## 5 . TRIGNOMETRY - Class 9

*What did we learn in the class ?*

*In a circle , the length of any chord is double the product of the radius and sin of the half the central angle .*

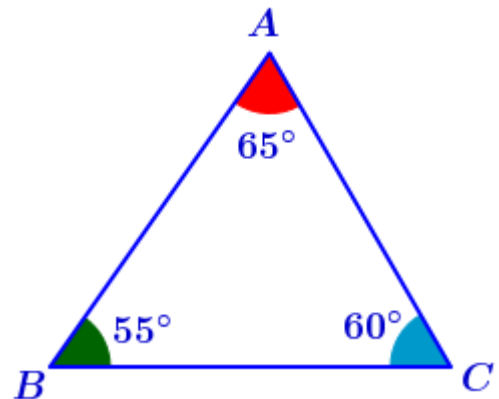*In a circle of radius $r$ , the length of an arc of central angle $x^0$*

$$\text{length of the chord} \ = \ 2 \, r \, sin \left(\frac{x}{2}\right)^{\circ}$$
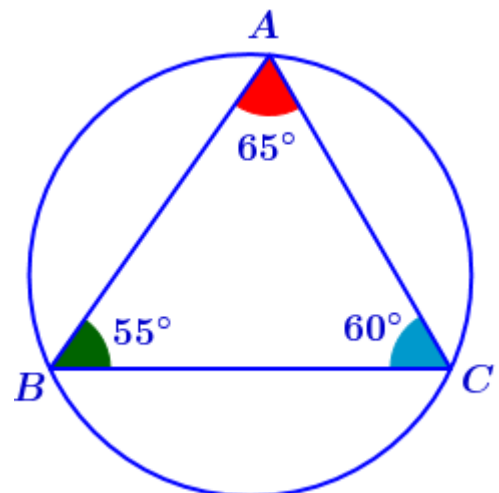
### *Triangle and circle*
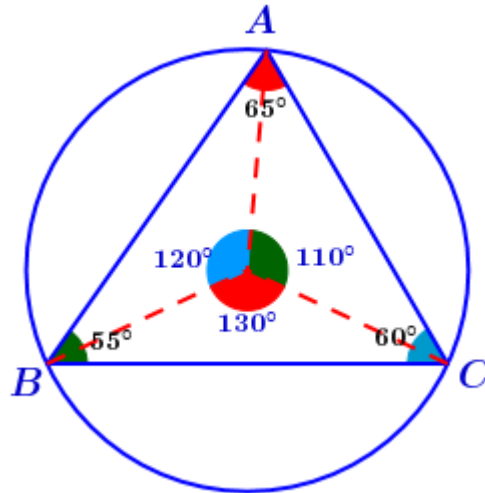
*The angles of a triangle determine the ratio of its sides*

*In triangle ABC , $< A = 65^0$ , $< B = 55^0$ , $< C = 60^0$*

*If we draw the circumcircle of the triangle , all its sides become chords of the circle .*

*All the central angle of each of these arc is double the angle opposite to it in the triangle .*
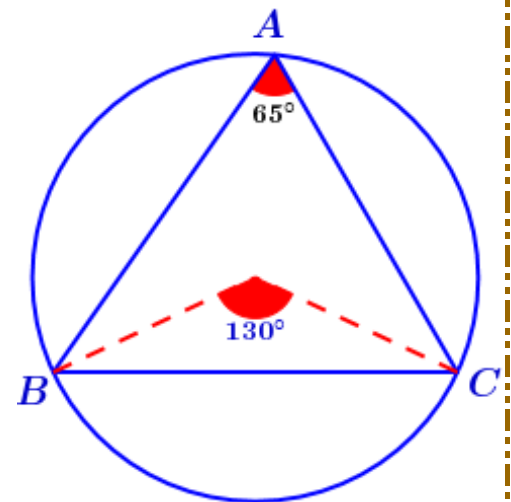
*If the radius of the circle is* $r$ ,

$$BC \ = \ 2 \, r \, sin \left( \frac{130}{2} \right)^{\circ} \ = \ 2 \, r \, sin \, 65°$$

*( In a circle , the length of any chord is double the product*

*of the radius and sin of the half the central angle . )*



*Similarly we can compute the length of other chords  ( other sides of the triangle ) .*

