

MATHEMATICS

Standard VII

Part - I



Government of Kerala
Department of Education

State Council of Educational Research and Training (SCERT), KERALA
2016

The National Anthem

Jana-gana-mana adhinayaka, jaya he
Bharatha-bhagya-vidhata.
Punjab-Sindh-Gujarat-Maratha
Dravida-Utkala-Banga
Vindhya-Himachala-Yamuna-Ganga
Uchchala-Jaladhi-taranga
Tava subha name jage,
Tava subha asisa mage,
Gahe tava jaya gatha.
Jana-gana-mangala-dayaka jaya he
Bharatha-bhagya-vidhata.
Jaya he, jaya he, jaya he,
Jaya jaya jaya, jaya he!

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give respect to my parents, teachers and all elders and treat everyone with courtesy.

I pledge my devotion to my country and my people. In their well-being and prosperity alone lies my happiness.

State Council of Educational Research and Training (SCERT)

Poojappura, Thiruvananthapuram 695012, Kerala

Website : www.scertkerala.gov.in

e-mail : scertkerala@gmail.com

Phone : 0471 - 2341883, Fax : 0471 - 2341869

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Dear children,

We've learned much math.
Let's now move up higher
To the world of Arithmetic
Full of interesting numbers
And strange relations
To the new levels of Geometry
To understand the logic of Math
And to find the new things
Let's move ahead with confidence.

Dr. P. A. Fathima
Director
SCERT

TEXTBOOK DEVELOPMENT TEAM

Participants

Anil Kumar M.K.

H.S.A., S.K.M.J.H.S.S. Wayanad

Arunlal M.J.

U.P.S.A, A. U. P. S. Aramangalam,
Kozhikode

Kunhabdulla M.

U.P.S.A, Muyipphoth M.U.P.S. Kozhikode

K.G. Thulasidharan Pillai

P.D. Teacher, G.H.S.S. Karukon, Kollam

Balagangadharan V.K.

G.M.H.S.S. Calicut University Campus,
Malappuram

Manikantan K.O.V.

U.P.S.A. Pattiyamma,A.U.P.S. Kannur

Rajesh K.P.

Lecturer, DIET, Kannur

R. Ramanujam

H.S.S.T., M.N.K. M.G.H.S.S., Pulapatta,
Palakkad

Sunil Kumar V.P.

H.S.A. Janatha H.S.S. Thembamood,
Thiruvananthapuram

Experts

Dr. E. Krishnan

Prof.(Rtd) University College, Thiruvananthapuram

Dr. Vijaya Kumar A.

Prof. Cochin University, Cochi

Artist

Dhaneshan M.V., Karivellur, Kannur

Academic Co-ordinator

Dr. Lidsonraj J. Research Officer, SCERT

ENGLISH VERSION

Dr. E. Krishnan

Prof.(Rtd) University College,
Thiruvananthapuram

S. Bindu

*Associate Professor of English (Rtd)
University College, Thiruvananthapuram*

Venugopal C.

*Asst. Professor, Govt. College
of Teacher Education, Thiruvananthapuram*

Academic Co-ordinator

Arun Jyothi. S. Research Officer, SCERT



State Council of Educational Research and Training (SCERT)

Vidya Bhavan, Thiruvananthapuram

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Certain icons are used in this textbook
for convenience



Computer Work



Additional Problems



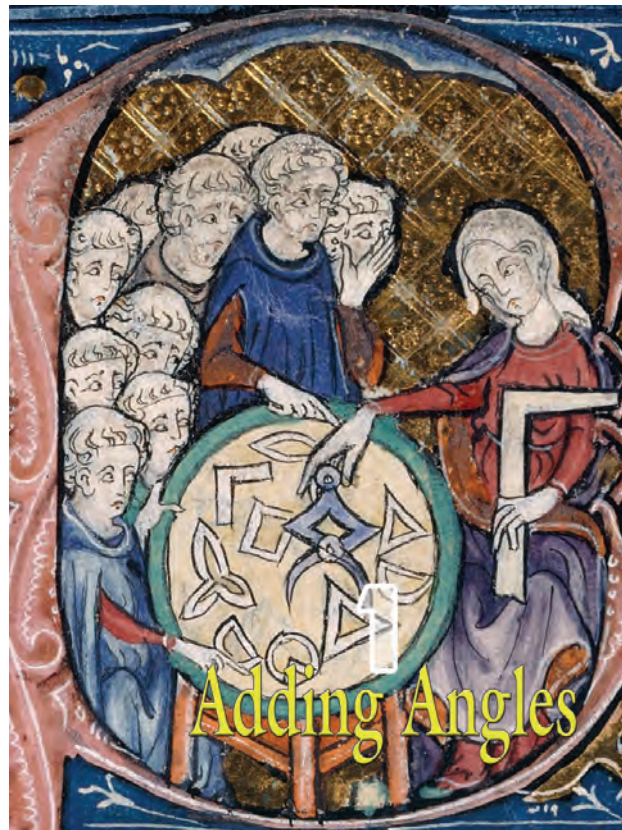
Project



Self Assessment

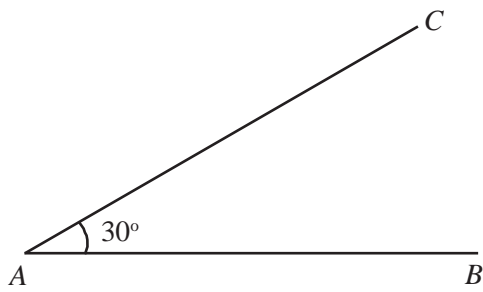
1

Adding Angles

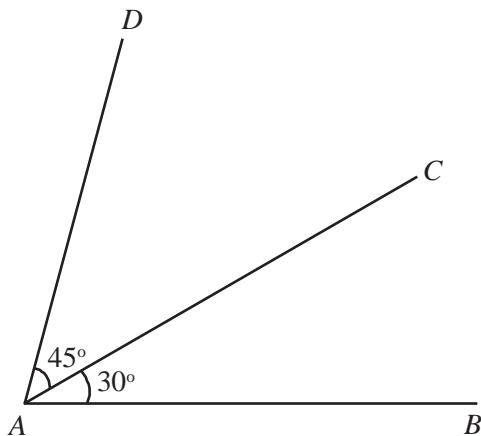


Joining angles

Can you draw this angle?



Draw another angle on top like this:



How many angles at A now?

$$\angle CAB = \dots\dots\dots$$

$$\angle DAC = \dots\dots\dots$$

Do you see one more large angle?

How much is it?

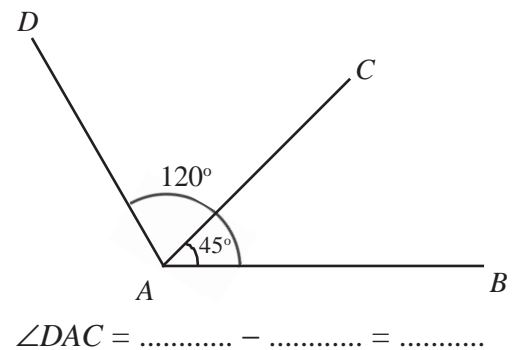
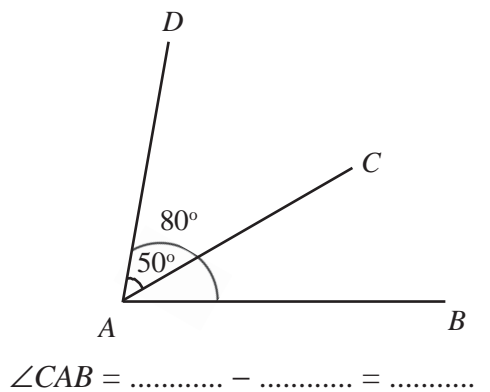
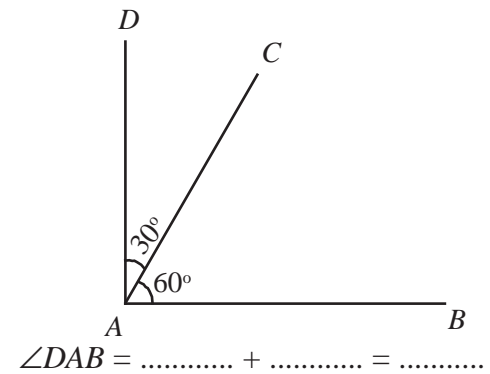
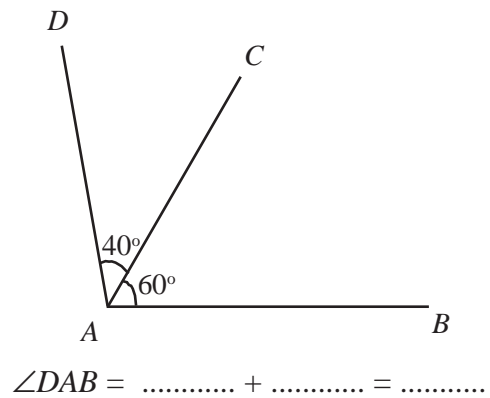
$$\angle DAB = \dots\dots\dots$$

How did you compute it?

$$\angle DAB = 45^\circ + 30^\circ = 75^\circ$$

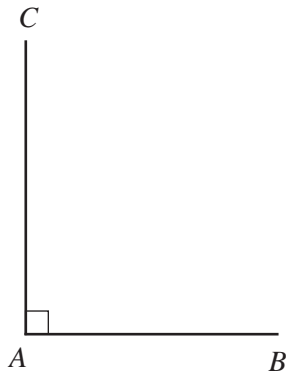
In the figures below, the measures of two angles are

shown. Write the third angle as a sum or difference and compute its measure:

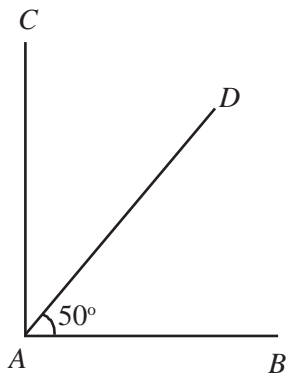


On both sides

Draw a line and a perpendicular to it as below:



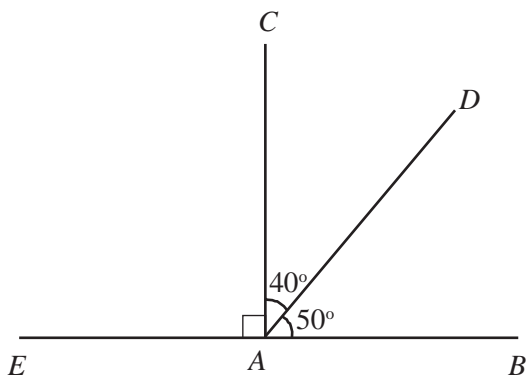
Now draw an angle within it like this:



What is the measure of $\angle DAC$?

$$\angle DAC = \dots - \dots = \dots$$

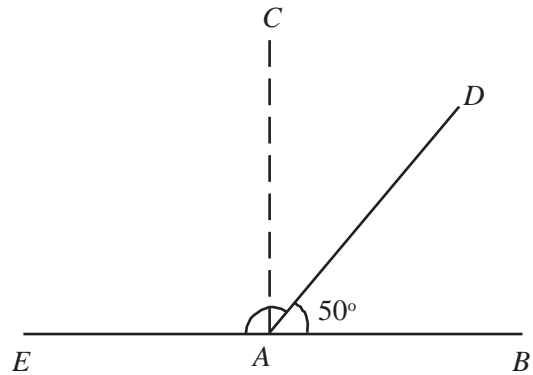
Let's stretch AB a bit to the left:



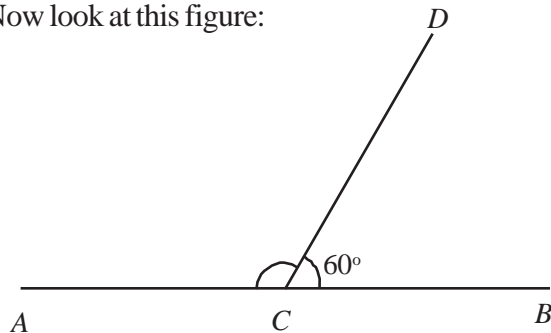
How much is $\angle DAE$?

$$\angle DAE = \dots + \dots = \dots$$

Do you see how $\angle DAE$ is related to $\angle DAB$?

