# CCE PR <br> UNREVISED <br> D <br>  KARNATAKA SECONDARY EDUCATION EXAMINATION BOARD, MALLESWARAM, BANGALORE - 560003 


S. S. L. C. EXAMINATION, JUNE, 2019

యృదరి లుత్రేగిక
MODEL ANSWERS

దినృంళ : 21. 06. 2019 ]
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## ఎిజయు : గణిక

## Subject : MATHEMATICS

( ळళి జఠ్యృひుము / Old Syllabus )

(ఇంగ్లిజ్ భలఱ్తంతర / English Version )

[ Max. Marks : 100

| Qn. <br> Nos. | Ans. <br> Key | Value Points | Marks <br> allotted |
| :---: | :---: | :---: | :---: |
| I. 1. |  | If $A$ and $B$ are two non-empty subsets of a universal set, then <br> De-Morgan's law is given by |  |
| (A) $(A \cup B)^{\prime}=A^{\prime} \cup B^{\prime}$ (B) $(A \cup B)^{\prime}=A^{\prime} \cap B^{\prime}$  <br> (C) $(A \cap B)^{\prime}=A^{\prime} \cap B^{\prime}$ (D) $(A \cup B)^{\prime}=(A \cap B)^{\prime}$  <br> Ans. :   <br> $(A \cup B)^{\prime}=A^{\prime} \cap B^{\prime}$  1 |  |  |  |


| Qn. <br> Nos. | Ans. <br> Key | Value Points |  |
| ---: | :--- | :--- | :---: |
| 2. |  | The value of ${ }^{n} C_{0} \times{ }^{n} C_{1}$ is |  |
|  |  | (B) $n$ |  |
|  |  | (C) $n!$ |  |

Ans. :
(B) $n$
3.
4.
5.
(C) 40

A quadratic equation whose roots are $3+2 \sqrt{5}$ and $3-2 \sqrt{5}$ is
(A) $x^{2}-6 x-11=0$
(B) $x^{2}+6 x-11=0$
(C) $x^{2}+6 x+11=0$
(D) $x^{2}-11 x+6=0$

Ans. :
(A) $x^{2}-6 x-11=0$

| Qn. <br> Nos. | Ans. <br> Key | Value Points | Marks <br> allotted |
| :---: | :---: | :---: | :---: |
| 6. | (A) | If $\tan A=\frac{3}{4}$ then $\sin A$ is <br> (A) $\frac{3}{5}$ <br> (B) $\frac{4}{3}$ <br> (C) $\frac{4}{5}$ <br> (D) $\frac{5}{3}$ <br> Ans. : $\frac{3}{5}$ | 1 |

7. 



The distance between the origin and point $(x, y)$ is
(A) $\sqrt{x^{2}-y^{2}}$
(B) $\sqrt{(x+y)^{2}}$
(C) $\sqrt{(x-y)^{2}}$
(D) $\sqrt{x^{2}+y^{2}}$

Ans. :
(D) $\sqrt{x^{2}+y^{2}}$

If $P$ is the mid-point of the line joining $A(1,4)$ and $B(3,6)$ then the co-ordinates of $P$ is
(A) $(4,10)$
(B) $(2,10)$
(C) $(2,5)$
(D) $(4,5)$

Ans. :
(C) $(2,5)$

| Qn. <br> Nos. | Value Points | Marks <br> allotted |
| :--- | :--- | :---: |
| II. | Answer the following: <br> (Question Numbers 9 to 14, give full marks to direct answers ) |  |
| Write the formula to find the Harmonic mean between two positive <br> integers $a$ and $b$. <br> Ans. : <br> Harmonic Mean $=\frac{2 a b}{a+b}$ | 1 |  |

10. State Euclid's Division Lemma.

Ans. :
Given positive integers $a$ and $b$ there exist unique integers $q$ and $r$
satisfying $a=b q+r, \quad 0 \leq r<b$.
12. In the figure, $P A$ and $P B$ are the tangents to the circle with centre $O$ and $\left\lfloor A P B=80^{\circ}\right.$. Find $\lfloor A O P$.


Ans. :

$$
\begin{aligned}
\lfloor A O B & =180^{\circ}-80^{\circ} \\
& =100^{\circ} \\
\lfloor A O P & =\frac{1}{2}\lfloor A O B \\
& =\frac{1}{2} \times 100^{\circ} \\
\lfloor A O P & =50^{\circ}
\end{aligned}
$$

| Qn. <br> Nos. | Value Points | Marks <br> allotted |
| :---: | :---: | :---: |

13. 

If the length of the diagonal of a square is $10 \sqrt{2} \mathrm{~cm}$, find the length of the side.

Ans. :

$A C^{2}=A B^{2}+B C^{2} \quad 1 / 2$
$(10 \sqrt{2})^{2}=A B^{2}+A B^{2}$
$200=2 A B^{2}$
$A B^{2}=\frac{200}{2}$
$A B^{2}=100$
$A B=10 \mathrm{~cm}$, Length of the side $=10 \mathrm{~cm}$
14. Write the formula to find the volume of the sphere whose radius is $r$ units.

Ans. :
Volume of sphere $=\frac{4}{3} \pi r^{3}$ cubic units
III. 15.

If $A=\{1,2,7\}$ and $B=\{5,7,12\}$ are two sets then verify
$A \cup B=B \cup A$.
Ans. :
$A=\{1,2,7\}, B=\{5,7,12\}$
$A \cup B=\{1,2,7\} \cup\{5,7,12\}$
$A \cup B=\{1,2,5,7,12\}$
... (i)
$B \cup A=\{5,7,12\} \cup\{1,2,7\}$
Qn.