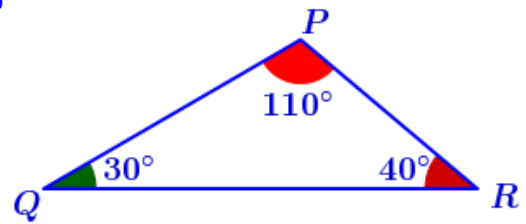$$AC \ = \ 2 \, r \, sin \left( \frac{110}{2} \right)^{\circ} \ = \ 2 \, r \, sin \, 55°$$

$$AB \ = \ 2 \, r \, sin \left( \frac{120}{2} \right)^{\circ} \ = \ 2 \, r \, sin \, 60°$$

$$AC \ : \ AB \ : \ BC = 2 \, r \, sin \, 55° \ : \ 2 \, r \, sin \, 60° \ : \ 2 \, r \, sin \, 65°$$

$$= \ sin \, 55° \ : \ sin \, 60° \ : \ sin \, 65°$$

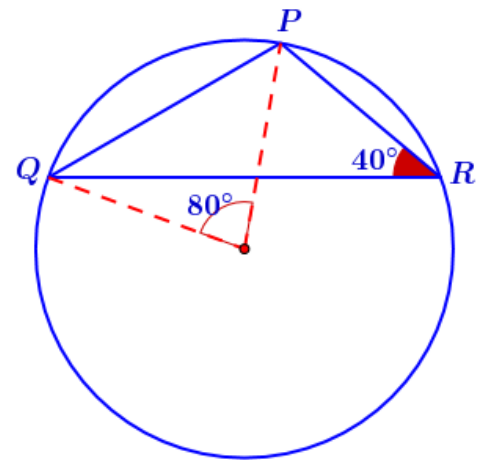SARATH .A .S , HST ,GHS ANCHACHAVADI

# Ratio of the sides of an obtuse triangle

In triangle  PQR ,   $< P = 110^0$ , $< Q = 30^0$ , $< R = 40^0$



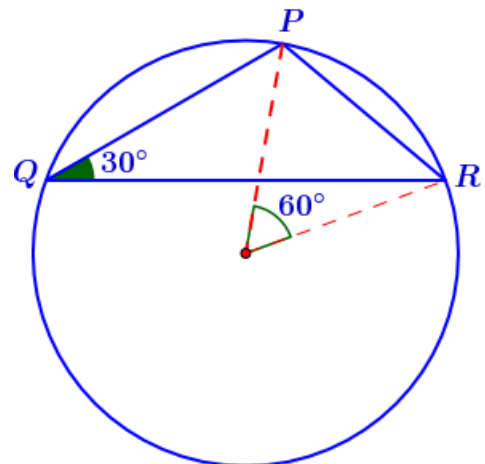If we draw the circumcircle of the triangle , all its sides  become chords of the circle .
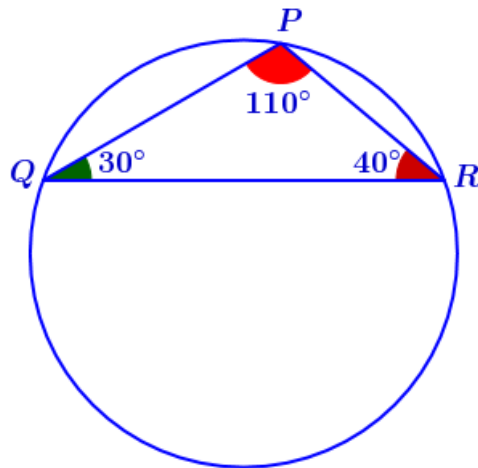


$$PQ = 2\,r\,sin\left(\frac{80}{2}\right)^{\circ} = 2\,r\,sin\,40^{\circ}$$

( In a circle , the length of any chord is double the product

of the radius and sin of the half the central angle .  )

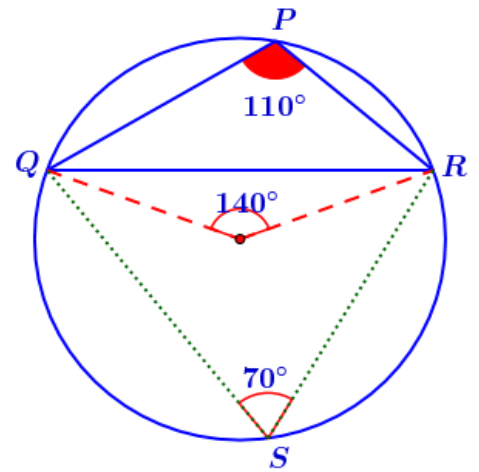$$PR = 2\,r\,sin\left(\frac{60}{2}\right)^{\circ} = 2\,r\,sin\,30^{\circ}$$