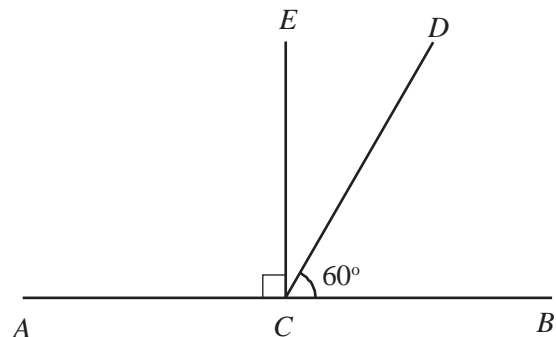


Now look at this figure:



Can you compute $\angle DCA$?

How about drawing a perpendicular at C and splitting this angle?

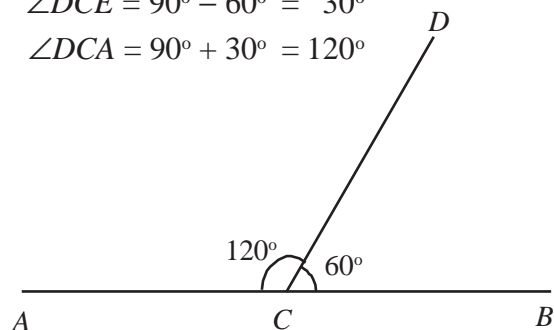


How much is $\angle DCE$?

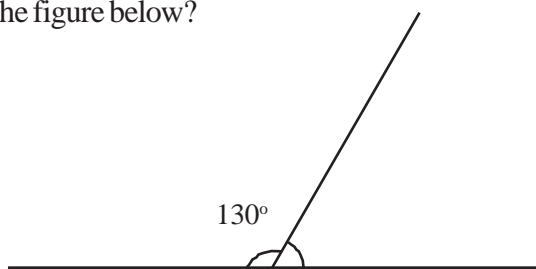
So, how much is $\angle DCA$?

$$\angle DCE = 90^\circ - 60^\circ = 30^\circ$$

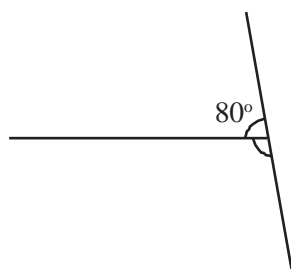
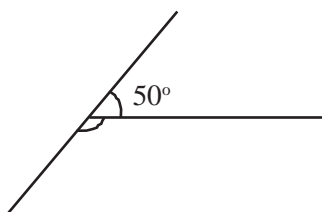
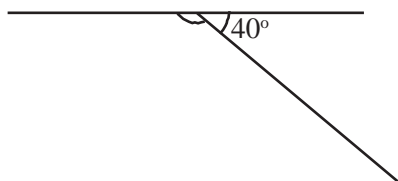
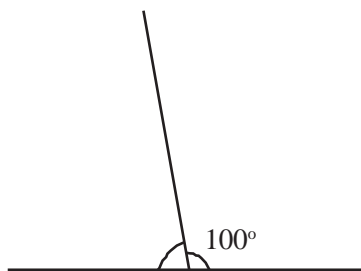
$$\angle DCA = 90^\circ + 30^\circ = 120^\circ$$



Like this, can you compute the angle on the right in the figure below?



In the figures below, the angles made on either side by joining two lines are shown; the measure of one angle is given. Compute the measure of the other:



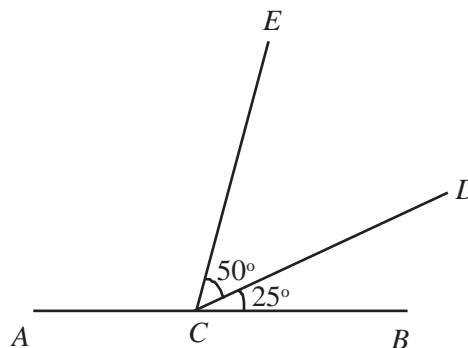
What do we see in all these?

If a line is drawn from another line, then the sum of the angles on either side is 180° .

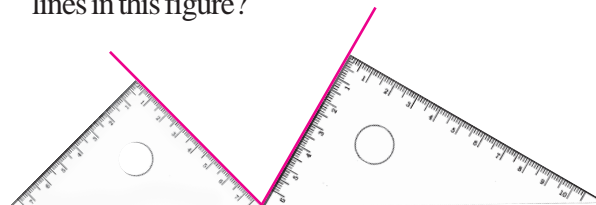
A pair of angles got like this is called a linear pair.

Angle calculation

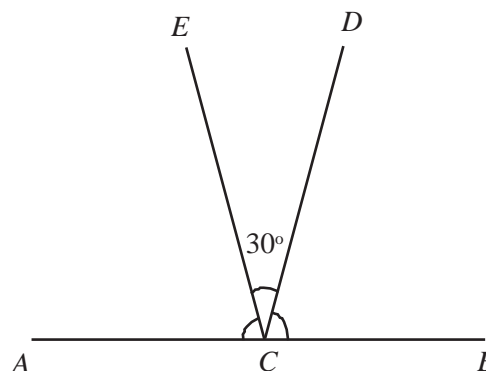
- How much is $\angle ACE$ in this figure?



- How much is the angle between the two slanted lines in this figure?

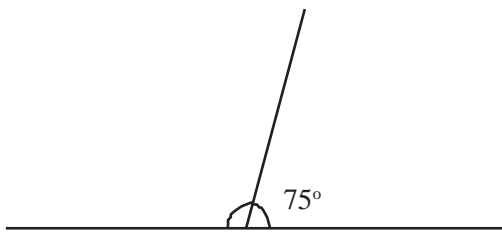


- In the figure below, $\angle ACD = \angle BCE$. How much is each?

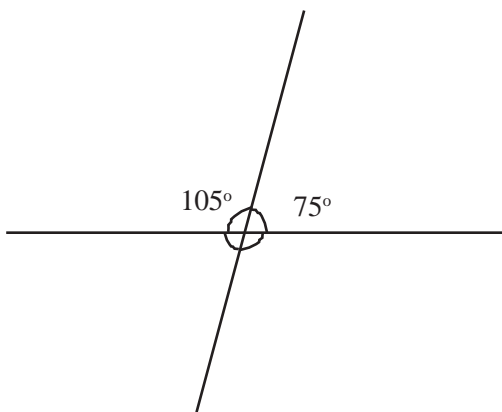


Cutting across

In the figure below, how much is the angle on the left?



Suppose we extend the upper line downwards:



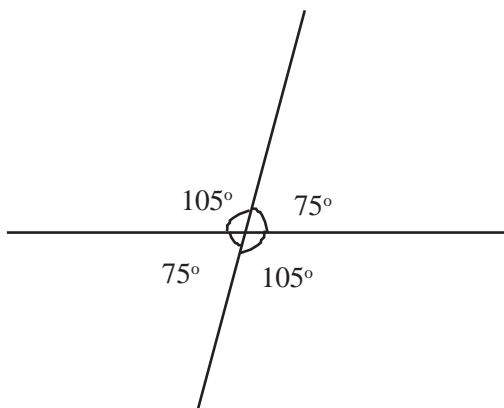
Now there are two more angles underneath.

How much is each?

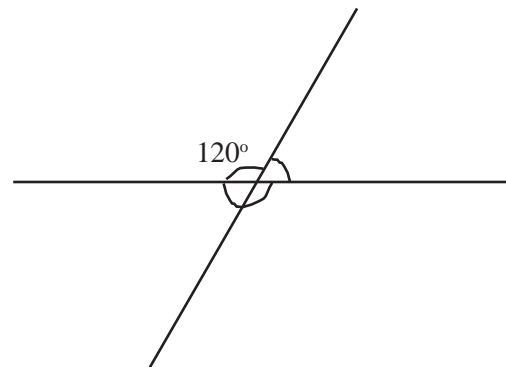
The angles at the top and bottom, on the left of the slanted line form a linear pair.

There is such a linear pair on the right also.

Now can't we compute these angles?



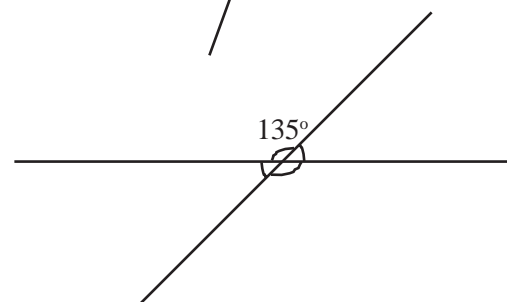
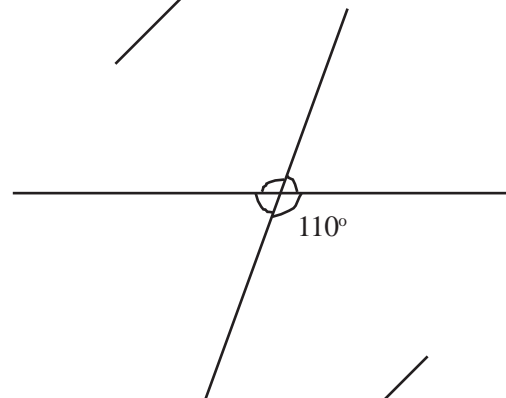
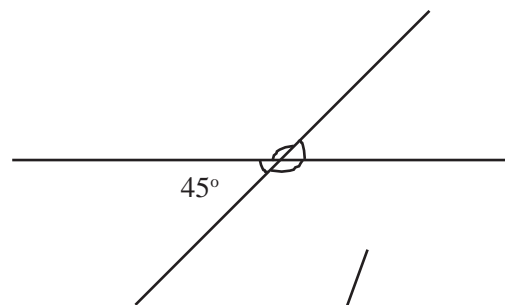
In the figure below also, two lines cut across each other. Can you compute the other three angles marked?



