SUMS-NOTE-2

PREVIOUS KNOWLADGE

- **Sequence**: A set of numbers by a law written as the first, second, third and so on.
- **Arithmetic sequence:** A sequence got by starting a fixed Number and adding or subtracting a fixed number repeatedly.
- **Common difference (d):** The constant difference got by subtracting from any term the just previous term is called the common difference of an arithmetic Sequence.
- $x_1, x_2, x_3, x_4, x_5, x_6, \ldots$ Are the terms of an arithmetic sequence and suffix denote position
- An arithmetic sequence with first term f and common difference d. the algebraic from for the arithmetic sequence is $x_n = dn + (f - d)$
- The algebraic from for the arithmetic sequence is $x_n = an + b$

  First term =f= $a+b$ common difference =d= a
- Sum of first n natural numbers

  $$1+2+3+ \ldots\ldots\ldots\ldots + n = \frac{n\,(n+1)}{2}$$

SUMS

- Sum of n terms of an arithmetic sequence

  Take an arithmetic sequence $x_n = an + b$

  Where $x_1 = a + b$

  $X_2 = 2a + b$

  $X_3 = 3a + b$

  ………………

  ………………

  Sum of n terms = $S_n = x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots + x_n$

  $S_n = (a+b) + (2a+b) + (3a+b) + \ldots\ldots\ldots\ldots + (na+b)$

  $S_n = (a+2a+3a+ \ldots\ldots +na) + (b+b+b+ \ldots\ldots +b)$

  (Take a outside) (Here adding b, n times)

  $S_n = a(1+2+3+ \ldots\ldots +n) + nb$

  (Sum of n natural numbers)

  $S_n = a\left(\frac{n\,(n+1)}{2}\right) + nb$

  From this we can say that

  Sum of n terms of an arithmetic sequence $x_n = an + b$ is

  $$S_n = a\left(\frac{n\,(n+1)}{2}\right) + nb$$

E.g.: find the sum of first 20 terms of an arithmetic sequence 11, 16, 21 …

The algebra of the arithmetic sequence is = $x_n = dn + (f - d)$

$X_n = 5n + (11-5)$

$= 5n + 6$

Here a = 5 and b = 6 then sum of 20 terms

$S_n = a\left(\frac{n(n+1)}{2}\right) + nb$

$S_{20} = 5\left(\frac{20(20+1)}{2}\right) + 20 \times 6$

$S_{20} = 5\left(\frac{420}{2}\right) + 120$

$S_{20} = 5(210) + 120 = 1050 + 120 = 1170$

$S_{20} = 1170$

E.g.: find the sum of first 10 terms of an arithmetic sequence 10, 13, 16 …

The algebra of the arithmetic sequence is = $x_n = dn + (f - d)$

$X_n = 3n + (10 - 3)$

$= \underline{\underline{3n + 7}}$

Here a = 3 and b = 7 then sum of 10 terms

$S_n = a\left(\frac{n(n+1)}{2}\right) + nb$

$S_{10} = 3\left(\frac{10(10+1)}{2}\right) + 10 \times 7$

$S_{10} = 3\left(\frac{110}{2}\right) + 70$

$S_{10} = 3(55) + 70 = 165 + 70 = 235$

$\underline{\underline{S_{10} = 235}}$

- Sum in another form

We have $S_n = a\left(\frac{n(n+1)}{2}\right) + nb$

$= \frac{1}{2}\, an(n+1) + nb$

$= \frac{an(n+1)}{2} + \frac{nb}{1} = \frac{an(n+1) + 2nb}{2} = \frac{n(a(n+1) + 2b)}{2}$

$= \frac{n(an + a + b + b)}{2} = \frac{n}{2}\left((an + b) + (a + b)\right)$

$S_n = \frac{n}{2}\,(x_n + x_1)$

$S_n = \frac{n}{2}\,(last\ term + first\ term)$

E.g.: find the sum first 20 terms of the arithmetic sequence 11, 16, 21 …

First term = x_1 = 11 last term = x_{20} = x_1 + (n-1) d

= 11+19x5 =11+95=106

The sum first 20 terms = $S_{20} = \frac{20}{2}\,(106+11)$

$S_{20} = 10 \times 117$

S_{20}= 1170

MORE EXAMPLES

1. Find the sum of first 25 terms of the arithmetic sequence 1, 4, 7 ………….

   First term = x_1 = 1 25^{th} term = x_{25} = x_1 + (n-1) d

   x_{25} = 1 + 24 x 3 =1+72 = 73

   The sum first 25 terms = $S_{25} = \frac{25}{2}\,(x_{25}+x_1)$

   $S_{25} = \frac{25}{2}(73+1)$

   $S_{25} = \frac{25}{2}(74)$

   S_{25} =25 x 37 =925

   S_{25} = 925

2. Look at the pattern given below

1				1^{st} line 1 term & last term 1
2	3			2^{nd} line 2 terms & last term 3 (1+2)
4	5	6		3^{rd} line 3 terms & last term 6 (1+2+3)
7	8	9	10	4^{th} line 4 terms & last term 10 (1+2+3+4)

……………………………………

a) The number of terms in 20^{th} line is 20

   Last term of the 20^{th} line is 1+2+3+……+20= $\frac{20(20+1)}{2}$ = 210

b) The number of terms in n^{th} line is n

   Last term of the n^{th} line is 1+2+3+……+n= $\frac{n(n+1)}{2}$

c) First term of the 20^{th} line = last term of 20^{th} line – 19x1= 210 -19 =191

   First term of the n^{th} line = last term of n^{th} line – (n-1) x d

d) Sum of terms in 20^{th} line = $S_{20} = \frac{\cancel{20}}{\cancel{2}}\,(210+191)$=10 x 401=4010

   Sum of terms in n^{th} line = $S_n = \frac{n}{2}$ (last term of n^{th} line + First term of the n^{th} line)

e) Sum of all terms up to 20^{th} line =1+2+3+…..+210 = $\frac{\cancel{210}(210+1)}{\cancel{2}}$ =105x211=22155

   Sum of all terms up to n^{th} line = 1+2+3+ …….. + $\frac{n(n+1)}{2}$

MORE QUESTIONS TO PRACTICE

1. Find the sum of the first 25 terms of each of the arithmetic sequences below.

   i) 11, 22, 33, ... ii) 12, 23, 34, ... iii) 21, 32, 43, ...
   iv) 19, 28, 37, ... v) 1, 6, 11, ...

   Refer more example 1

2. What is the difference between the sum of the first 20 terms and the next 20 terms of the arithmetic sequence 6, 10, 14, ...?

3. Calculate the difference between the sums of the first 20 terms of the arithmetic sequences 6, 10, 14, ... and 15, 19, 23, ...

Click here and watch the video class for better understanding of the problems and concepts

4. Find the sum of all three digit numbers, which are multiples of 9.

5.
   1
   2 3
   4 5 6
   7 8 9 10
   ....
   ....

   (i) Write the next two lines of the pattern above
   (ii) Write the first and the last numbers of the 10th line
   (iii) Find the sum of all the numbers in the first ten lines.

   Do this problem as shown in the above example

6. Find the sum of first 100 terms of arithmetic sequence
   a) 1, 4, 7 ....
   b) 4, 10, 16 ....