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$$QR \;=\; 2\,r\,sin\left(\frac{140}{2}\right)^{\circ} \;=\; 2\,r\,sin\,70^{\circ}$$

{ $< P = 110\,^{0}$ ==> $< S = 180\text{-}110 = 70\,^{0}$ *(Angles on the*

*alternate arcs are supplementary* )

*Central angle of the chord QR* $= 2 \times 70 = 140\,^{0}$ }

$$PR \;:\; PQ \;:\; QR \;=\; 2\,r\,sin\,30^{\circ} \;:\; 2\,r\,sin\,40^{\circ} \;:\; 2\,r\,sin\,70^{\circ}$$

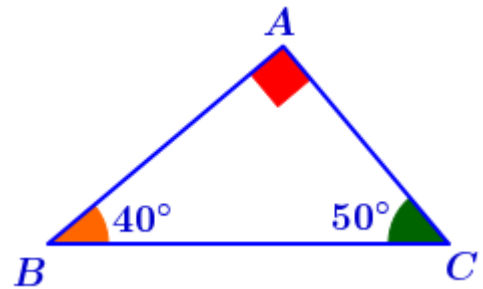$$=\; sin\,30^{\circ} \;:\; sin\,40^{\circ} \;:\; sin\,70^{\circ}$$
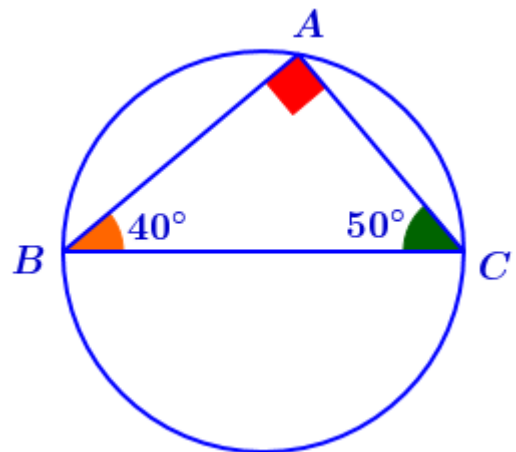
**NOTE :**

*If there is an angle greater than $90\,^{\circ}$ in a triangle , take the sine of its supplementary*

*angle to find the length of the sides using the angles .*

### Ratio of the sides of a right triangle

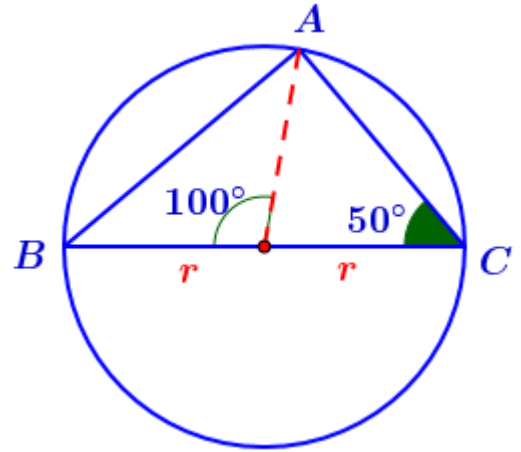In triangle ABC , $< A = 90\,^{0}$ , $< B = 40\,^{0}$ , $< C = 50\,^{0}$

If we draw the circumcircle of the triangle , all its

sides become chords of the circle . Here the hypotenuse
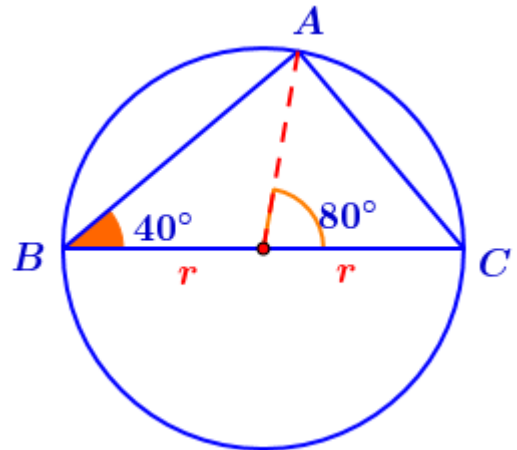
BC is the diameter of the circle .