What do we see from all these?

Among the four angles made by two lines cutting across each other, the sum of each pair of nearby angles is 180° . Each pair of opposite angles are equal.

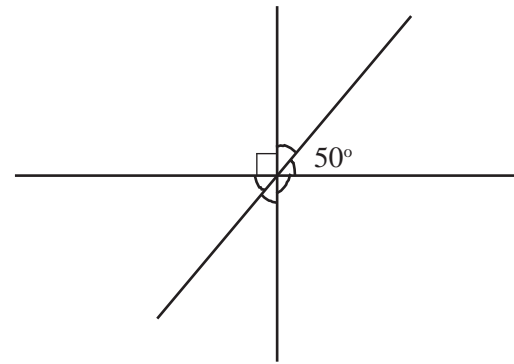
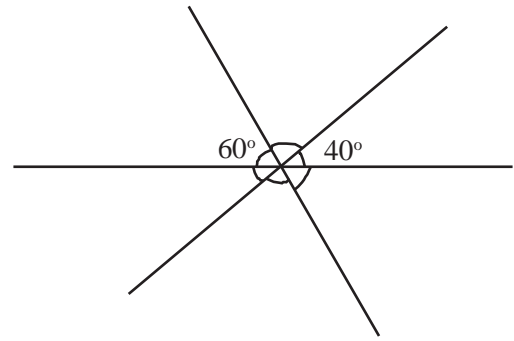
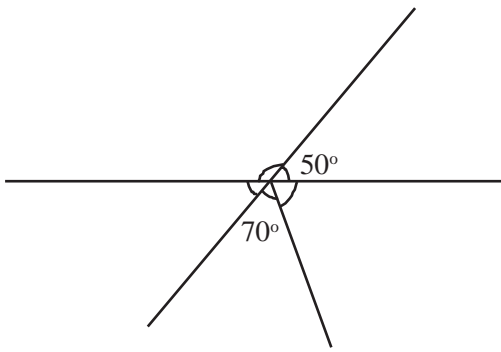
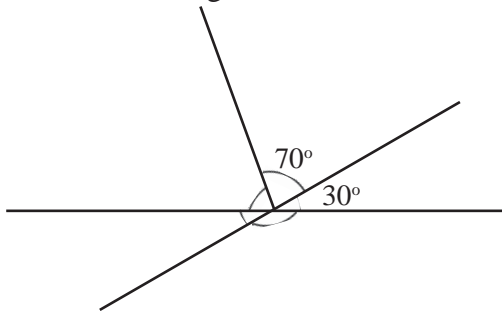
Now can't you calculate the angles marked in each of the figures below? Write them in the figure.





Let's do it!

In each figure some angles are given. Find all other angles.

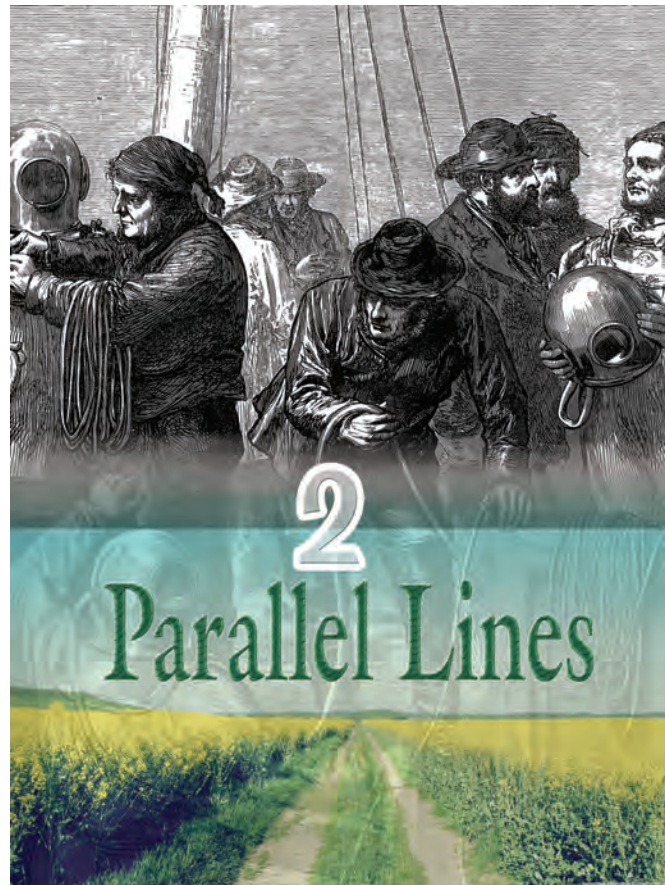


Looking back

Achievements	On my own	With teacher's help	Must improve
<ul style="list-style-type: none"> Applying the geometrical ideas learnt in new contexts. 			
<ul style="list-style-type: none"> Explaining the concepts of linear pair and opposite angles, using ideas relating to angles. 			
<ul style="list-style-type: none"> Solving problems using the knowledge relating to angles. 			

2

Parallel Lines

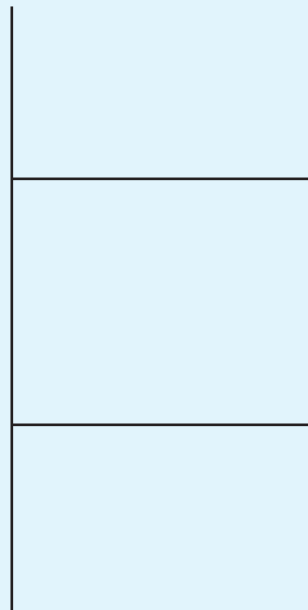
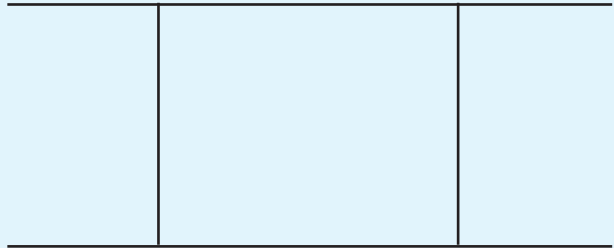




Parallel Lines

We get a line by joining any two points; on the other hand, do any two lines meet at a point?

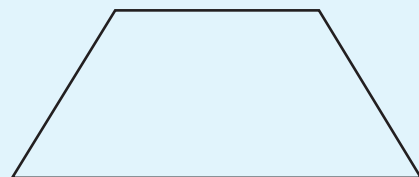
What about the lines got by extending a pair of opposite sides of a rectangle?



Do they meet however much we extend them?

Why?

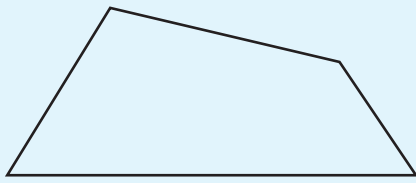
Look at this quadrilateral:



If we extend the top and bottom sides, do they meet?

What about the left and right sides?

What if we draw a quadrilateral like this?



Does any pair of opposite sides meet if we extend them?

Why?

Lines which are at the same distance everywhere, and do not meet anywhere, are called parallel.

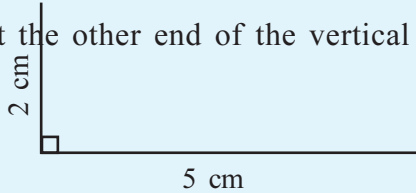
Same distance

Don't you know how to draw a rectangle?

How do we draw a rectangle of length 5 centimetre and breadth 3 centimetre?

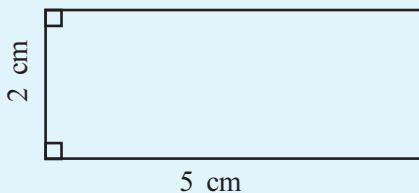
There are several ways, right?

First draw a horizontal line 5 centimetre long and draw a vertical line at one end, 2 centimetre high:



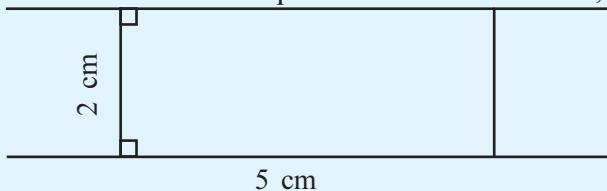
Next at the other end of the vertical line, draw a

perpendicular 5 centimetres long. Joining the other end of this perpendicular to the end of the first line makes our rectangle:



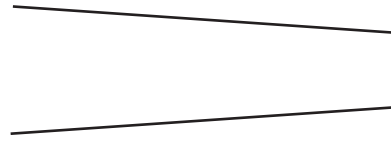
get a pair of parallel lines:

So if we take a line and a point 2 centimetres from it, how



Distance

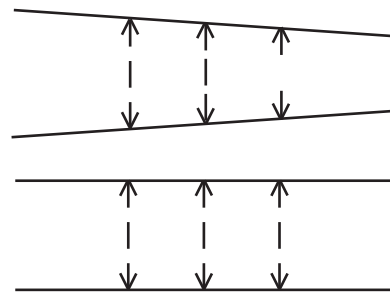
Would these lines meet, if extended?



How about these?



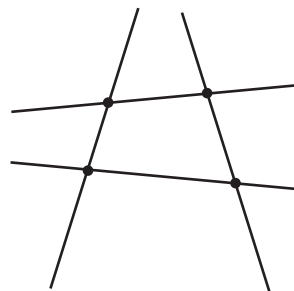
Look at the distance between the lines in both the figures:



What can we say about the distance between two parallel lines?



In GeoGebra, draw a quadrilateral. Use the **Line through two points** tool to extend the sides.



Do they meet?

Use move tool to drag the corners of the quadrilateral. When do the sides fail to meet?

Perpendicular and parallel

See the figure:



Look at the perpendicular lines to the horizontal.

Are they parallel?

Now see this:



A perpendicular is drawn to the horizontal line and then a perpendicular to the perpendicular is drawn.

Are the horizontal lines parallel?

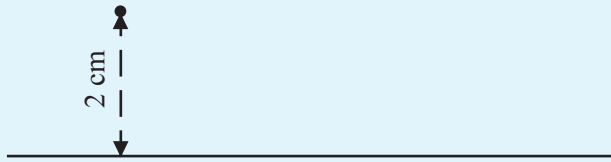


There are tools in GeoGebra to draw parallel and perpendiculars to a line. First draw a line and mark a point on it. Select the **Perpendicular line** tool, and click on the line and point, to get a perpendicular to the line through the point. Such a perpendicular can be drawn through a point not on the line also. Try drawing a perpendicular to this perpendicular.

