MALAPPURAM EDUCATIONAL DISTRICT
EM_1.05
MATHEMATICS
Class - X
Chapter-1
ARITHMETIC SEQUENCES

## SUMS

Previous Knowledge: - In any arithmetic sequence the sum of each pair of terms equidistant from both ends is equal. Or Sum of any number of terms $=$ Number of pair sum $x$ Pair Sum.

## Sum of first ' $n$ ' natural numbers

Sum of first ' $n$ ' natural numbers,

$$
\begin{aligned}
& \mathbf{S}_{\mathrm{n}}=1+2+3+4+--------------------+(\mathrm{n}-1)+\mathrm{n} \\
& \mathbf{S}_{\mathrm{n}}=\frac{n(n+1)}{2} \quad \begin{array}{l}
\text { (because number of pair sum }=n / 2 \\
\text { Pair Sum }=n+1 \text { ) }
\end{array} \\
& \mathbf{S}_{\mathrm{n}}=\frac{n(n+1)}{2}
\end{aligned}
$$

Eg: - Sum of first 100 natural numbers $=\frac{100 \times 101}{2}=5050$

## Sum of first ' $n$ ' even numbers

Sum of first ' n ' even numbers, $\quad \mathbf{S}_{\mathbf{n}}=2+4+6+$ $\qquad$ $+2 n$

$$
\begin{aligned}
& S_{\mathrm{n}}=2(1+2+3+4+-\cdots-\cdots-\cdots+\cdots)=2 n(n+1) / 2 \\
& S_{n}=n(n+1)
\end{aligned}
$$

Eg: - Sum of first 20 even numbers $=20 \times 21=420$

## Sum of first ' $n$ ' odd numbers

Sum of first ' $n$ ' odd numbers, $\mathbf{S}_{\mathbf{n}}=1+3+5+7+$ $\qquad$ $+(2 \mathrm{n}-1)$

$$
\begin{array}{ll}
\mathbf{S}_{\mathbf{n}}=n / 2(1+2 n-1) & \begin{array}{l}
\text { (because number of pair sum }=n / 2, \\
\text { Pair Sum }=1+2 n-1=2 n)
\end{array} \\
\mathbf{S}_{\mathbf{n}}=n / 2 & \\
\mathbf{S}_{\mathbf{n}}=\mathbf{n}^{2} & 2 n)
\end{array}
$$

Eg: - Sum of first 50 odd numbers $=50^{2}=2500$

## Sum of first ' n ' terms of an Arithmetic Sequence

Sum of first ' $n$ ' terms of an Arithmetic Sequence,

$$
\begin{aligned}
& \mathbf{S}_{\mathbf{n}}=\mathbf{X}_{\mathbf{1}}+\mathbf{X}_{\mathbf{2}}+\mathbf{X}_{\mathbf{3}}+\mathbf{X}_{\mathbf{4}}+\ldots-------+\mathbf{X}_{\mathbf{n}} \\
& \mathbf{S}_{\mathbf{n}}=n / 2(X 1+X n) \quad \begin{array}{l}
\text { (because number of pair sum }=n / 2 \text { and } \\
\text { Pair Sum } \left.=X_{1}+X_{n}\right)
\end{array} \\
& S=\frac{\text { Number of terms }}{2} \times(\text { First term }+ \text { Last term })
\end{aligned}
$$

Eg: - Sum of first 40 of 5, 8, 11, -------- $=40 / 2\left(X_{1}+X_{40}\right)=20(5+122)=\mathbf{2 5 4 0}$

## Another method of finding Sum

General form of an arithmetic sequence is $\mathbf{X}_{\mathbf{n}}=\mathbf{a n}+\mathbf{b}$
for $x_{1}=a+b, x_{2}=2 a+b, x_{3}=3 a+b, x_{4}=4 a+b,-----,------,-----, x_{n}=a n+b$ then


$$
\begin{aligned}
& \mathbf{S}_{\mathrm{n}}=(\mathrm{a}+2 \mathrm{a}+3 \mathrm{a}+\cdots---+\mathrm{na})+\left(\mathrm{b}+\mathrm{b}+\mathrm{b}+\mathrm{b}+\ldots . . \mathrm{n}^{\prime} \text { times }\right) \\
& \mathbf{S}_{\mathrm{n}}=\mathrm{a}(1+2+3+4+\cdots+\mathrm{n})+\mathrm{nb} \\
& \mathbf{S}_{\mathrm{n}}=a \frac{n(n+1)}{2}+n b
\end{aligned}
$$