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**If the radius of the circle is $r$ , BC = 2 $r$**

$$AB = 2\,r\,sin\left(\frac{100}{2}\right)^{\circ} = 2\,r\,sin\,50^{\circ}$$



$$AC = 2\,r\,sin\left(\frac{80}{2}\right)^{\circ} = 2\,r\,sin\,40^{\circ}$$



$$AC : AB : BC = 2\,r\,sin\,40^{\circ} : 2\,r\,sin\,50^{\circ} : 2\,r$$

$$= sin\,40^{\circ} : sin\,50^{\circ} : 1$$

*( We can find that $sin\,90^{\circ} = 1$ , text book page number 124 )*

**<u>Conclusion</u>**

> *The ratio of the sides of a triangle is the ratio of the sine of their opposite angles .*
>
> *If one angle is greater than $90^{\circ}$ , then take the sine of its supplementary angle .*

If one side of a triangle is **4 cm** and angles on both ends of that side are

**70 °** and **80 °** respectively .

 **a ) Find the measure of the third angle  ?**

 **b) Find the circum radius  ?**

 **c) Find the measure of other two sides   ?**



_**Answer**_

**In triangle  ABC  ,  BC = 4 cm   , < B = 80 ° , < C = 70 °**

**a)  < A = 180 - ( 80  + 70 ) = 180 – 150 = 30 $^0$**

 ( *Sum of the angles of a triangle is  180 °* )

*b)* $\quad BC = 2\,r\,sin\,30°$

$\qquad 2\,r\,sin\,30° = 4$

$\qquad 2\,r \times \dfrac{1}{2} = 4$

$\qquad r = 4\,cm$

*c)* $AB = 2\,r\,sin\,70° = 2 \times 4 \times 0.9397 = 7.5176\,cm$

$\quad AC = 2\,r\,sin\,80° = 2 \times 4 \times 0.9848 = 7.8784\,cm$
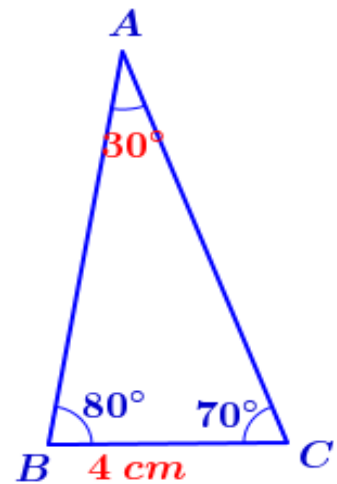
_**More activty**_

The angles of a triangle are  **50 °** ,  **60 °** and **70 °**  and the circum radius  is  **5 cm**

 **a ) Find the ratio of the sides of the triangle    ?**

 **b) Find the measure of the three sides of the triangle  ?**

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## WORKSHEET

**( 1 )** *In the figure* $BC = 5\sqrt{3}$ *cm , < B = 50$^0$ , < C = 70$^0$*
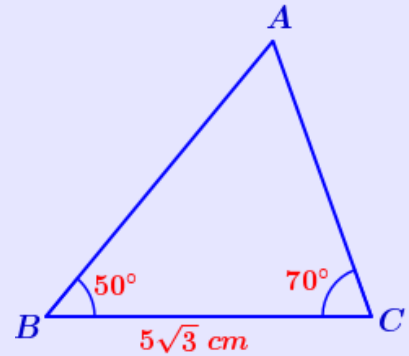
a) *What is the measure of < A ?*

b) *What is the circum diameter of the triangle ABC ?*

c) *What is the length of AB ?*

d) *What is the length of AC ?*

**( 2 )** *In the figure QR = 7.6 cm , PQ = PR , < Q = 65$^0$ $^0$*
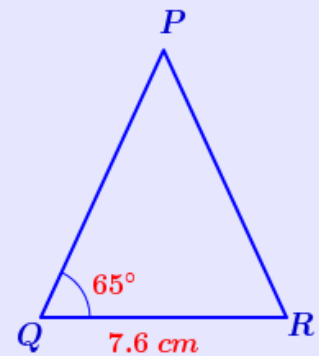
a) *What is the measure of < P ?*