Nos.

| Value Points |  |  |
| ---: | :--- | ---: |
|  | $\ldots$ (ii) | $1 / 2$ |

From (i) and (ii)
$A \cup B=B \cup A$

In a Harmonic progression 5th term is $\frac{1}{12}$ and 11 th term is $\frac{1}{15}$. Then find the 25th term.

Ans. :
$T_{5}=\frac{1}{12}$
$T_{11}=\frac{1}{15}$
$T_{25}=$ ?
Corresponding terms in A.P. will be
$T_{5}=12$
$T_{11}=15$
$d=\frac{T_{p}-T_{q}}{p-q}$
$=\frac{T_{5}-T_{11}}{5-11}$
$=\frac{12-15}{-6}$
$=\frac{-3}{-6}$

Marks allotted
16. Define Arithmetic progression. Write the general form of arithmetic progression.

Ans. :
An arithmetic progression is a sequence in which each term is obtained by adding a fixed number to the preceding term ( except the first term ).

The general form of AP is $a, a+d, a+2 d, a+3 d, \ldots \ldots \ldots$.

| Qn. <br> Nos. | Value Points | Marks <br> allotted |
| :---: | :---: | :---: |

$a=12-2$
$a=10$
Now $T_{n}=a+(n-1) d$
$T_{25}=a+24 d$
$=10+24\left(\frac{1}{2}\right)$
$=10+12$
$T_{25}=22$

Corresponding term in Harmonic Progression is

$$
T_{25}=\frac{1}{22}
$$

Alternate method:
In A.P. $T_{5}=12$

$$
T_{11}=15
$$

$$
T_{n}=a(n-1) d
$$

$\therefore \quad T_{5}=a+4 d$
i.e. $12=a+4 d$

Similarly $T_{11}=a+10 d$

$$
\begin{equation*}
15=a+10 d \tag{ii}
\end{equation*}
$$

| Qn. <br> Nos. | Value Points | Marks <br> allotted |
| :---: | :---: | :---: |

From (i)

$$
\begin{aligned}
& a+4 d=12 \\
& a+4\left(\frac{1}{2}\right)=12 \\
& a+2=12 \\
& a=12-2 \\
& a=10
\end{aligned}
$$

Now $T_{25}=a+24 d$

$$
\begin{aligned}
& =10+24\left(\frac{1}{2}\right) \\
& =10+12 \\
T_{25} & =22
\end{aligned}
$$

$\therefore \quad$ Corresponding term in H.P. is $T_{25}=\frac{1}{22}$

$$
d=\frac{1}{2}
$$

18. Prove that $5-\sqrt{3}$ is an irrational number.

Ans. :
Let us assume $5-\sqrt{3}$ is a rational number
$\Rightarrow \quad 5-\sqrt{3}=\frac{p}{q}$, where $p, q \in z, \quad q \neq 0$
$5-\frac{p}{q}=\sqrt{3}$
$\Rightarrow \quad \frac{5 q-p}{q}=\sqrt{3}$

| Qn. <br> Nos. | Value Points |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\Rightarrow$ | $\sqrt{3}$ is a rational number, $\because \frac{5 q-p}{q}$ is rational. | $1 / 2$ |

But we know that $\sqrt{3}$ is not a rational number.
This leads to contradiction.
$\therefore \quad$ Our assumption that $5-\sqrt{3}$ is a rational number is wrong.
$\Rightarrow \quad 5-\sqrt{3}$ is an irrational number.
19. Find, how many three-digit even numbers can be formed using the digits $3,5,7,8$ and 9 , without repeating any digit.

Ans. :

| Hundred place | Ten place | Unit place |
| :---: | :---: | :---: |
| 4 ways $\left[\begin{array}{c}o r \\ 4^{2}\end{array}\right]$ | 3 ways $\left[\begin{array}{c}o r \\ { }^{3} P_{1}\end{array}\right]$ | 1 way \{8] |

To form 3-digit even number, unit place can be filled only in one way i.e. by 8 .

Hundred place can be filled in 4 ways
Ten's place can be filled in 3 ways
$\therefore \quad$ By F.P.C., number of three digit even number that can be formed using the digits $3,5,7,8$ and 9 is
$=4 \times 3 \times 1$
or $\quad{ }^{4} P_{1} \times{ }^{3} P_{1} \times 1$
$=12$
$\therefore \quad$ Totally 12,3 digit even numbers can be formed.
20. There are eight teachers in a school, including headmaster. Find in how many ways, can a committee of 5 members be formed so as to include headmaster in the committee.

Ans. :

There are 8 teachers including Headmaster
A committee of 5 members is to be formed and headmaster is one of the members.
$\therefore \quad$ Only 4 members are to be selected from remaining 7 teachers. $1 / 2$
$\therefore \quad$ The possible number of such committees $=1 \times{ }^{7} C_{4}$

$$
\begin{aligned}
& =\frac{7 \times 6 \times 5 \times 4}{4!} \\
& =\frac{7 \times \not \subset \times 5 \times \not \subset}{A \times B \times Z \times 1} \\
& =35 \text { ways. }
\end{aligned}
$$

$\therefore \quad$ The committee can be formed in 35 ways.
500 lottery tickets are sold. Of these 5 tickets are allotted prizes. Sanjay purchased one lottery ticket. What is the probability that Sanjay gets lottery prize ?