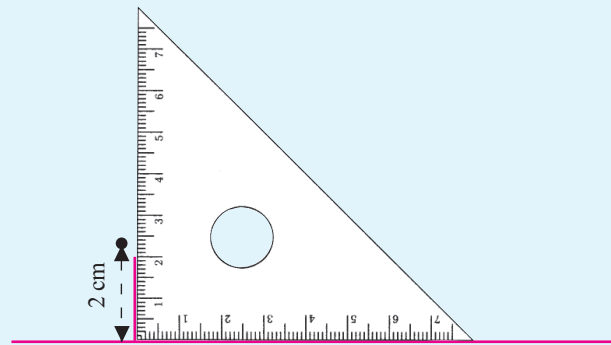
To draw a line parallel to another, the **Parallel line** tool is used. Mark a point not on the line. Select this tool and click on the line and the point, to get the parallel through the point. Drag the point using the **Move** tool. What happens if the point is on the line?

do we a draw a parallel to the line through the point?

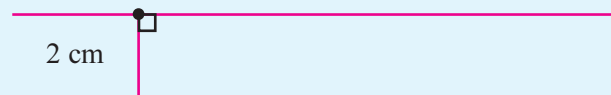
First draw the perpendicular to the line through the point:



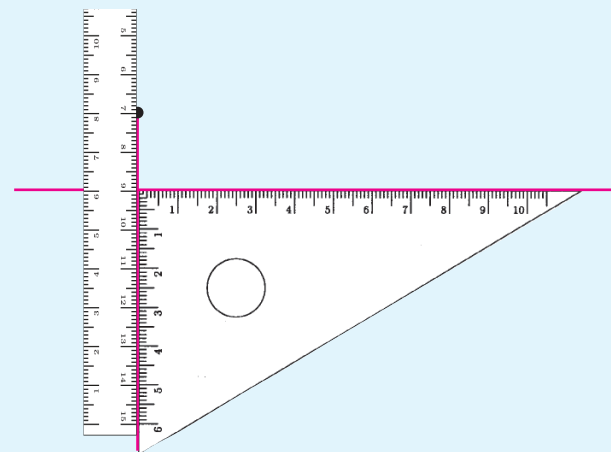
Then draw the perpendicular to this perpendicular through



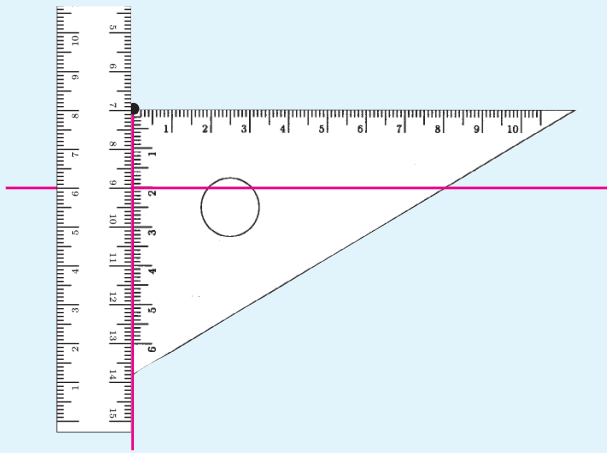
the point:



Instead of actually drawing the first perpendicular, we can use a ruler:



Now by shifting the set square upward and putting its square corner at the point, we can draw the parallel:



What if the point is below the line?

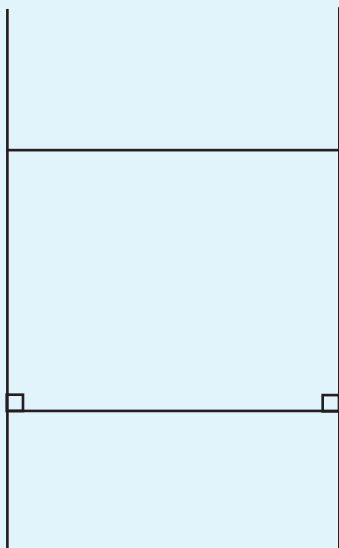
What did we see here?

Through any point not on a line, we can draw a parallel line.

How many parallels can be drawn through a point not on a line?

Same direction

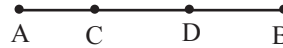
Opposite sides of a rectangle are parallel:



This can be said in a different manner: Two perpendiculars to the same line are parallel.

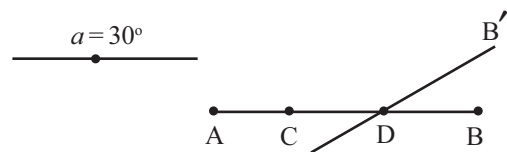


Draw line AB in GeoGebra and mark two other points C, D on it.

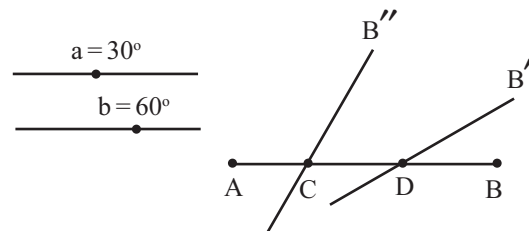


Now select the **Slider** tool and click on the GeoGebra screen. In the dialog box, select the **Angle** option by clicking on the small circle beside it. Against the **Name**, type a. Now click **Apply**.

Select the **Angle with given size tool** and click on B and D. In the dialog box, type a as **Angles** and click **OK**. Now we get a point B' . Join D and B' by a line.



Now make another slider named b. Select the **Angle** tool and click on B and C in that order. In the dialog box, type b as the angle and click **OK**. Join the new point B'' with C.

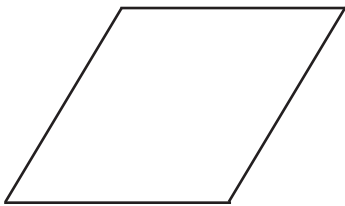
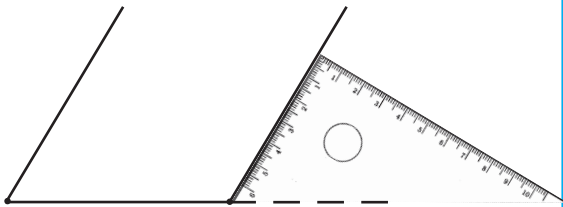
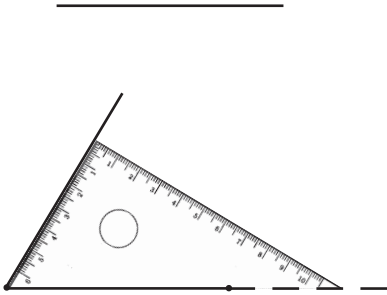


Use the **Move** tool to change the values of a and b. What happens to the lines? When do they fail to meet?

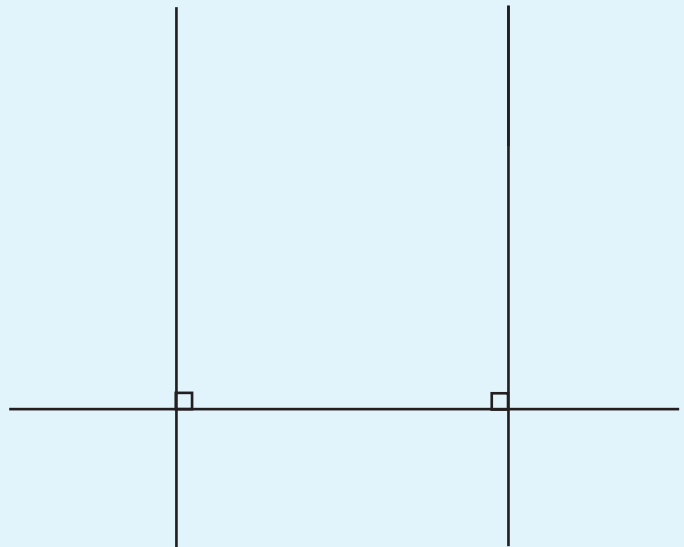
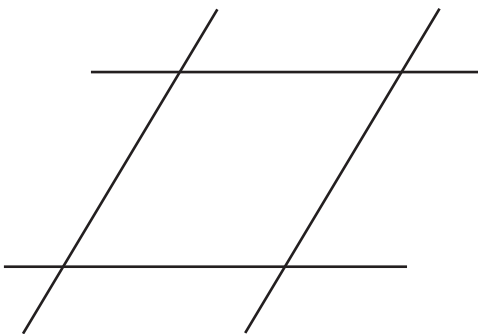
Try these with a single slider for the angles at C and D.

Not a square, but...

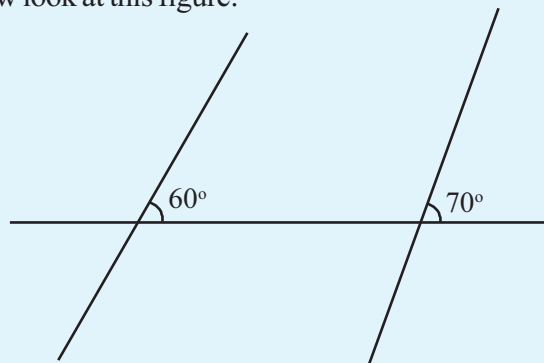
You know how to draw rectangles, using a set square. What if we use another corner, instead of the square one?



Does any pair of opposite sides meet, when extended?



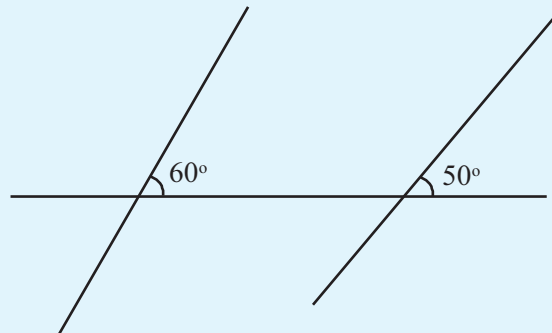
Now look at this figure:



Are the slanted lines parallel?

What would happen if these lines are extended upwards?

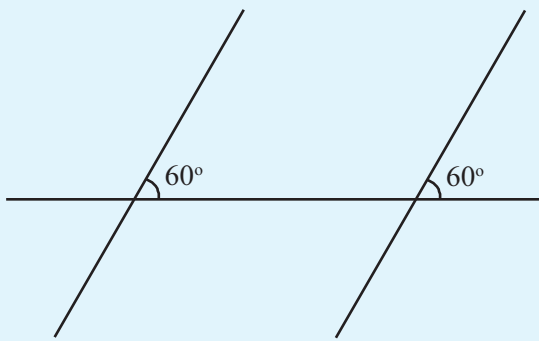
What if the lines are like this?