b) *What is the circum diameter of the triangle PQR ?*

c) *What is the length of PR ?*

d) *What is the perimeter of the triangle PQR ?*

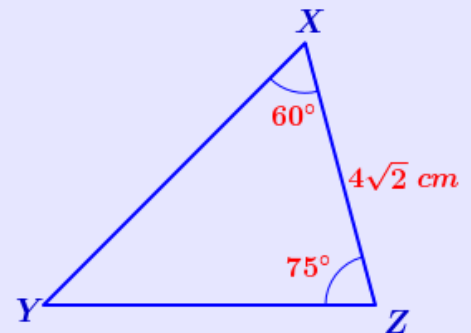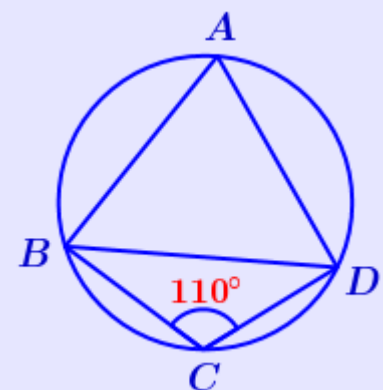**( 3 )** *In the figure XZ = 4 $\sqrt{2}$ cm , < X = 60$^0$ , < Z = 75$^0$*

a) *What is the measure of < Y ?*

b) *What is the circum diameter of the triangle XYZ ?*

c) *What is the length of XY ?*

d) *What is the length of YZ ?*

**( 4 )** *In the figure radius of the circle is 5 cm , < BCD = 110$^0$*

*and AB = AD*

a) *What is the measure of < BAD ?*

b) *What is the length of BD ?*

c) *What is the measure of < ABD ?*

d) *What is the length of AB ?*

*hints :*

| Angle | 50$^0$ | 55$^0$ | 65$^0$ | 70$^0$ | 75$^0$ |
|-------|--------|--------|--------|--------|--------|
| sin   | 0.76   | 0.82   | 0.91   | 0.94   | 0.96   |

## 5 . TRIGNOMETRY - Class 10

*What did we learn in the class ?*

**The ratio of the sides of a triangle is the ratio of the sine of their opposite angles .**

**If one angle is greater than 90 ° , then take the sine of its supplementary angle .**
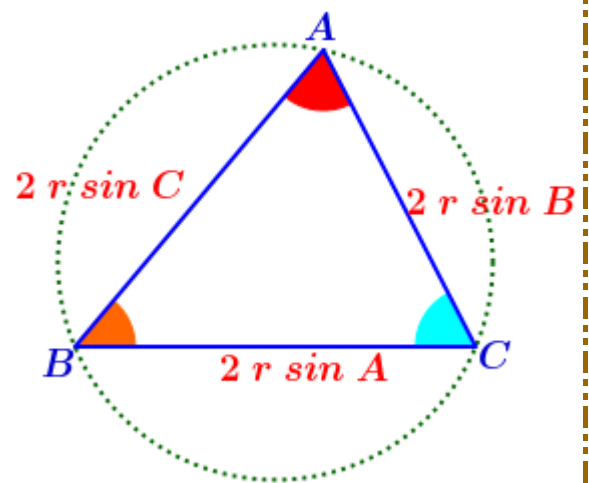
### Activity 1