Ans. :

500 lottery tickets are sold

$$
\therefore \quad n(S)=500 \quad 1 / 2
$$

Sanjay purchased 1 ticket.
Let $A$ be the event of Sanjay getting lottery prize.

$$
\begin{array}{ll} 
& \text { Then } n(A)={ }^{5} C_{1}=5 \\
\therefore \quad & P(A)=\frac{n(A)}{n(S)} \\
& P(A)=\frac{5}{500}
\end{array}
$$

$$
\text { OR } \quad P(A)=\frac{1}{100}
$$

$\therefore \quad$ The probability of Sanjay getting prize is $\frac{5}{500}$ or $\frac{1}{100}$

Rationalise the denominator and simplify $\frac{2}{\sqrt{5}-\sqrt{3}}$.
$2 \sqrt{a}+7 \sqrt{a}-3 \sqrt{a} \quad 1 / 2$
22.

Find the sum of $2 \sqrt{a}, 7 \sqrt{a},-3 \sqrt{a}$.
Ans. :
$=9 \sqrt{a}-3 \sqrt{a}$
$=6 \sqrt{a}$.

Ans. :

$$
\begin{align*}
& \frac{2}{\frac{2}{\sqrt{5}-\sqrt{3}}} \begin{aligned}
\frac{2}{\sqrt{5}-\sqrt{3}} & =\frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} \\
& =\frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^{2}-(\sqrt{3})^{2}} \quad \therefore \quad(a+b)(a-b)=a^{2}-b^{2} \\
& =\frac{2(\sqrt{5}+\sqrt{3})}{5-3} \\
& =\frac{22(\sqrt{5}+\sqrt{3})}{2} \\
& =\sqrt{5}+\sqrt{3}
\end{aligned} \quad 1 / 2 \\
&
\end{align*}
$$

24. 

Find the remainder obtained when $P(x)=x^{3}+3 x^{2}-5 x+8$ is divided by $g(x)=(x-1)$.

Ans. :
$P(x)=x^{3}+3 x^{2}-5 x+8, \quad g(x)=x-1$
By remainder theorem, remainder is $P(1)$

## Qn.

Value Points | Marks |
| :---: | :---: |
| allotted |

$P(x)=x^{3}+3 x^{2}-5 x+8$
$P(1)=1^{3}+3(1)^{2}-5(1)+8$

$$
=1+3-5+8
$$

$$
=12-5
$$

$P(1)=7$
$\therefore \quad$ The remainder is 7

$$
\begin{array}{ll}
x-1) & x^{3}+3 x^{2}-5 x+8\left(x^{2}+4 x-1\right. \\
& \\
& \begin{array}{l}
x^{3}-x^{2} \\
(-) \quad(+)
\end{array} \\
\begin{array}{ll}
4 x^{2}-5 x+8 \\
4 x^{2}-4 x \\
(-) \quad(+)
\end{array} & 1 / 2 \\
\frac{1+x+8}{(+)(-)} \\
&
\end{array}
$$

$\therefore \quad$ The remainder is 7 .
Alternate method:

Divide $3 x^{3}+11 x^{2}+34 x+106$ by $(x-3)$, using synthetic division and find the quotient and remainder.

| Qn. <br> Nos. | Value Points |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | If $(x-5)$ is a factor of $x^{3}-3 x^{2}+a x-10$ |  |  |  |  |
|  | 3 | 3 | 11 | 34 | 106 |
|  |  | $\downarrow$ | 9 | 60 | 282 |
|  |  | 3 | 20 | 94 | 388 |

$\therefore \quad$ The quotient is $3 x^{2}+20 x+94$ and the remainder is 388

## OR

$(x-5)$ is a factor of $P(x)=x^{3}-3 x^{2}+a x-10$
$\Rightarrow \quad P(5)=0$
Now $P(x)=x^{3}-3 x^{2}+a x-10$

$$
\begin{aligned}
& P(5)=5^{3}-3(5)^{2}+5 \cdot a-10 \\
& 0=125-75-5 \cdot a-10 \\
& 0=40+5 a \\
& \therefore \quad 5 a=-40 \\
& a=\frac{-40}{5} \\
& \therefore \quad a=-8 \\
& \\
& \therefore \quad a=-8
\end{aligned}
$$

Marks allotted

2

| Qn. | Value Points | Marks |
| :---: | :---: | :---: |
| Nos. | allotted |  |

26. Draw a chord $A B$ of length 5 cm in a circle of radius 3 cm . Construct a tangent at the point $B$.

Ans. :

$$
\begin{aligned}
& r=3 \mathrm{~cm} \\
& A B=5 \mathrm{~cm}
\end{aligned}
$$


$B Q$ is the required tangent

| Circle - | $1 / 2$ |
| :--- | :--- |
| Chord - | $1 / 2$ |

27. In the figure if $D E \| B C$ and $D P \| B E$ then prove that

$$
A E^{2}=A P . A C
$$



OR

| Qn. | Value Points | Marks <br> Nos. |
| :---: | :---: | :---: |

If the areas of two similar triangles are equal, then prove that they are
congruent.
Ans. :
Given: $D E \| B C$

$$
D P \| B E
$$

To prove : $A E^{2}=A P . A C$
Proof: $\quad \triangle A D P \sim \triangle A B E$

$$
\begin{array}{lll}
\because & \lfloor A=\lfloor A \\
& \text { and }\lfloor A D P \\
& \boxed{A B E} \text { as } D P \| B E \\
\therefore & \frac{A D}{A B}=\frac{A P}{A E} & \ldots \text { (i) } \quad \because \quad \text { Thales theorem }
\end{array}
$$

Similarly $\quad \triangle A D E \sim \triangle A B C$
$\therefore \quad\lfloor A=\lfloor A$

$$
\begin{array}{rlll} 
& \lfloor A D E & \lfloor A B C & \text { as } D E \| B C \\
\therefore & \frac{A D}{A B}=\frac{A E}{A C} & \ldots \text { (ii) } & \because \\
\text { Thales theorem } & 1 / 2
\end{array}
$$

From (i) and (ii)

$$
\begin{aligned}
& \frac{A P}{A E}=\frac{A E}{A C} \\
& A E^{2}=A P . A C
\end{aligned}
$$

Direct proof may be given full marks.