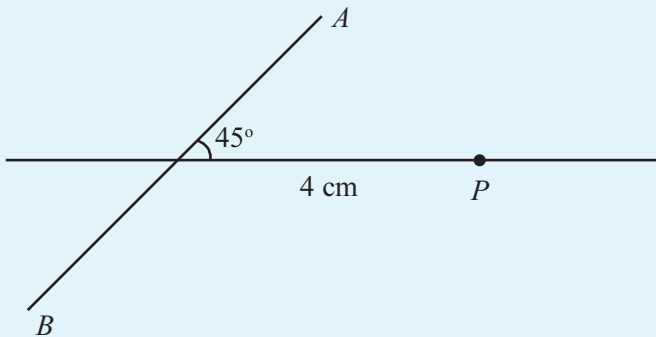
Would the lines meet if extended upwards?

Suppose we extend them downwards?

If the lines are not to meet either way, by how much should the right line be slanted?

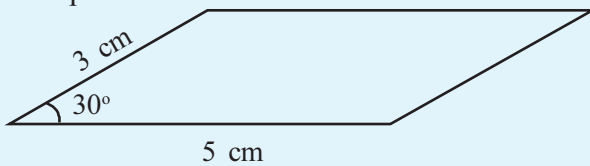


Now draw a figure like this in your notebook:



Is there a quick and easy way to draw a line parallel to AB through P ?

In the quadrilateral shown below, both pairs of opposite sides are parallel:

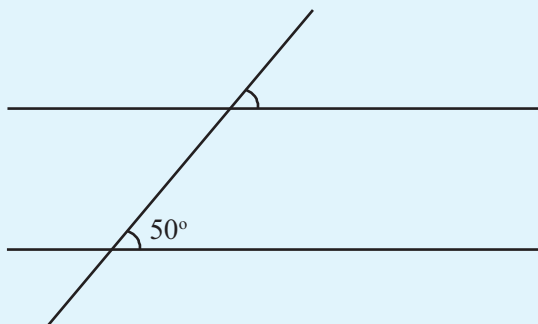


Can you draw this with the same lengths and angle?

A quadrilateral like this, with opposite sides parallel, is called a *parallelogram*.

Parallels and angles

In the figure below, the top and bottom lines are parallel:



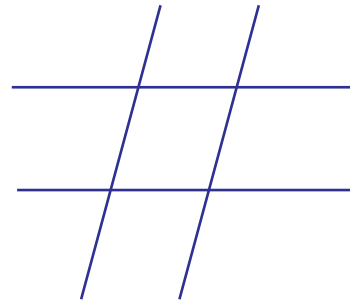
How much is the angle marked on the top?

When parallels cut

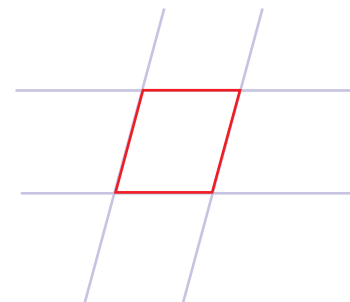


Draw a pair of parallel lines.

And another pair cutting across them.



Look at the figure made between them:



What is its name?

Rectangle and parallelogram

Cut out a cardboard rectangle:



Now cut a triangle through the bottom corner as shown below:



Place the triangle on the other side like this.

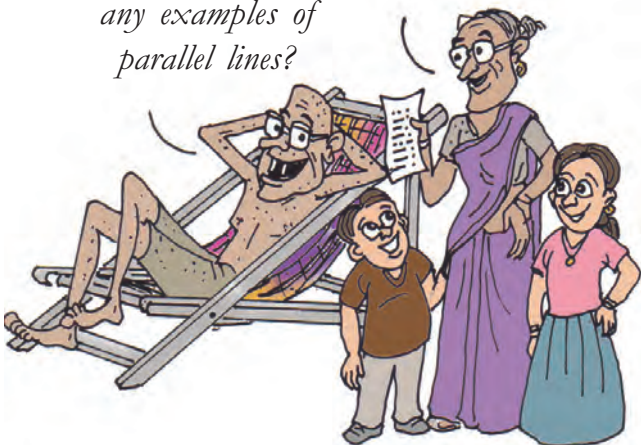


Is this a parallelogram?

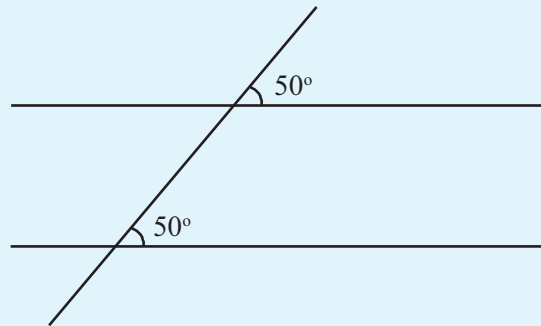
Why?

*Sure they did! Arms
and legs of chairs,
eye glasses,...*

*Did the kids get
any examples of
parallel lines?*

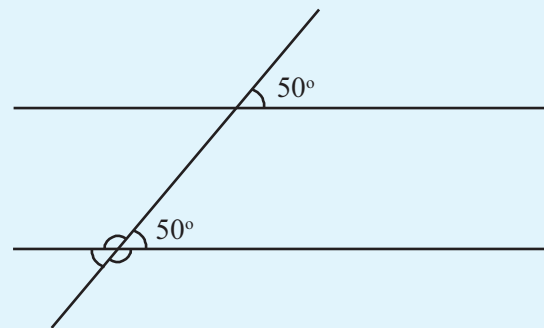


Parallel lines should have the same slant with any other line:

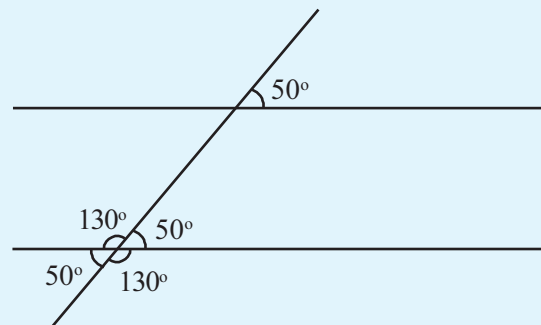


There are other angles here. Can you figure out those?

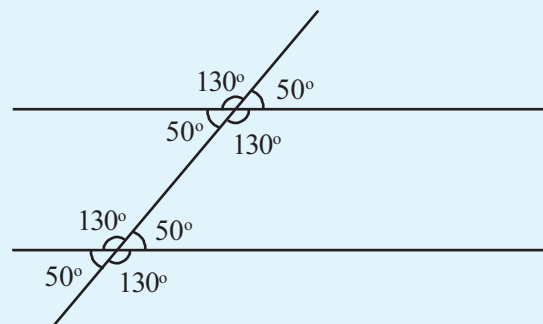
First look at the three angles below:



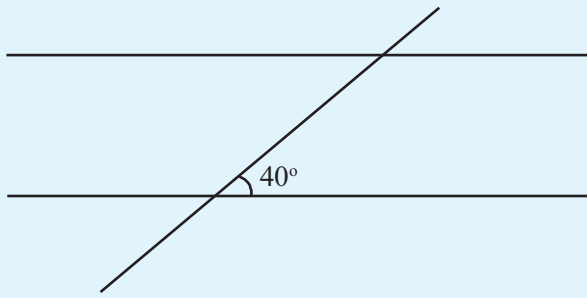
What are the relations between the four angles made by two lines cutting across each other?



Can't you find out the angles at the top also like this?



In the next figure also, the lines at the top and bottom are parallel:

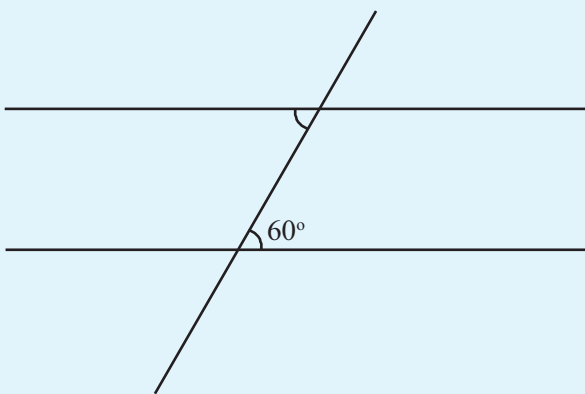
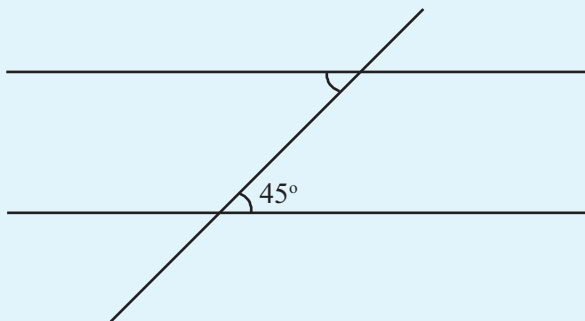


Write down the measures of the other seven angles in this figure.

What we have seen here can be written thus:

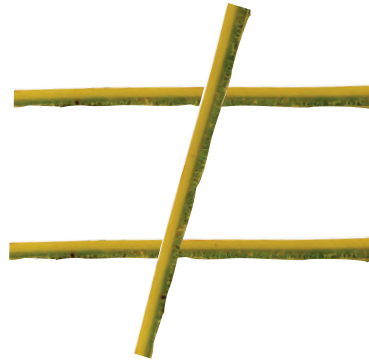
Parallel lines make equal angles with any other line.

In the figures below, there is a pair of parallel lines and a third line cutting across them. In each figure, the measure of one angle is given and another angle is marked. Find out its measure and write it in the figure itself.

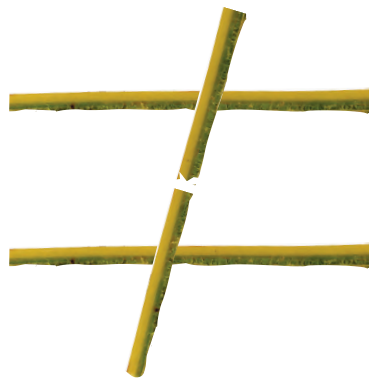


Unchanging shapes

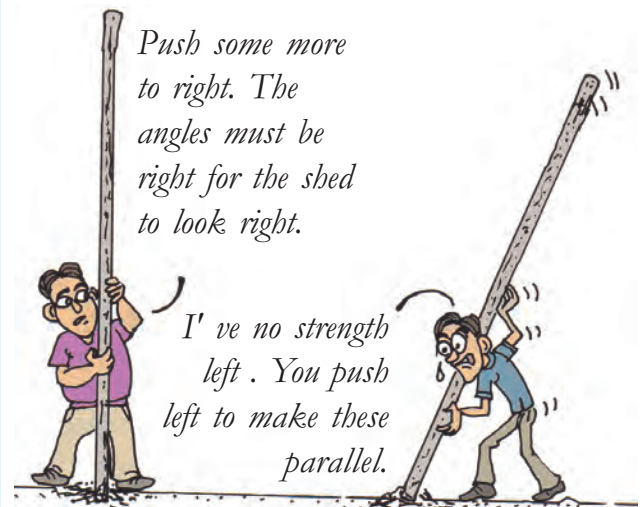
Place two straight and slender twigs parallel to each other. Place another one across and paste them together:



Now break at the middle to get two pieces:

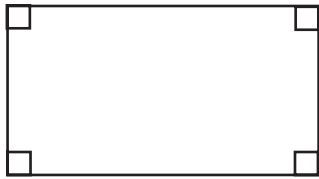


Place one piece over the other. All these angles match, don't they?