General form of the Sum
$\mathbf{S}_{\mathrm{n}}=n / 2(X 1+X n)$
If $\mathrm{X}_{1}=\mathrm{f}$ and common difference is ' d ',

$$
\begin{aligned}
& \mathbf{S}_{\mathbf{n}}=n / 2[\mathrm{f}+\mathrm{f}+(\mathrm{n}-1) \mathrm{d}] \\
& \mathbf{S}_{\mathbf{n}}=n / 2(2 \mathrm{f}+\mathrm{dn}-\mathrm{d}) \\
& \mathbf{S}_{\mathbf{n}}=\mathrm{fn}+d / 2 \mathrm{n}^{2}+d / 2^{n} \\
& \mathbf{S}_{\mathbf{n}}=d / 2 n^{2}+\left(f-\frac{d}{2}\right) n
\end{aligned}
$$

Algebra of Sum of an arithmetic sequence is a second degree polynomial in the form of $a^{2}+{ }^{2}$ bn
The coefficient of $\boldsymbol{n}^{2}$ in the algebraic form is half the common difference of the sequence
The sum of the coefficients in algebraic form is the first term of the Arithmetic Sequence.

## WORKSHEET 1.05

1) In the following table find the sum by suitably filling the columns

| Arithmetic Sequence | No of terms (n) | Formula used | Sum (Sn) |
| :---: | :---: | :---: | :---: |
| $1+2+3+4+--------------+200$ | 200 | $\frac{n(n+1)}{2}$ | $(200 \times 201) / 2=20100$ |
| $2+4+6+$---------------------+200 | --- | $n(n+1)$ | --- |
| $1+3+5+-----------------+199$ | --- | $n^{2}$ | --- |
| $11+21+31+----------------+201$ | --- |  | --- |

2. Write the algebra of sum of the sequences by completing the table. Also find the sum of given number of terms.

| Arithmetic Sequence | Common Difference (d) | First Term (f) | d/2 | f-d/2 | Algebra of Sum $\mathbf{S n}=\mathbf{d} / \mathbf{2} \mathbf{n}^{2}+(\mathbf{f}-\mathbf{d} / \mathbf{2}) \mathbf{n}$ | Sum of given number of terms (Sn) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7,11,15,19,..... | 4 | 7 | 2 | 3 | $2 n^{2}+3 n$ | $\begin{gathered} \mathrm{S}_{10}=2 \times 10^{2}+3 \times 10 \\ =230 \end{gathered}$ |
| 10,16,22,28,..... | $\ldots$ | $\ldots$ | .... | $\ldots$ | .... | $\mathrm{S}_{15}=\ldots . . . .$. |
| 1/2, 1, 11/2, 2,..... | .... | $\ldots$ | .... | $\ldots$ | $\ldots$ | $\mathrm{S}_{20}=\ldots . . . .$. |
| 100,90,80,70,....... | .... | $\ldots$ | $\cdots$ | $\ldots$ | .... | $\mathrm{S}_{30}=\ldots . . . .$. |

3. In the following table, algebra of sum of some terms of Arithmetic Sequences are given. Write the algebra of sequences by completing the table. Also compute the required terms in the last column.

| Algebra of Sum (Sn) | Common Differnece (d) | Firts Term (f) | Algebra of Sequence (Xn) | Term required |
| :---: | :---: | :---: | :---: | :---: |
| $5 n^{2}+3 n$ | 5 x 2 | $5+3=8$ | 10n-2 | $\mathrm{X}_{15}=10 \times 15--2=148$ |
| $3 n^{2}+2 n$ | .... | .... |  | $\mathrm{X}_{25}=.-\mathrm{-}-\mathrm{-}-\mathrm{-}-{ }^{\text {- }}$ |
| $1 / 2 \mathbf{n}^{2}+1 / 2 n$ | .... | .... |  | $\mathbf{x}_{30}=.--------{ }^{\text {- }}$ |
| $\mathbf{n}^{2}-3 \mathrm{n}$ | $\ldots$ | $\ldots$ |  |  |

4. Find the sum of all terms of the following sequences
a) $\mathbf{5 , 8 , 1 1}$ ,200
b) $\mathbf{1 0 0 , 9 5 , 9 0}$, ,0
c) $1,11 / 2,2,21 / 2$, 100
d) $\mathbf{- 1 0 , - 1 5 , - 2 0 ,}$ ,-100
5. Prove that 9 added to the sum of any number of consecutive terms of $7,9,11,13,--$ from the beginning is a prefect square.