## OR

Let $\triangle A B C \sim \triangle D E F$
Given that $\operatorname{ar}(\triangle A B C)=\operatorname{ar}(\triangle D E F)$
To prove : $\triangle A B C \cong \triangle D E F$


Proof. $\triangle A B C \sim \triangle D E F$

$$
\begin{array}{ll}
\Rightarrow & \frac{\operatorname{ar}(\triangle A B C)}{\operatorname{ar}(\triangle D E F)}=\frac{B C^{2}}{E F^{2}} \\
\therefore \quad & \frac{\operatorname{ar}(\triangle A B C)}{\operatorname{ar}(\triangle A B C)}=\frac{B C^{2}}{E F^{2}} \\
& 1=\frac{B C^{2}}{E F^{2}} \\
\therefore \quad & B C^{2}=E F^{2} \\
\Rightarrow \quad B C=E F
\end{array}
$$

If $A=60^{\circ}, B=30^{\circ}$ then prove that $\cos (A+B)=\cos A \cdot \cos B-\sin A \cdot \sin B$.2

Ans. :
$A=60^{\circ}$
$B=30^{\circ}$
To prove : $\cos (A+B)=\cos A \cdot \cos B-\sin A \cdot \sin B$
Consider $\cos (A+B)$
$=\cos \left(60^{\circ}+30^{\circ}\right)$
$=\cos \left(90^{\circ}\right)$
$=0$

Now $\cos A . \cos B-\sin A \cdot \sin B$
$=\cos 60^{\circ} \cdot \cos 30^{\circ}-\sin 60^{\circ} \cdot \sin 30^{\circ}$
$=\frac{1}{2} \cdot \frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2} \cdot \frac{1}{2}$

| Qn. <br> Nos. | Value Points | Marks <br> allotted |
| :---: | :---: | :---: |

29. The distance between the points $(3,1)$ and $(0, x)$ is 5 units. Find $x$.

Ans. :
$(3,1) \Rightarrow\left(x_{1}, y_{1}\right)$
$(0, x) \Rightarrow\left(x_{2}, y_{2}\right)$
$d=5$ units
$d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
$5=\sqrt{(0-3)^{2}+(x-1)^{2}}$
$5=\sqrt{9+x^{2}+1-2 x}$
Squaring on both the sides

$$
\begin{array}{lll}
25= & 10+x^{2}-2 x & 1 / 2 \\
\text { i.e. } & x^{2}-2 x-15=0 \\
\therefore & x^{2}-5 x+3 x-15=0 \\
& x(x-5)+3(x-5)=0 \\
& (x-5)(x+3)=0 \\
x-5=0 \quad \text { or } \quad x+3=0 \\
x= & \text { or } \quad x=-3 \\
\therefore \quad & x=5 \quad \text { or } \quad x=-3
\end{array}
$$

| Qn. <br> Nos. | Value Points |  |  |
| :---: | :---: | :---: | :---: |
| 30. | Draw a plan using following information : <br> ( Scale $20 \mathrm{~m}=1 \mathrm{~cm}$ ) |  |  |
|  |  |  | To D ( in metres ) |

Ans. :
Scale : $20 \mathrm{~m}=1 \mathrm{~cm}$
$\therefore \quad 40 \mathrm{~m}=\frac{40}{20}=2 \mathrm{~cm}$
$120 \mathrm{~m}=\frac{120}{20}=6 \mathrm{~cm}$
$140 \mathrm{~m}=\frac{140}{20}=7 \mathrm{~cm}$
$200 \mathrm{~m}=\frac{200}{20}=10 \mathrm{~cm}$
$60 \mathrm{~m}=\frac{60}{20}=3 \mathrm{~cm}$
$30 \mathrm{~m}=\frac{30}{20}=1.5 \mathrm{~cm}$


| Qn. | Value Points | Marks <br> allotted |
| :---: | :---: | :---: |

Find the sum of $1+2+4+\ldots$ up to 10 terms.
Ans. :
$1+2+4+\ldots$ up to 10 terms
The terms are in Geometric progression
$a=1$
$r=\frac{T_{2}}{T_{1}}=\frac{2}{1}=2$
$n=10$
$S_{10}=$ ?
$S_{n}=\frac{a\left(r^{n}-1\right)}{r-1}$
$S_{10}=\frac{1\left(2^{10}-1\right)}{2-1}$
= 1024 - 1
$S_{10}=1023$
$\therefore \quad 1+2+3+\ldots$ up to 10 terms $=1023$ Shading -
.