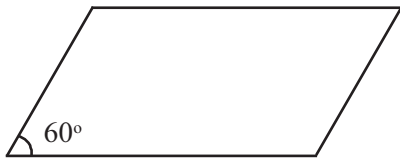


Angles of a parallelogram

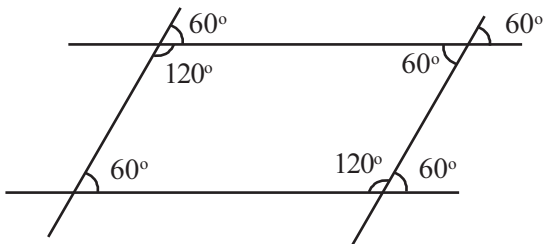
All angles in a rectangle are right angles.



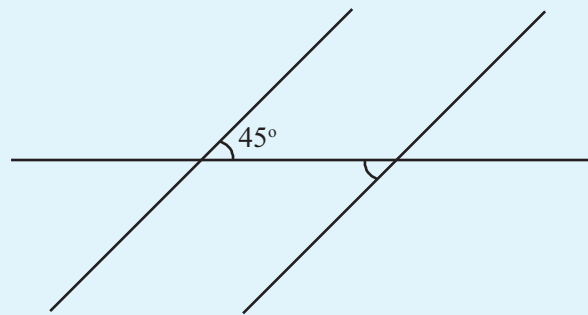
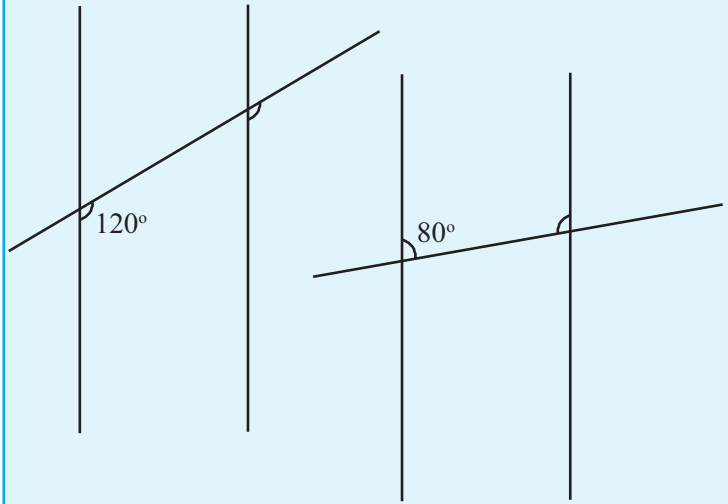
What about the angles in a parallelogram?



Can you find the angles in the first parallelogram?

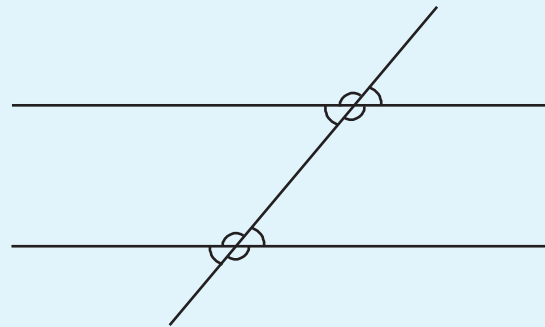


Now find the angles in the second one.



Matching angles

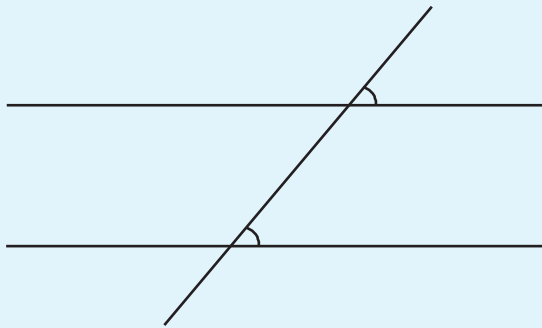
When a line cuts across a pair of parallel lines, eight angles are formed:



In the figure, the bottom line makes four angles, and the top line makes another four angles with the line cutting across them.

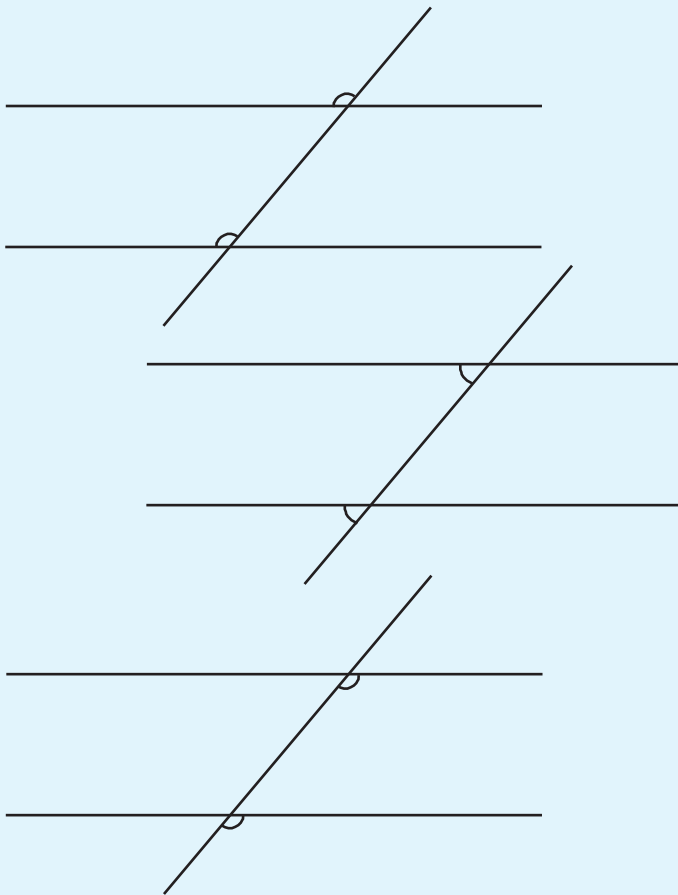
We can pair one angle at the bottom with one at the top in several different ways. Some such pairs are equal; others are supplementary (meaning their sum is 180°).

Let's look at the pairs of equal angles. For convenience, they are divided into two types. Look at the pair of angles marked in the figure below:



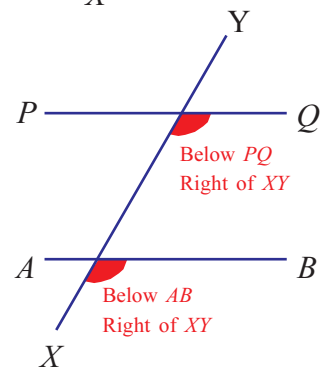
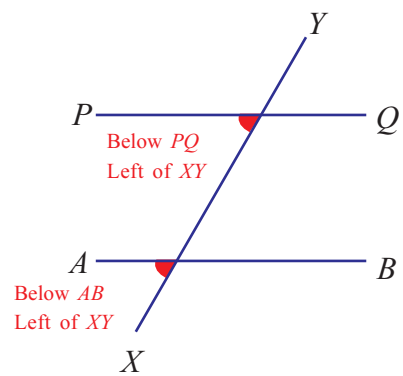
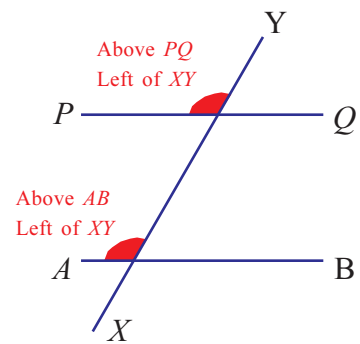
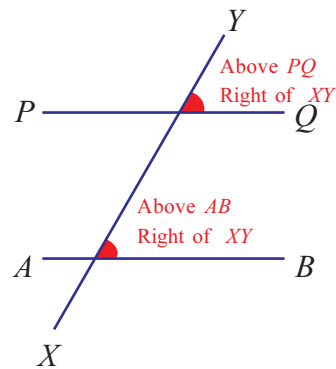
Of these, the angle at the bottom is on top of the horizontal line, and on the right of the slanted line; the angle at the top also is on top of its horizontal line, and on the right of the slanted line.

There are three more pairs of angles which are at similar positions at the bottom and top:



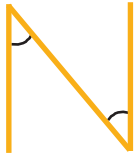
Angles in each such pairing, done according to similar positions, are called *corresponding angles*.

Corresponding angles



Alphabet angles

Draw the letter N as below:



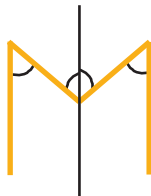
What is the relation between the two angles marked?

Now look at the letter M.

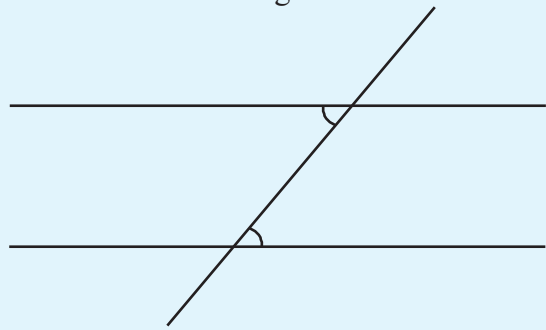


See any relation between the angles?