*If the circumradius of the triangle ABC is* $r$ *,*

*the sides of the triangle are*

$$BC = 2\,r\,sin\,A$$

$$AB = 2\,r\,sin\,C$$

$$AC = 2\,r\,sin\,B$$

*If we take the side opposite to* $<A$ *as* $a$ *,*

*the side opposite to* $<B$ *as* $b$ *and*

*the side opposite to* $<C$ *as* $c$ *, then*

$$a = 2\,r\,sin\,A$$

$$b = 2\,r\,sin\,B$$

$$c = 2\,r\,sin\,C$$
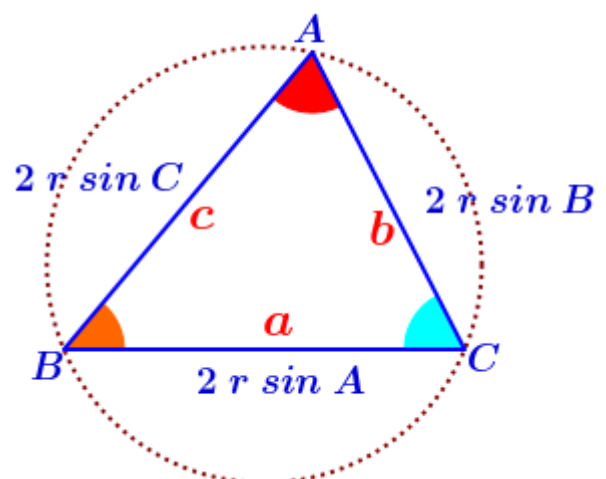
$$a = 2\,r\,\sin A \quad ==> \quad \frac{a}{\sin A} = 2\,r$$

$$b = 2\,r\,\sin B \quad ==> \quad \frac{b}{\sin B} = 2\,r$$

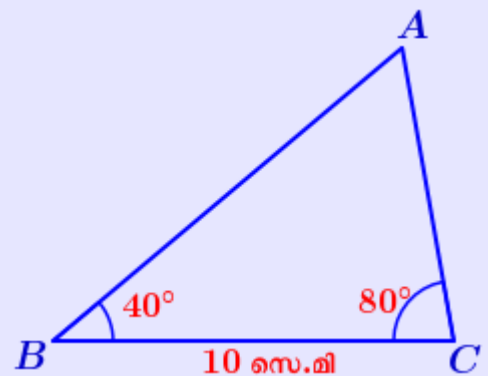$$c = 2\,r\,\sin C \quad ==> \quad \frac{c}{\sin C} = 2\,r$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2\,r$$

**NOTE :** *If one angle is greater than $90^\circ$ , then take the sine of its supplementary angle .*

*<u>Let's solve some problems related to this idea</u>*

( 1 ) *In the figure BC = 10 cm , < B = 40 ° and < C = 80 °*

  **a )** *What is the measure of < A    ?*

  **b)** *What is the circum diameter of the triangle ?*

  **c)** *Compute the length of the other two sides of the*

    *triangle ?*



40°     80°

B    10 സെ.മി    C

*<u>Answer</u>*

**a )** < A = 180 - ( 40 + 80 ) = 180 – 120 = $60^0$

                  *( Sum of the angles of a triangle is $180^\circ$ )*

**b )**

$$\frac{BC}{\sin A} = \frac{AC}{\sin B} = \frac{AB}{\sin C} = 2\,r$$
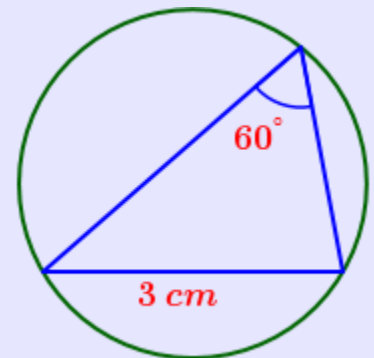
$$\frac{10}{\sin 60^0} = 2\,r$$

$$2\,r = 10 \div \frac{\sqrt{3}}{2} \quad = 10 \times \frac{2}{\sqrt{3}} = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3} \quad cm$$

b) $AC = 2\,r \sin 40^\circ \quad = \frac{20\sqrt{3}}{3} \times \sin 40^\circ \quad = \frac{20\sqrt{3}}{3} \times 0.6428$

$$= \frac{20 \times 1.73 \times 0.6428}{3} = 7.4136 \quad cm$$

$$AB = 2\,r \sin 80^\circ \quad = \frac{20\sqrt{3}}{3} \times \sin 80^\circ \quad = \frac{20\sqrt{3}}{3} \times 0.9848$$

$$= \frac{20 \times 1.73 \times 0.9848}{3} = 11.358 \quad cm$$

( 2 ) *The figure shows a triangle and its circumcircle*

   *What is the radius of the circle   ?*



**<u>Answer</u>**

$$2\,r = \frac{a}{\sin A} = \frac{3}{\sin 60^0} = 3 \div \frac{\sqrt{3}}{2} \quad = 3 \times \frac{2}{\sqrt{3}} = 2\sqrt{3} \quad സെ.മി$$
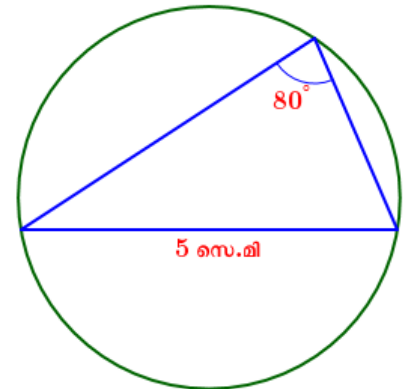
$$r = \frac{2\sqrt{3}}{2} = \sqrt{3} \quad cm$$

