STD - X

MATHEMATICS

Class - 04

1) In each of the arithmetic sequence below, some terms are missing and their positions are marked with ___ . Find them.

i) 24, 42, 60, 78, . . .

Ans) $x_1 = 24$

$d = x_2 - x_1$

$= 42 - 24$

$= 18$

$\therefore x_3 = 60$

$x_4 = 78$

ii) 6, 24, 42, 60, . . .

$x_2 = 24$

$x_3 = 42$

$\therefore d = 42 - 24$

$= 18$

$x_1 = 24 - 18$

$= 6$

$x_4 = 42 + 18$

$= 60$

iii) -12, 6, 24, 42, . . .

$x_3 = 24$

$x_4 = 42$

$\therefore d = 18$

$x_2 = 6$

$x_1 = 6 - 18$

$= -12$

iv) 24, 33, 42, 51, ...

Ans) $x_1 = 24$

$x_3 = 42$

$\therefore x_2 = 24 + 9$

$= \underline{33}$

$x_4 = 42 + 9$

$= \underline{51}$

$\therefore d = \frac{\text{term difference}}{\text{position difference}}$

$= \frac{42 - 24}{3 - 1}$

$= \frac{18}{2}$

$d = \underline{9}$

v) 15, 24, 33, 42, ...

Ans) $x_2 = 24$

$x_4 = 42$

$x_1 = 24 - 9$

$= \underline{15}$

$x_3 = 42 - 9$

$= \underline{33}$

$\therefore d = \frac{42 - 24}{4 - 2}$

$= \frac{18}{2}$

$d = \underline{9}$

vi) 24, 30, 36, 42, ...

$x_1 = 24$

$x_4 = 42$

$\therefore x_2 = 24 + 6$

$= \underline{30}$

$x_3 = 30 + 6 = \underline{36}$

$d = \frac{42 - 24}{4 - 1}$

$= \frac{18}{3} = \underline{6}$

2. The terms in two positions of some arithmetic sequences are given below. Write the first five terms of each:

i) 3rd term 34
6th term 67

Ans) $x_3 = 34$

$x_6 = 67$

$\therefore d = \frac{67 - 34}{6 - 3}$

$= \frac{33}{3}$

$d = 11$

$\therefore x_2 = 34 - 11$

$= 23$

$x_1 = 23 - 11$

$= 12$

$\therefore$ sequence = 12, 23, 34, 45, 56, ...

ii) 3rd term 43
6th term 76

Ans) $x_3 = 43$

$x_6 = 76$

$\therefore d = \frac{76 - 43}{6 - 3} = \frac{33}{3} = 11$

$\therefore x_2 = 43 - 11$

$= 32$

$x_1 = 32 - 11 = 21$

$\therefore$ Sequence = 21, 32, 43, 54, 65 ...

iii) 3rd term 2
5th term 3

Ans) $x_3 = 2$
$x_5 = 3$

$d = \frac{3-2}{5-3} = \frac{1}{2} = 0.5$

$\therefore x_2 = 2 - 0.5$
$= 1.5$

$x_1 = 1.5 - 0.5$
$= 1$

$\therefore$ sequence = 1, 1.5, 2, 2.5, 3, 3.5, ...

iv) 4th term 2
7th term 3

Ans) $x_4 = 2$
$x_7 = 3$

$\therefore d = \frac{3-2}{7-4} = \frac{1}{3}$

$\therefore x_1 = 2 - 3 \times \frac{1}{3}$
$= 2 - 1$
$x_1 = 1$

$\therefore$ sequence = $1, 1\frac{1}{3}, 1\frac{2}{3}, 2, 2\frac{1}{3}, \ldots$

v) 2nd term 5
5th term 2

Ans) $x_2 = 5$
$x_5 = 2$

$d = \frac{2-5}{5-2} = \frac{-3}{3} = -1$

$\therefore x_1 = 5 - -1 = 6$

$\therefore$ Sequence = 6, 5, 4, 3, 2, 1, ...

3. The 5th term of an arithmetic sequence is 38 and the 9th term is 66. What is its 25th term?

Ans) $x_5 = 38$, $x_9 = 66$ } Given

$\therefore d = \frac{66-38}{9-5} = \frac{28}{4} = \underline{7}$

$\therefore x_{25} = x_5 + 20d$

$= 38 + 20 \times 7$

$= 38 + 140$

25th term = $\underline{178}$

4) Is 101 a term of the arithmetic sequence 13, 24, 35, ...? What about 1001?

Ans) 13, 24, 35, ...

$x_1 = 13$

$d = 24 - 13 = \underline{11}$

Difference between 101 and the first term = $101 - 13 = \underline{88}$

88 is the multiple of 11 ($88 \div 11 = 8$)

So 101 is a term of this sequence

$1001 - 101 = 900$

This difference is not a multiple of 11.

So 1001 is not a term of this sequence.

5) How many three-digit numbers are there, which leave a remainder 3 on division by 7?

Ans) 3-digit numbers are = 100, 101, 102, ... 999

3-digit numbers which leave a remainder 3 on division by 7

= 101, 108, 115, ... - 997

No. of terms, $n = \frac{\text{last term} - \text{first term}}{\text{common difference}} + 1$

$= \frac{997 - 101}{7} + 1$

$= 128 + 1$

$n = 129$

$$7 \overline{)100}\ 14,\ 7,\ 30,\ 28,\ 2$$

$$7 \overline{)101}\ 14,\ 7,\ 31,\ 28,\ 3$$

$$7 \overline{)999}\ 142,\ 7,\ 29,\ 28,\ 19,\ 14,\ 5$$

$$7 \overline{)998}\ 142,\ 7,\ 29,\ 28,\ 18,\ 14,\ 4$$