Draw a line down the middle and see:



Equal angles from the bottom and top can be paired in another manner. See the angles below:

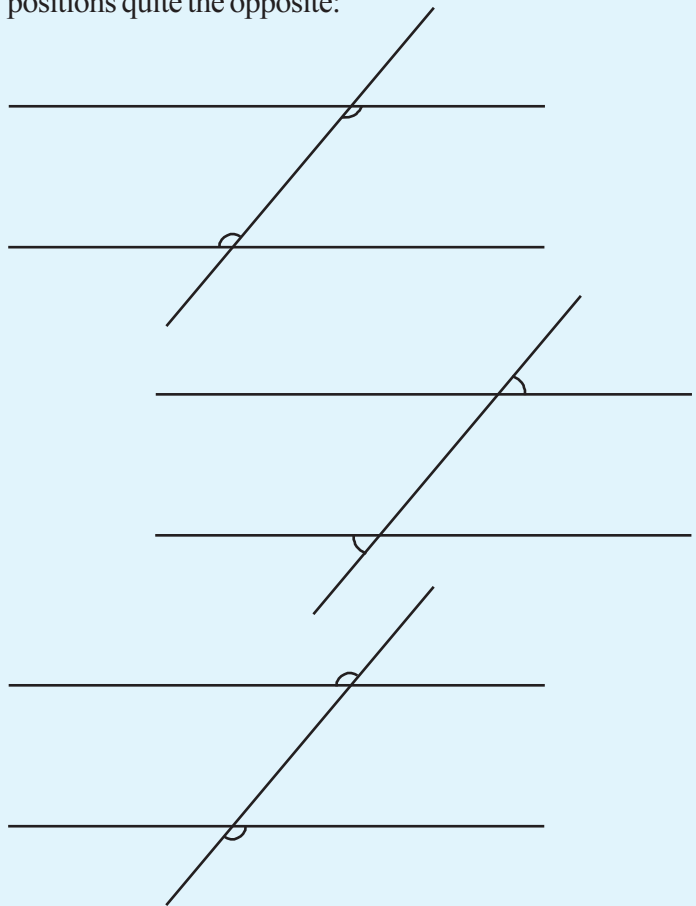


The bottom angle is on top of the horizontal line, and on the right of the slanted line

What about the top angle?

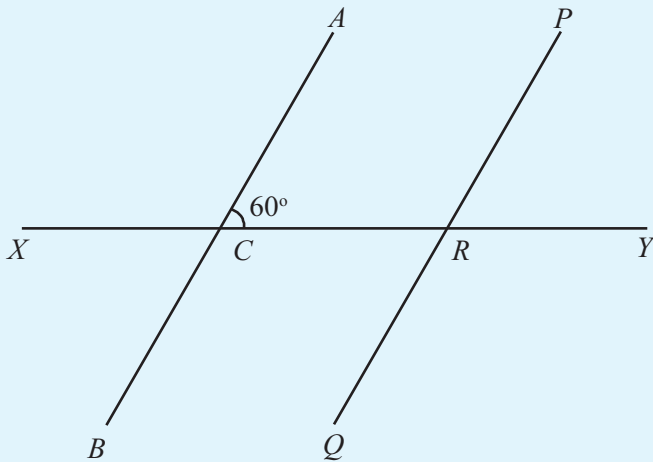
At the bottom of the horizontal line, and on the left of the slanted line.

We can pair the equal angles in three other ways, with the positions quite the opposite:



Angles in each such pairing, done with reverse positions, are called *alternate angles*.

In the figure below, the pair of parallel lines and the cutting line are all named. The measure of one angle is also given. Complete the tables below by writing the names and measures of all pairs of corresponding and alternate angles:



Corresponding angles	
Names	Measure
$\angle ACY, \angle PRY$	60°

Alternate angles	
Names	Measure
$\angle ACY, \angle QRX$	60°

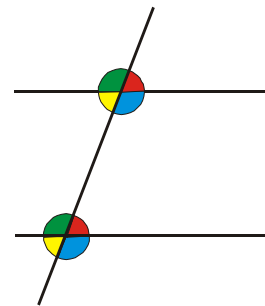
In short,

The angles formed by a line cutting across two parallel lines can be paired in several ways, choosing one angle of the four made with one line and one of the four made with the other. Of these, eight pairs have equal angles. Based on the positions with respect to the lines, angles in four such pairs are called corresponding and angles in the other four are called alternate.

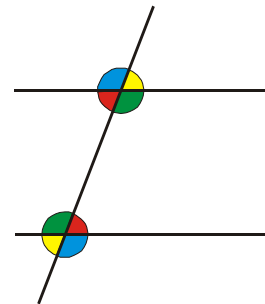
Corresponding and alternate

Look at the picture.

Pairs of corresponding angles are of the same colour.



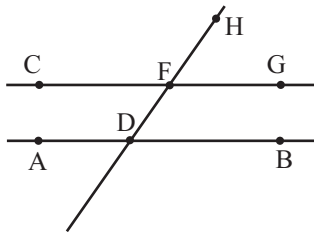
What about this picture?





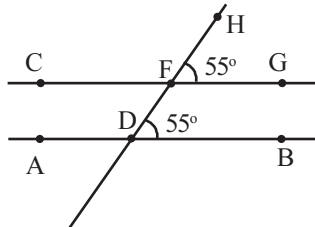
In GeoGebra draw a line AB and a parallel line through C.

Mark points D and F on these and join them. Mark two points G and H as in the figure.



Next select the **Angle** tool and click on G, F, H in that order and then on B, D, F.

Now we can see the measure of these angles.



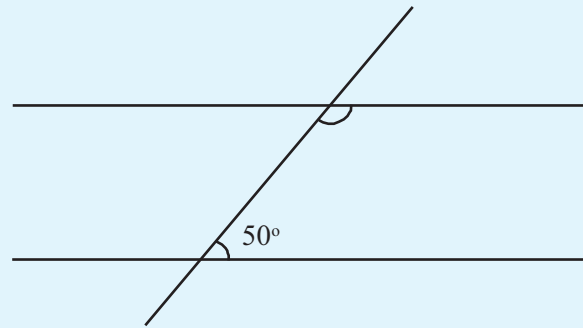
Use the **Move** tool to change the position of F.

Find the other angles at F and D like this.

We can also colour these angles. Right click on any angle and select **Object properties**. In the menu, click on **Color** and choose a colour. Choose the same colour for equal angles at the top and bottom.

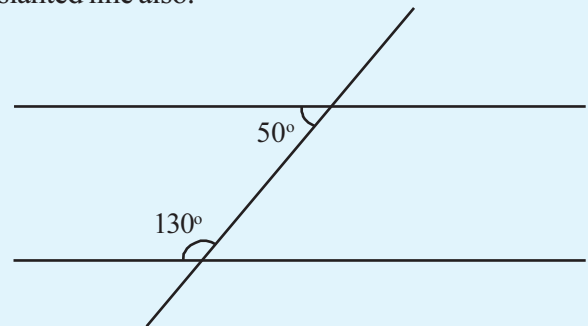
Supplementary angles

Let's have another look at a picture of a two parallel lines cut by a third line:



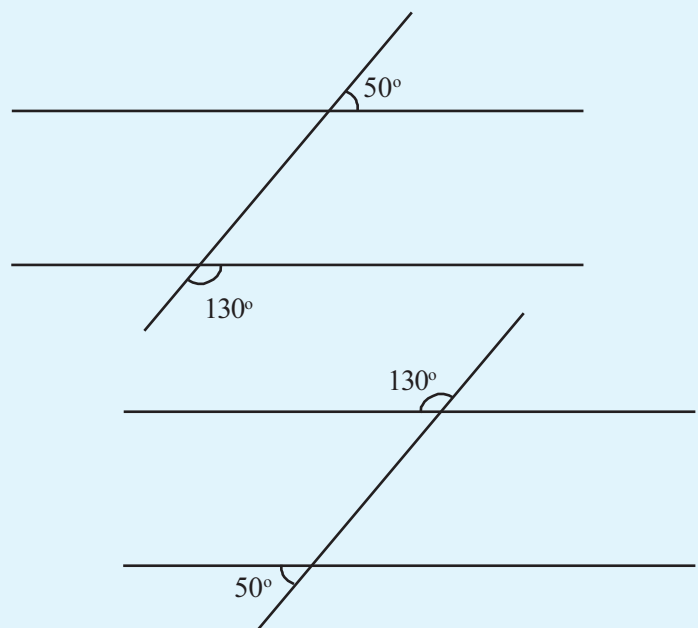
How much is the marked upper angle?

There is such a pair of supplementary angles on the left of the slanted line also.

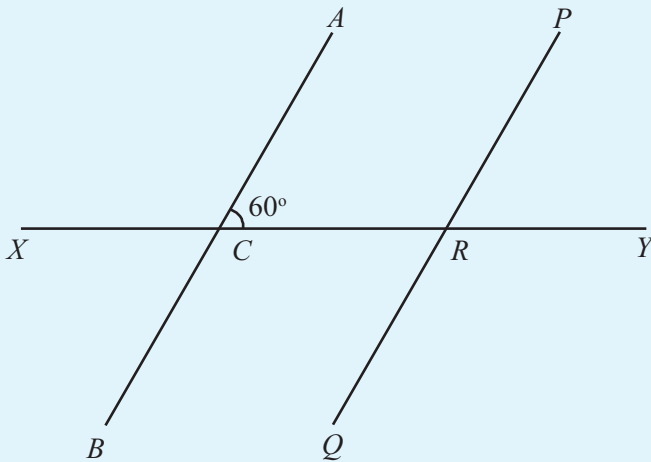


The angles in each of these two pairs are called *co-interior*.

There are also two pairs of *co-exterior* angles.



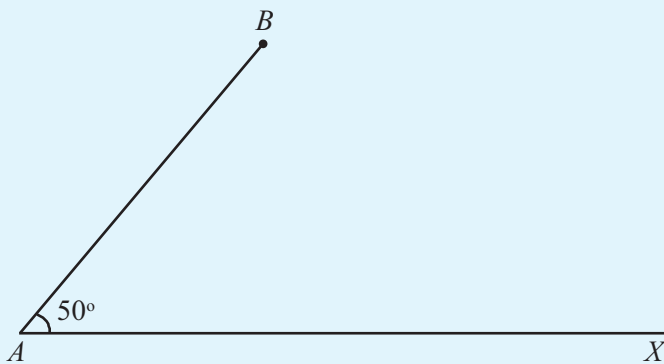
In the figure below, the lines AB and PQ are parallel and the line XY cuts them at C and R . Find all the pairs of co-interior and co-exterior angles and write down their names and measures.



Co-interior angles	Co-exterior angles

Parallel lines and triangles

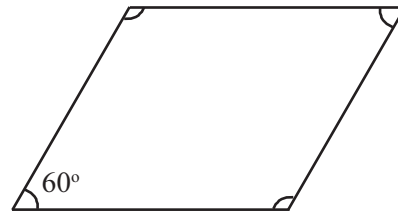
See this figure:



A line starting from B is to be drawn, parallel to AX .

Sums of angles

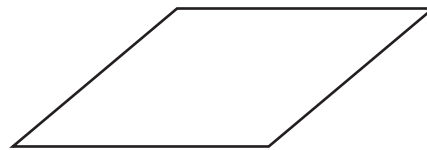
Look at the parallelogram:



Can you write the measures of the other three angles?