**( 3 )** *A circle is to be drawn , passing through the ends of a line , 5 cm long and the angle on the circle on one side of the line should be 80° .What should be the radius of the circle ?*

Answer
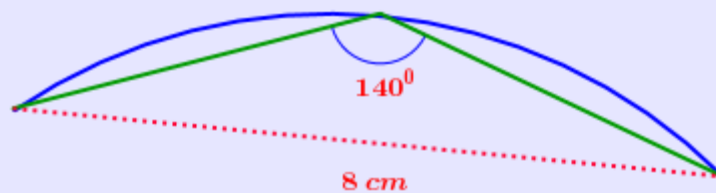


$$2\,r = \frac{a}{\sin A} = \frac{5}{\sin 80°} = \frac{5}{0.9848}$$

$$r = \frac{5}{2 \times 0.9848} \quad cm$$

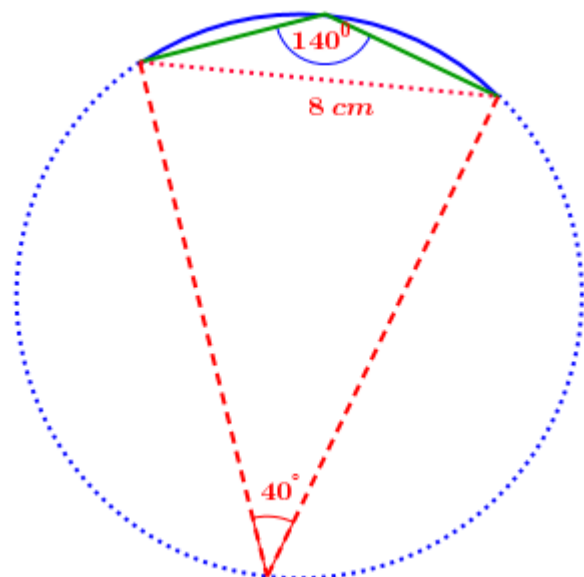**( 4 )** *The picture below shows part of a circle*



140⁰

8 cm

*What is the radius of the circle ?*

Answer



140⁰

8 cm

40°

$$2\,r = \frac{a}{\sin A}$$

$$= \frac{8}{\sin 40°}$$

*( If one angle is greater than 90° , then take the sine of its supplementary angle .)*

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$$2\,r \;=\; \frac{8}{sin\ 40^\circ} \;=\; \frac{8}{sin\ 40^\circ} \;=\; \frac{8}{0.6428}$$

$$r \;=\; \frac{8}{2\ \times\ 0.6428} \;=\; \frac{4}{0.6428}\quad cm$$

**( 5 )** *A regular pentagon is drawn with all its vertices on a circle of radius 15 cm . Calculate the length of the sides of the this pentagon ?*
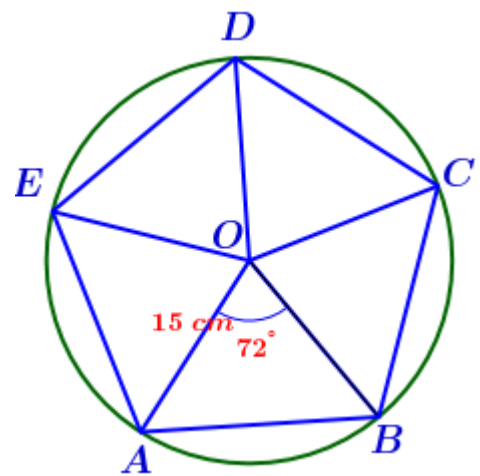
*Answer*

