

## ALGEBRA OF ARITHMETIC SEQUENCES

- **SEQUENCE:** A set of numbers by a law written as the first, second, third and so on.
- **ARITHMETIC SEQUENCE:** A sequence got by starting a fixed Number and adding or subtracting a fixed number repeatedly.
- **COMMON DIFFERENCE (d):** The constant difference got by subtracting from any term the just previous term is called the common difference of an arithmetic Sequence.
- $x_1, x_2, x_3, x_4, x_5, x_6, \dots$  Are the terms of an arithmetic sequence and suffix denote position
- $\text{common difference} = \frac{\text{Term Difference}}{\text{positional difference}}$

- Consider an arithmetic sequence

10, 13, 16, 19, 22, 25, 28.....

Here

$$x_1 = f = 10$$

$$x_2 = f + d = 10 + 3 = 13$$

$$x_3 = f + 2d = 10 + 2 \times 3 = 16$$

$$x_4 = f + 3d = 10 + 3 \times 3 = 19$$

$$x_{20} = f + 19d = 10 + 19 \times 3 = 67$$

if continue this  $n^{\text{th}}$  term of this arithmetic sequence

$$x_n = f + (n-1)d = 10 + (n-1) \times 3$$
$$= 10 + 3n - 3$$

$$x_n = 3n + 7$$

From this we can say that the  $n^{\text{th}}$  term of this arithmetic sequence is

$$x_n = 3n + 7$$

This is also known as the algebra of the sequence

- From the above sequence we can see that the  **$n^{\text{th}}$  term or algebra** of any arithmetic sequence with first term is  $f$  and common difference  $d$  is

$$x_n = f + (n-1)d$$

$$x_n = f + dn - d$$

$$x_n = dn + (f - d)$$

E.g.: find the  $n^{\text{th}}$  term of this arithmetic sequence 1, 5, 9, 13.....

$$F=1 \quad d=4$$

$$x_n = dn + (f - d)$$

$$x_n = 4n + (1 - 4)$$

$$= 4n + (-3)$$

$$X_n = 4n - 3$$

➤ If the  $n^{\text{th}}$  term of any arithmetic sequence is  $X_n = an + b$  then

First term =  $a + b$     common difference =  $a$

E.g.: the  $n^{\text{th}}$  term of an arithmetic sequence is  $X_n = 5n + 3$ . what is the sequence

First term =  $f = 5 + 3 = 8$     common difference = 5

The arithmetic sequence is 8, 13, 18, 23, 28.....

#### MORE QUESTIONS TO PRACTICE

1.

The 8<sup>th</sup> term of an arithmetic sequence is 12 and its 12<sup>th</sup> term is 8.  
What is the algebraic expression for this sequence?

Common difference is .....use(  $common\ difference = \frac{Term\ Difference}{positional\ difference}$  )

First term .....

Algebra of the sequence is ..... Use ( $x_n = dn + (f - d)$ )

2.

Prove that the arithmetic sequence with first term  $\frac{1}{3}$  and common difference  $\frac{1}{6}$  contains all natural numbers.

Algebra of the sequence is ..... Use ( $x_n = dn + (f - d)$ )

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3.

Prove that the arithmetic sequence with first term  $\frac{1}{3}$  and common difference  $\frac{2}{3}$  contains all odd numbers, but no even number.

Algebra of the sequence is ..... Use ( $x_n = dn + (f - d)$ )

4.

Prove that the squares of all the terms of the arithmetic sequence 4, 7, 10, ... belong to the sequence.

Common difference is .....

First term .....

Algebra of the sequence is ..... Use  $(x_n = dn + (f - d))$

Square the algebraic equation.....

5.

Prove that the arithmetic sequence 5, 8, 11, ... contains no perfect squares.

Common difference is .....

First term .....

Algebra of the sequence is ..... Use  $(x_n = dn + (f - d))$

Square the algebraic equation.....

6.

Write the whole numbers in the arithmetic sequence  $\frac{11}{8}, \frac{14}{8}, \frac{17}{8}, \dots$

Do they form an arithmetic sequence?

Common difference is .....

First term .....

Algebra of the sequence is ..... Use  $(x_n = dn + (f - d))$