Drawing 2 circles (intersecting circles ) -

Qn.
Nos.
 -
33. Using Euclid's division algorithm, find the HCF of 45 and 60. Ans. :
$a=60, \quad b=45$

According Euclid's division lemma $a=b q+r$
i) $\quad \therefore 60=45 \times 1+15$
ii) $45=15 \times 3+0$
$\therefore \quad \mathrm{HCF}$ of 45 and 60 is $15 \quad 1 / 2$

The following table shows how the students come to school. Draw a pie chart to represent the data:

| Walk | Bicycle | Bus | School Van |
| :---: | :---: | :---: | :---: |
| 12 |  |  |  |
|  | 8 | 6 | 10 |

Ans. :

| Students come to <br> school by | Number of students | Central angle |
| :---: | :---: | :---: |
| Walk | 12 | $12 \times 10^{\circ}=120^{\circ}$ |
| Bicycle | 8 | $8 \times 10^{\circ}=80^{\circ}$ |
| Bus | 6 | $6 \times 10^{\circ}=60^{\circ}$ |
| School van | 10 | $10 \times 10^{\circ}=100^{\circ}$ |
|  | 36 | $360^{\circ}$ |

36 students corresponds to $360^{\circ}$
$\Rightarrow \quad 1$ student corresponds to $\frac{360^{\circ}}{36}=10^{\circ}$

| Drawing circle $-1 / 2$ |  |
| :--- | ---: |
| Plotting - | $1 / 2$ |

35. If $f(x)=2 x^{3}+3 x^{2}+8 x-5$ then find the values of
i) $\quad f(0)$
ii) $f(1)$.
Ans. :
$f(x)=2 x^{3}+3 x^{2}+8 x-5$
i) $f(0)=2(0)^{3}+3(0)^{2}+8(0)-5$

$$
=0+0+0-5
$$

$$
f(0)=-5
$$

ii) $f(1)=2(1)^{3}+3(1)^{2}+8(1)-5$

$$
=2+3+8-5
$$

$$
f(1)=8
$$

$$
1 / 2
$$

2 Marks
allotted
36.

Solve the equation using formula :

$$
x^{2}-3 x+2=0
$$

Ans. :

## Value Points

Marks allotted

In the given figure, $A E \| D B, D C=4 \mathrm{~cm}, C E=12 \mathrm{~cm}$ and $B D=5 \mathrm{~cm}$.
Find the length of $A E$.


Ans. :

In $\triangle A E C$ and $\triangle B D C$

$$
\begin{array}{llll} 
& \lfloor A C E=\lfloor B C D & \ddots & \text { Vertically opposite angles } \\
& \lfloor A=\lfloor B & \because & \text { Alternate angle } \\
\therefore & \triangle A E C \sim \triangle B D C & \because & A A \text { criteria } \\
\therefore & \frac{A E}{B D}=\frac{E C}{C D} & & \\
\hline & & & 1 / 2 \\
& & &
\end{array}
$$

$\left.\begin{array}{c|cc|c}\hline \begin{array}{c}\text { Qn. } \\ \text { Nos. }\end{array} & \text { Value Points } & & \begin{array}{c}\text { Marks } \\ \text { allotted }\end{array} \\ \hline & \frac{A E}{5} & =\frac{12}{4} & \\ \\ & A E & =\frac{12 \times 5}{4} & 1 / 2\end{array}\right]$
38. In right angled triangle $A B C,\left\lfloor B=90^{\circ}, A B=12 \mathrm{~cm}\right.$ and $A C=13 \mathrm{~cm}$.

Find $B C$.


Ans. :
In
$\triangle A B C$

$$
\begin{array}{rlr}
A C^{2} & =A B^{2}+B C^{2} & 1 / 2 \\
13^{2} & =12^{2}+B C^{2} \\
B C^{2} & =13^{2}-12^{2} \\
& =169-144
\end{array}
$$

$$
B C^{2}=25
$$

$B C^{2}=25$

$$
B C=\sqrt{25}
$$

$B C=\sqrt{25}$

$$
B C=5 \mathrm{~cm}
$$

$B C=5 \mathrm{~cm}$
,
Value Points
$\frac{\sin 54^{\circ}}{\cos 36^{\circ}}$.
39.

Find the value of $\frac{\sin 36^{\circ}}{\cos 54^{\circ}}-\frac{\sin 54^{\circ}}{\cos 36^{\circ}}$. allotted

Ans. :

$$
\begin{aligned}
& \frac{\sin 36^{\circ}}{\cos 54^{\circ}}-\frac{\sin 54^{\circ}}{\cos 36^{\circ}} \\
& =\quad \frac{\cos \left(90^{\circ}-36^{\circ}\right)}{\cos 54^{\circ}}-\frac{\cos \left(90^{\circ}-54^{\circ}\right)}{\cos 36^{\circ}}
\end{aligned}
$$

$$
\because \quad \sin A=\cos \left(90^{\circ}-A\right)
$$

$$
=\frac{\cos 54^{\circ}}{\cos 54^{\circ}}-\frac{\cos 36^{\circ}}{\cos 36^{\circ}}
$$

$$
=\quad 1-1
$$

$=0$
40. Find the distance between the points (2, 3) and (6, 6), using the formula.

Ans. :
$(2,3) \Rightarrow\left(x_{1}, y_{1}\right)$
$(6,6) \Rightarrow\left(x_{2}, y_{2}\right)$
$d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
$=\sqrt{(6-2)^{2}+(6-3)^{2}}$
$=\sqrt{4^{2}+3^{2}}$
$=\sqrt{16+9}$
$=\sqrt{25} \quad \therefore \quad d=5$ units
$1 / 2$

| $\begin{array}{c}\text { Qn. } \\ \text { Nos. }\end{array}$ |
| :---: |
| IV. 41. |

Find three positive integers in Arithmetic progression such that their sum is 24 and product is 480 .

OR
If the 4th and 8th terms of a Geometric progression are 24 and 384 respectively, find the first term and common ratio.

