

## Ratio & Proportion, Mixture & Allegations Pre Class Notes

Ratio of two quantities is the fraction that one quantity is relative to the other. Ratio of a to b is  $\frac{a}{b}$  or a : b.

'a', the numerator is called **antecedent** & the denominator 'b' is called **consequent**. It should be noted that both 'a' & 'b' must be of same unit.

**Eg-** In a certain class, the ratio of passing grades to failing grades is 7 to 5. How many of the 36 students failed the course?

The ratio, "7 to 5" (or 7 : 5 or  $\frac{7}{5}$ ), tells you that, of every 7 + 5 = 12 students, five failed. That is,  $\frac{5}{12}$  of the class flunked. Then  $(\frac{5}{12})(36) = \mathbf{15 \text{ students failed.}}$

### Comparison Ratio:

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

**Example:** Are the ratios 3 to 4 and 6:8 equal?

The ratios are equal if  $\frac{3}{4} = \frac{6}{8}$ .

These are equal if their cross products are equal; that is, if  $3 \times 8 = 4 \times 6$ .

Since both of these products equal 24, the answer is yes, the ratios are equal.

### Important Notes:

If a ratio  $\frac{a}{b}$  is given such that  $\frac{a}{b} > 1$ , or  $a > b$  & there is an integer x such that x is positive, then

$$\frac{a+x}{b+x} < \frac{a}{b} \text{ \& \> } \frac{a-x}{b-x} > \frac{a}{b}$$

If a ratio  $\frac{a}{b}$  is given such that  $\frac{a}{b} < 1$ , or  $a < b$ ,

$$\text{Then } \frac{a+x}{b+x} > \frac{a}{b} \text{ \& \> } \frac{a-x}{b-x} < \frac{a}{b}$$

- Compounded Ratio** of two ratios a/b and c/d is ac/bd, i.e., ac : bd.
- Duplicate ratio** of a : b is  $a^2 : b^2$
- Triplicate ratio** of a : b is  $a^3 : b^3$
- Sub-duplicate ratio** of a : b is  $\sqrt{a} : \sqrt{b}$
- Sub-triplicate ratio** of a : b is  $\sqrt[3]{a} : \sqrt[3]{b}$
- Reciprocal ratio** of a : b is b : a

### Proportion:-

A proportion is an equality that exists between two ratios. Eg- 1:4 :: 2:8 is a proportional consisting of four terms. The first & the last are called extremes, & the second & third are called the means.

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### Direct proportion:

Two variables are directly proportional if their corresponding values have constant ratio. If one quantity is multiplied or divide by the same number, the ratio of the variable remain unchanged.

Eg-

Price of 1 gallon of gas = \$2 , Price of 10 gallon of gas = \$20

Gallons of gas increases by a factor of 10 so does price

Hence, the price of gas and gallons of gas are in direct proportion

### Inverse Proportional

Two variables are inversely proportional, if they are so related that an increase by multiplication in one variable results in corresponding decrease in the other, & a decrease by division in one variable results in a corresponding increase in the other.

Eg- Speed of a car is inversely proportional to time taken

### Notes:

- a. **Invertendo.** If  $a : b :: c : d$  then  $b : a :: d : c$
- b. **Alternendo.** If  $a : b :: c : d$  then  $a : c :: b : d$
- c. **Componendo.** If  $a : b :: c : d$  then  $(a + b) : b :: (c + d) : d$
- d. **Dividendo.** If  $a : b :: c : d$  then  $(a - b) : b :: (c - d) : d$
- e. **Componendo and dividendo.**

If  $a : b :: c : d$  then  $(a + b) : (a - b) :: (c + d) : (c - d)$

i.e.,  $a/b = c/d \Rightarrow (a + b)/(a - b) = (c + d)/(c - d)$

### Alligation:-

The world allegation means 'Linking'. & states that when different quantities of same or different ingredients of different value are mixed together to produced a mixture of a mean value, the ratio of their quantities is in inversely proportional to the differences in their value from the mean value. This is also applicable in case of their cost prices.

### Rule of Allegation:

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of Dearer}} = \frac{\text{C.P of dearer} - \text{Mean Price}}{\text{Mean Price} - \text{C.P of Cheaper}}$$

Eg- A shopkeeper mixes two varieties of sugar costing Rs 25/Kg and Rs 20/Kg in a certain ratio such that the cost of the mixture is Rs 23/Kg then find the ratio in which the 2 types of sugar were mixed?

Let the ratio in which sugar of rs 25/Kg and Rs 20/Kg be p:q then total cost of mixture =

$$\frac{25p + 20q}{p + q} = 23$$

Solving for p, q we get  $\frac{p}{q} = \frac{23 - 20}{25 - 23}$ . This is called allegation

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### Removal By Equal Amount:

Consider a vessel containing only ingredients 'X' of 'a' unit. From this 'a' is taken out & replaced by an equal amount of ingredient 'Y'. This process is repeated 'n' times, then after 'n' operations

$$\frac{\text{Amount Of X left}}{\text{Amount of X originally Present}} = \left(\frac{a-b}{a}\right)^n$$

**Note:** The final volume of mixture remains contents and is same as initial volume same as initial of liquid X