*In the figure ABCDE is a regular pentagon .*

*O is the centre of the circle .*

$$<\ AOB \;=\; \frac{360}{5} \;=\; 72^\circ$$

*Length of the chord AB* $\quad = 2\,r\,sin\,\left(\dfrac{72}{2}\right)^{\!\circ}$

$$=\ 2\ \times\ 15\ \times\ sin\ 36^\circ$$

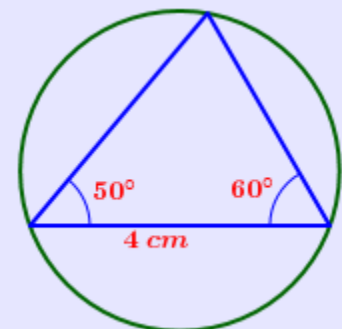$$=\ 30\ \times\ 0.5878\ =\ 17.634\quad cm$$

*More activities*

**( 1 )** *What is the circumradius of an equilateral triangle of sides 8 cm ?*

**( 2 )** *The figure shows a triangle and its circumcircle .*

**a )** *Compute the diameter of the circle ?*

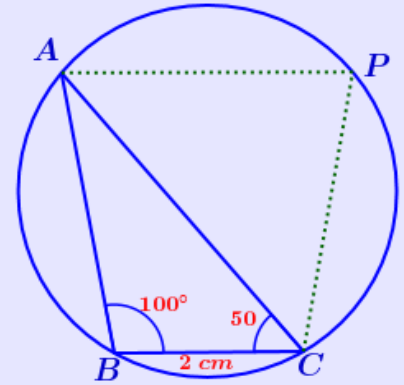**b )** *Compute the lengths of the other two sides of the triangle ?*

## WORKSHEET

**( 1 )** *In the figure BC = 2 cm , < B = 100° , < C = 50°*

**a )** *What is the measure of < BAC ?*

**b)** *What is the radius of the circle ?*

**c)** *What is the measure of < APC ?*

**d)** *What is the length of AC ?*

( *hint :* sin 50⁰ = 0.76 , sin 80⁰ = 0.98 )

*(figure: circle with inscribed points A, P, B, C; angle at B = 100°, angle at C = 50, BC = 2 cm)*

**( 2 )** *In the figure QR = 5 $\sqrt{2}$ cm*

*< Q = 25⁰ , < R = 20⁰*

**a )** *What is the measure of < P ?*

**b)** *What is the circumradius of the triangle PQR ?*

**c)** *What is the length of PQ ?*

**d)** *What is the length of PR ?* ( *hint :* sin 20⁰ = 0.34 , sin 25⁰ = 0.42 )

*(figure: triangle PQR with angle Q = 25°, angle R = 20°, QR = 5√2 cm)*
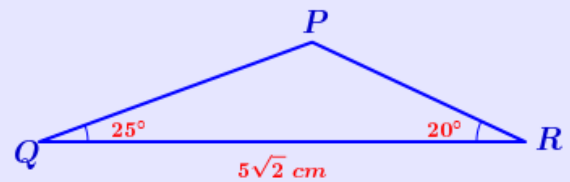
**( 3 )** *In the figure YZ = 9 cm , XY = XZ , < Y = 30⁰*

**a )** *What is the measure of < Z ?*

**b)** *What is the measure of < X ?*

**c)** *What is the circumradius of the triangle XYZ ?*

**d)** *What is the length of XZ ?*

**e)** *What is the perimeter of the triangle XYZ ?*

*(figure: triangle XYZ with angle Y = 30°, YZ = 9 cm)*
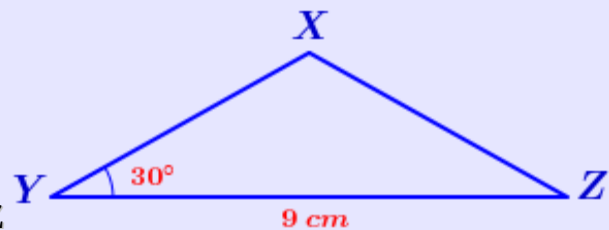
**( 4 )** *In a circle of radius 4 cm ,*

**a )** *What is the length of the chord of central angle 60° ?*

**b)** *What is the length of the chord of central angle 120° ?*

**c)** *Is to possible to draw a chord of length 2 cm having central angle 30⁰ in this circle ?*

*Why ?* ( *hint :* sin 15⁰ = 0.26 )

SSAARRAATTHH ..AA ..SS ,,

HHSSTT .. GGHHSS AANNCCHHAACCHHAAVVAADDII