Ans. :
Let the three positive integers in A.P. be $a-d, a, a+d$
Given that

$$
\begin{gathered}
a-d+a+a+d=24 \\
3 a=24 \\
a=8
\end{gathered}
$$

Also,

$$
\begin{gathered}
(a-d)(a)(a+d)=480 \\
a\left(a^{2}-d^{2}\right)=480 \\
a^{2}-d^{2}=\frac{480}{a} \\
8^{2}-d^{2}=\frac{480}{8} \\
64-d^{2}=60 \\
d^{2}=64-60 \\
d^{2}=4 \\
d= \pm 2
\end{gathered}
$$

If $a=8, \quad d=+2$
$\therefore \quad$ Three terms are $6,8,10$
OR

$$
\text { If } \quad a=8, \quad d=-2
$$

$\therefore \quad$ Three terms are $10,8,6$
OR
Qn.

| Value Points | Marks <br> allotted |
| :---: | :---: | :---: |

$T_{4}=24$
$T_{8}=384$
$a=$ ?
$r=$ ?
In G.P. $\quad T_{n}=a r^{n-1}$
Consider $\quad \frac{T_{8}}{T_{4}}=\frac{384}{24}$
$\frac{\alpha r^{7}}{\phi r^{3}}=\frac{384}{24}_{1}^{16}$
$r^{4}=16$
$r^{4}=2^{4}$
$\therefore \quad r=2$
We know that $T_{4}=24$

$$
\begin{aligned}
& \text { i.e. } a r^{3}=24 \\
& a(2)^{3}=24 \\
& a=\frac{24}{8}=3 \\
& a=3
\end{aligned}
$$

$\therefore \quad$ The first term is $a=3$
The common ratio is $r=2$
42. Calculate the standard deviation of the following scores :
$2,4,6,8,10$.
Ans. :

| Qn. <br> Nos. | Value Points |  |  |
| :---: | :---: | :---: | :---: |
|  | i) Direct method : |  |  |
|  |  | $x$ | $x^{2}$ |
|  |  | 2 | 4 |
|  |  | 4 | 16 |
|  |  | 6 | 36 |
|  |  | 8 | 64 |
|  |  | 10 | 100 |
|  |  | $\sum x=30$ | $\sum x^{2}=220$ |

$n=5$
Standard deviation $\quad \sigma=\sqrt{\frac{\sum x^{2}}{n}-\left(\frac{\sum x}{n}\right)^{2}}$

$$
\begin{aligned}
\sigma & =\sqrt{\frac{\sum x^{2}}{n}-\left(\frac{\sum x}{n}\right)^{2}} \\
\sigma & =\sqrt{\frac{220}{5}-\left(\frac{30}{5}\right)^{2}} \\
& =\sqrt{44-36}
\end{aligned}
$$

$$
\sigma=\sqrt{8} \quad 1 / 2
$$

$$
\sigma \simeq 2 \cdot 8 \quad 1 / 2
$$

ii) Actual mean method:

| $x$ | $d=x-\bar{x}$ | $d^{2}$ |
| :---: | :---: | :---: |
| 2 | -4 | 16 |
| 4 | -2 | 4 |
| 6 | 0 | 0 |
| 8 | 4 | 4 |
| 10 |  | $\sum d^{2}=40$ |
| $\sum x=30$ |  |  |

$$
n=5
$$

Qn.

| Mean $=\bar{x}$ | $=\frac{\sum x}{n}$ |
| ---: | :--- |
|  | $=\frac{30}{5}$ |
| $\bar{x}$ | $=6$ |

Standard deviation $\quad \sigma=\sqrt{\frac{\sum d^{2}}{n}}$
$=\sqrt{\frac{40}{5}}$
$=\sqrt{8}$
$\sigma \approx 2 \cdot 8$
iii) Assumed Mean method:

| $x$ | $d=x-\mathrm{A}$ | $d^{2}$ |
| :---: | :---: | :---: |
| 2 | -4 | 16 |
| 4 | -2 | 4 |
| 6 | 0 | 0 |
| 8 | 2 | 4 |
| 10 | 4 | 16 |
|  | $\sum d=0$ | $\sum d^{2}=40$ |

Let us assume $A=6$
$n=5$

$$
\begin{aligned}
& \sigma=\sqrt{\frac{\sum d^{2}}{n}-\left(\frac{\sum d}{n}\right)^{2}} \\
& =\sqrt{\frac{40}{5}-\left(\frac{0}{5}\right)^{2}} \\
& =\sqrt{8} \\
& \sigma \approx 2.8
\end{aligned}
$$

| Qn. <br> Nos. | Value Points | Marks <br> allotted |
| :---: | :---: | :---: |

iv) Step deviation method :

| $x$ | $d=x-A$ | Step deviation <br> $d=\frac{x-A}{c}$ | $d^{2}$ |
| :---: | :---: | :---: | :---: |
| 2 | -4 | -2 | 4 |
| 4 | -2 | -1 | 1 |
| 6 | 0 | 0 | 0 |
| 8 | 2 | 1 | 1 |
| 10 | 4 | 2 | 4 |
|  |  | $\sum d=0$ | $\sum d^{2}=10$ |

