\text{perpendicular}}{\text{base}} = \frac{BC}{AB}$	(iii) tangent C = $\frac{\text{perpendicular}}{\text{base}} = \frac{AB}{BC}$
(iv) cosecant A = $\frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{BC}$	(iv) cosecant C = $\frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{AC}{AB}$
(v) secant A = $\frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{AB}$	(v) secant C = $\frac{\text{hypotenuse}}{\text{base}} = \frac{AC}{BC}$
(v) cotangent A = $\frac{\text{base}}{\text{perpendicular}} = \frac{AB}{BC}$	(v) cotangent C = $\frac{\text{base}}{\text{perpendicular}} = \frac{BC}{AB}$

Note from above six relationships:

$$\text{cosecant } A = \frac{1}{\sin A}, \text{ secant } A = \frac{1}{\cos A}, \text{ cotangent } A = \frac{1}{\tan A},$$

However, it is very tedious to write full forms of t-ratios, therefore the abbreviated notations are:

sine A is $\sin A$

cosine A is $\cos A$

tangent A is $\tan A$

cosecant A is cosec A
 secant A is sec A
 cotangent A is cot A

TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratio of angle(s) is called a trigonometric identity, if it is true for all values of the angles involved. These are:

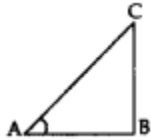
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

- $\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta \Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$
- $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \Rightarrow \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta \Rightarrow \cot^2 \theta = \operatorname{cosec}^2 \theta - 1$
- $\sec^2 \theta - \tan^2 \theta = 1 \Rightarrow \sec^2 \theta = 1 + \tan^2 \theta \Rightarrow \tan^2 \theta = \sec^2 \theta - 1$
- $\sin \theta \operatorname{cosec} \theta = 1 \Rightarrow \cos \theta \sec \theta = 1 \Rightarrow \tan \theta \cot \theta = 1$

ALERT:

A t-ratio only depends upon the angle 'θ' and stays the same for same angle of different sized right triangles.

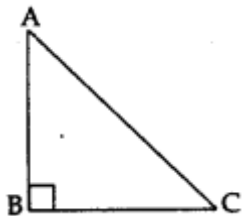


Value of t-ratios of specified angles:

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
cosec A	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
cot A	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

The value of $\sin \theta$ and $\cos \theta$ can never exceed 1 (one) as opposite side is 1. Adjacent side can never be greater than hypotenuse since hypotenuse is the longest side in a right-angled Δ .

't-RATIOS' OF COMPLEMENTARY ANGLES



If $\triangle ABC$ is a right-angled triangle, right-angled at B, then
 $\angle A + \angle C = 90^\circ$ [$\because \angle A + \angle B + \angle C = 180^\circ$ angle-sum-property]
or $\angle C = (90^\circ - \angle A)$

Thus, $\angle A$ and $\angle C$ are known as complementary angles and are related by the following relationships:

$$\begin{aligned}\sin(90^\circ - A) &= \cos A; \operatorname{cosec}(90^\circ - A) = \sec A \\ \cos(90^\circ - A) &= \sin A; \sec(90^\circ - A) = \operatorname{cosec} A \\ \tan(90^\circ - A) &= \cot A; \cot(90^\circ - A) = \tan A\end{aligned}$$