Let $A=6$
Common factor $c=2$

$$
n=5
$$

$$
\sigma=\sqrt{\frac{\sum d^{2}}{n}-\left(\frac{\sum d}{n}\right)^{2}} \times c
$$

$$
1 / 2
$$

$$
=\sqrt{\frac{10}{5}-0} \times 2
$$

$$
=\sqrt{2-0} \times 2
$$

$$
=2 \sqrt{2}
$$

$$
1 / 2
$$

$$
\sigma \approx 2 \cdot 8
$$

43. If one root of the quadratic equation $x^{2}-6 x+q=0$ is twice the other, find the value of $q$.

## OR

If $m$ and $n$ are the roots of equation $x^{2}-3 x+1=0$, find the value of
i) $m^{2} n+m n^{2}$
ii) $\frac{1}{m}+\frac{1}{n}$.

| Qn. <br> Nos. |  | Value Po |
| :---: | :--- | :--- |
|  | Ans. : |  |
|  | $x^{2}-6 x+q=0$ |  |
|  | $a=1, \quad b=-6, \quad c=q$ |  |

Let $m$ and $n$ be the roots and $m=2 n$
Sum of the roots $m+n=\frac{-b}{a}$

$$
\begin{aligned}
& 2 n+n=\frac{-(-6)}{1} \\
& 3 n=6 \\
& n=2 \\
& \therefore \quad m=2 n \\
& m=2(2) \\
& m=4 \\
& m \cdot n=\frac{c}{a} \\
& (2 n)(n)=\frac{q}{1} \\
& 2 n^{2}=q \\
& 2(2)^{2}=q \\
& \therefore \quad q=8 \\
& 1 / 2 \\
& 1 / 2
\end{aligned}
$$

OR
$x^{2}-3 x+1=0$
$a=1$,

$$
b=-3, \quad c=1
$$

Sum of the roots $m+n=\frac{-b}{a}$
$m+n=\frac{-(-3)}{1}$
$m+n=3$
Product of the roots $=m n=\frac{c}{a}$

| Value Points | Marks <br> allotted |
| :---: | :---: |

Prove that "if two circles touch each other externally, the centres and the point of contact are collinear".

Ans. :


Data: $\quad A$ and $B$ are the centres of touching circles. $P$ is the point of contact.

To prove : $\quad A, P$ and $B$ are collinear.
Construction: Draw the tangent XPY

| Qn. <br> Nos. | Value Points | Marks allotted |
| :---: | :---: | :---: |
|  | Proof: In the figure $\begin{aligned} &\lfloor A P X=90^{\circ} \quad \therefore \quad \text { Radius drawn at the point of contact } \\ &\lfloor B P X=90^{\circ} \quad \text { is perpendicular to the tangent } \\ &\lfloor A P X+\lfloor B P X=90^{\circ}+90^{\circ} \\ &\lfloor A P X+\lfloor B P X=180^{\circ} \\ &\left\lfloor A P B=180^{\circ}\right. \end{aligned}$ <br> $\therefore \quad A P B$ is a straight line. | 3 |

In the figure if $A D \perp B C$, prove that $A B^{2}+C D^{2}=B D^{2}+A C^{2}$. 3


OR

In the figure, $O$ is any point inside a rectangle $A B C D$. Prove that $O B^{2}+O D^{2}=O A^{2}+O C^{2}$.


Ans. :
粦 (21)1302-PR(D)

| Qn. <br> Nos. | Value Points | Marks <br> allotted |
| :---: | :---: | :---: |

In $\triangle A D C$,

From (i) and (ii)

$$
\begin{aligned}
& A B^{2}-B D^{2}=A C^{2}-C D^{2} \\
& A B^{2}+C D^{2}=A C^{2}+B D^{2}
\end{aligned}
$$

## OR

$E F \| D C$
$\therefore \quad E F \perp A D$ and $E F \perp B C$
In $\triangle$ OEA,
$O A^{2}=A E^{2}+O E^{2}$
... (i)

In $\triangle O B F, \quad O B^{2}=B F^{2}+O F^{2}$
In $\triangle O F C, \quad O C^{2}=O F^{2}+C F^{2}$
... (iii)
In $\triangle O E D, \quad O D^{2}=O E^{2}+D E^{2}$
... (iv)
$1 / 2$

Adding (ii) and (iv)

$$
\begin{aligned}
O B^{2}+O D^{2} & =B F^{2}+O F^{2}+O E^{2}+D E^{2} \\
& =A E^{2}+O F^{2}+O E^{2}+F C^{2} \quad \because \quad B F=A E \\
& D E=F C \\
& =A E^{2}+O E^{2}+O F^{2}+F C^{2} \\
& =O A^{2}+O C^{2}
\end{aligned}
$$

$$
\therefore \quad O B^{2}+O D^{2}=O A^{2}+O C^{2}
$$

$\therefore \quad O B^{2}+O D^{2}=O A^{2}+O C^{2}$

$$
A B^{2}=B D^{2}+A D^{2}
$$

$$
\begin{array}{lll}
A C^{2}=A D^{2}+C D^{2} & & 1 / 2 \\
A D^{2}=A C^{2}-C D^{2} & \ldots \text { (ii) } & 1 / 2 \tag{ii}
\end{array}
$$

$1 / 2$

$$
\begin{equation*}
A D^{2}=A B^{2}-B D^{2} \tag{i}
\end{equation*}
$$

| Qn. <br> Nos. | Value Points |
| ---: | :---: |
| 46. | Prove that |$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2 \sec A$.

OR
The shadow of a tower when sun's altitude is $30^{\circ}$, is 40 m longer than its shadow when the sun's altitude was $60^{\circ}$. Find the height of the tower.


Ans. :
L.H.S. $=\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$
$=\frac{\cos A(\cos A)+(1+\sin A)(1+\sin A)}{\cos A(1+\sin A)}$
$=\frac{\cos ^{2} A+(1+\sin A)^{2}}{\cos A(1+\sin A)}$
$=\frac{\cos ^{2} A+1+\sin ^{2} A+2 \sin A}{\cos A(1+\sin A)}$
$=\frac{2+2 \sin A}{\cos A(1+\sin A)}$
$=\frac{2[1+\sin A]}{\cos A[1+\sin A]}$
$=\frac{2}{\cos A}$
$=2 \sec A=$ RHS
$\therefore \quad \frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2 \sec A$.


| Qn. <br> Nos. | Value Points | Marks <br> allotted |
| :---: | :---: | :---: |

Alternate method:
$\left.\begin{array}{c}y=x^{2} \\ \hline x\end{array} \left\lvert\, \begin{array}{c|c|c|c|c|c|c|}\hline 0 & 1 & 2 & 3 & -1 & -2 & -3 \\ \hline y & 0 & 1 & 4 & 9 & 1 & 4 \\ 9 \\ \hline x & 0 & 1 & 2 & 3 & -1 & -2 \\ \hline\end{array} \begin{array}{|c|c|c|c|c|}\hline y & 2 & 1 & 0 & -1\end{array}\right.\right) 3$

Table -
Drawing line - $\quad 1 / 2$
Drawing parabola -
Identifying roots -1
$1 / 2$
4


| Qn. <br> Nos. | Value Points |
| :---: | :---: |
|  |  |


| Qn. <br> Nos. | Value Points | Marks <br> allotted |
| :---: | :---: | :---: |

48. Construct a direct common tangent to two circles of radii 4 cm and 2 cm whose centres are 8 cm apart.

Ans. :
$R=4 \mathrm{~cm}$
$r=2 \mathrm{~cm}$
$d=8 \mathrm{~cm}$
$R-r=4-2=2 \mathrm{~cm}$

$P Q$ and $R S$ are required tangents

| Drawing $A B$, marking mid-point - | $1 / 2$ |
| :--- | ---: |
| Drawing $C_{1}, C_{2}, C_{3}, C_{4}-$ | 2 |

Joining $B X / B Y — \quad 1 / 2$
Joining $P Q / R S-$
49. Prove that, "the areas of similar triangles are proportional to the squares of their corresponding sides".

Ans. :


Data: $\quad \triangle A B C \sim \triangle D E F$

$$
\frac{A B}{D E}=\frac{B C}{E F}=\frac{A C}{D F}
$$

To prove: $\frac{\operatorname{ar}(\triangle A B C)}{\operatorname{ar}(\triangle D E F)}=\frac{B C^{2}}{E F^{2}}$

Construction: Draw $A L \perp B C, \quad D M \perp E F$

Proof : In $\triangle A L B$ and $\triangle D M E$

$$
\begin{array}{lll}
\lfloor A B L & =\lfloor D E M & \because
\end{array} \text { Data } \quad \text { Construction }
$$



Hence the theorem is proved.
50. A 20 m deep well with diameter 7 m is dug and the mud from digging is evenly spread out to form a platform of cuboid shape, of length 22 m and breadth 14 m . Find the height of the platform.

| Qn. | Value Points | Marks <br> Nos. |
| :---: | :---: | :---: |

A cylindrical vessel of height 32 cm and base radius 18 cm is completely filled with sand. Then the sand in the vessel is poured on the plane ground to form a conical heap of sand of height 24 cm . Find the base radius of conical heap of sand.

Ans. :
Shape of the well is a cylinder with $h_{c y}=20 \mathrm{~m}$ and $r=\frac{7}{2} \mathrm{~m}$
$\therefore \quad$ Amount of mud obtained by digging well is $\pi r^{2} h$.
This mud is spread to form cuboid shaped platform and volume of cuboid is $l \times b \times h$
$\therefore \quad$ Volume of mud in both the cases is same

$$
\begin{array}{rlr}
\therefore \quad \pi r^{2} h & =l \times b \times h & 1 \\
\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 50 & =22 \times 14 \times h & 1 \\
\therefore \quad h & =\frac{7 \times 5}{14} & \\
h & =\frac{5}{2} \mathrm{~m} & 1 / 2 \\
h & =2.5 \mathrm{~m}
\end{array}
$$

$\therefore \quad$ Height of the platform is 2.5 m .

$$
\begin{aligned}
& h_{c y}=32 \mathrm{~cm} \\
& r_{c y}=18 \mathrm{~cm} \\
& h_{\text {cone }}=24 \mathrm{~cm} \\
& r_{\text {cone }}=?
\end{aligned}
$$

| Qn. Nos. | Value Points |  | Marks allotted |
| :---: | :---: | :---: | :---: |
|  | Volume of sand in cylindrical vessel = |  |  |
|  | Volume of sand in conical shape | $1 / 2$ |  |
|  | $\therefore \quad \pi r_{c y}^{2} h_{c y}=\frac{1}{3} \pi \cdot r_{\text {cone }}^{2} \cdot h_{\text {cone }}$ | 1 |  |
|  | $18 \times 18 \times 32=\frac{1}{3} \times r_{\text {cone }}^{2} \times 244^{8}$ | 1 |  |
|  | $r_{\text {cone }}^{2}=\frac{18 \times 18 \times 32}{-8} 4$ | $1 / 2$ |  |
|  | $r_{\text {cone }}^{2}=18^{2} \times 2^{2}$ |  |  |
|  | $\therefore \quad r=\sqrt{18^{2} \times 2^{2}}$ |  |  |
|  | $r=36 \mathrm{~cm}$ |  |  |
|  | $\therefore \quad$ Radius of cone is 36 cm | 1 | 4 |