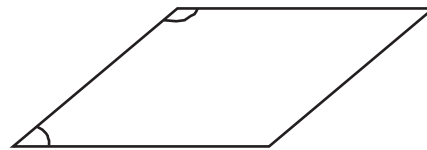
What is the sum of all the angles?

Now look at this parallelogram:

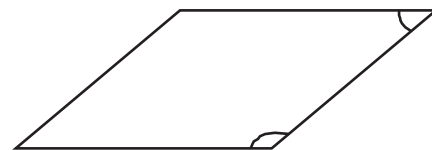


No angle is given.

But can't you say what the sum of the two angles on the left is?



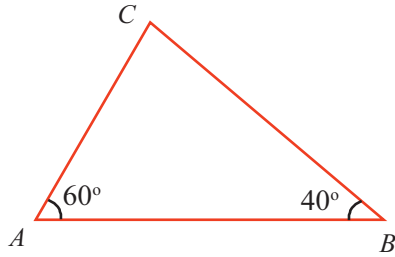
What about those on the right?



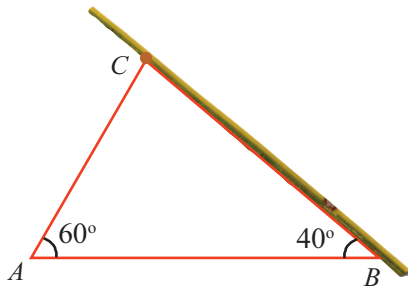
So, what is the sum of all four angles?

Triangle and parallel lines

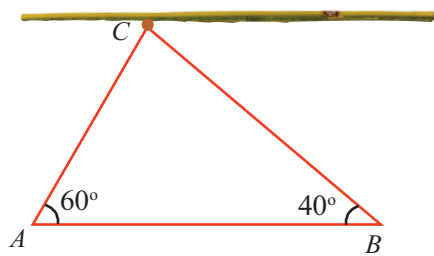
Draw a triangle like this in a piece of cardboard.



Now place a long and thin stick along the side BC and stick a pin through it at C .



Rotate the stick upwards till it is parallel to AB .



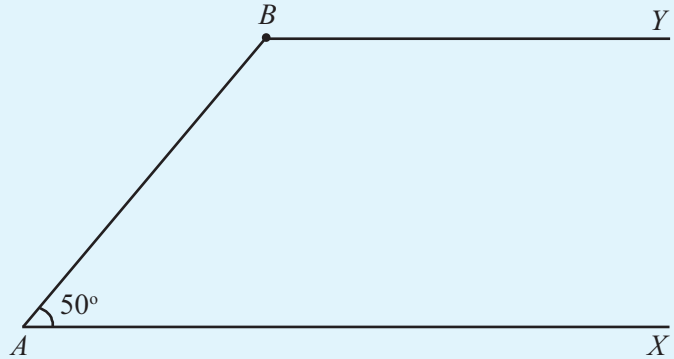
Now what angle does the stick make with BC ?

And with AC ?

So, how much is the angle at C in the triangle?

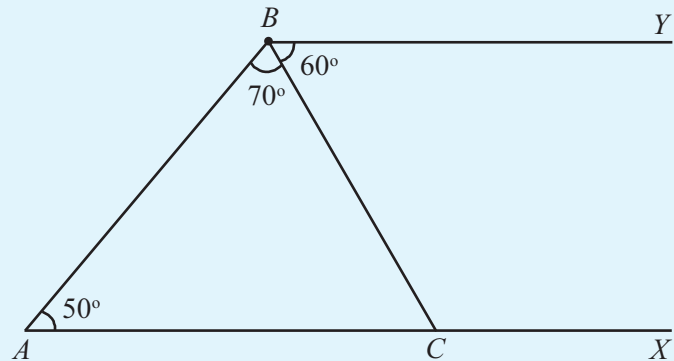
How do we do this?

The angles at A and B are co-interior, right?



Draw this in your notebook.

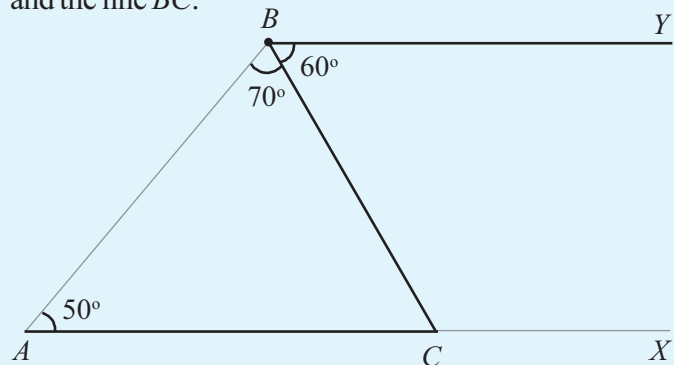
Next draw a slanted line through B in this figure. Let the angle with AB be 70° . This line is not parallel to AX . Let the point at which this line meets AX be named C :



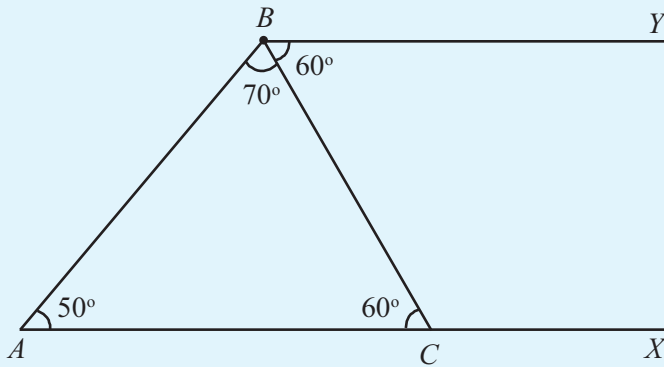
Now ABC is a triangle. And we know the angles at A and B .

How much is the angle at C ?

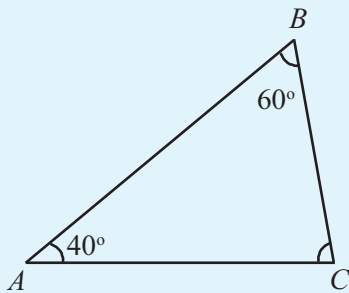
The lines AC and BY are parallel. Concentrate on these and the line BC :



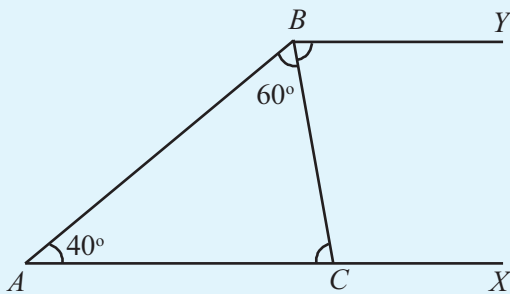
$\angle ACB$ and $\angle CBY$ are alternate angles:



Likewise, can you compute the angle at C in the triangle below?



How about extending AC and drawing a line from B parallel to it, like the first figure?



We have to calculate $\angle ACB$.

First, we note that it is equal to $\angle CBY$ (Why?).

To find $\angle CBY$, we need only know $\angle ABY$; and this angle together with $\angle A$ make a co-interior pair.

So,

$$\angle ABY = 180^\circ - 40^\circ = 140^\circ$$

From this we get

$$\angle CBY = 140^\circ - 60^\circ = 80^\circ$$

Thus we find

$$\angle ACB = \angle CBY = 80^\circ$$

Parallelogram and triangle

Look at this parallelogram:

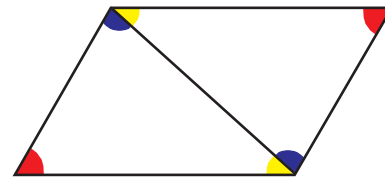


What is the relation between the angles marked red?

And the angles marked green?

And those marked in different colours?

Now join the opposite corners to make two triangles:



What is the relation between the angles marked blue?

And those marked yellow?

What is the sum of three angles of different colours?

What is the sum of the three angles of each triangle?

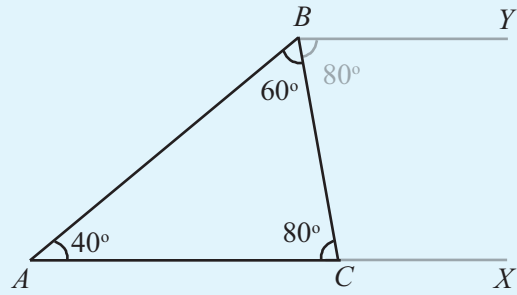
Theorem and proof

How do we conclude that the sum of the angles in any triangle is 180° ? Is it enough if we draw several triangles, measure the angles and check the sum? How can we say for sure that for a triangle not among these, the sum is 180° ?

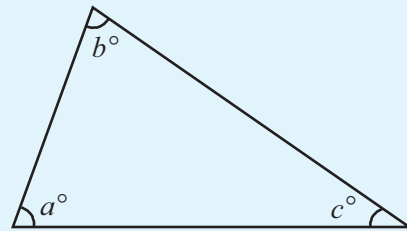
In any triangle, we can draw a line through one vertex, parallel to the opposite side. And then using the relations between angles made by parallel lines, we can see that the sum of the angles of a triangle is 180° .

By doing this, we achieve much.

- Even if we change the triangle, the arguments used do not change. So, the conclusion of these arguments also is true for the changed triangle.
- Properties of parallel lines can be easily recognized. But the fact that sum of the angles of a triangle is 180° , is not immediately obvious. This is an example of establishing complex ideas, starting from simple truths.
- When arguments are linked one after another using ideas of parallel lines, we not only get the theorem that sum of the angles of a triangle is 180° , but also see why it is so.

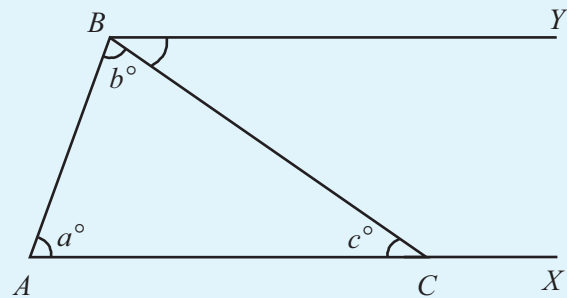


Now see this triangle:



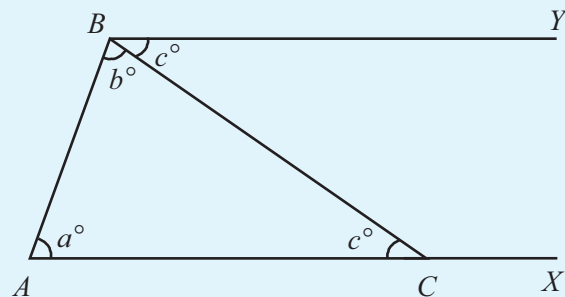
The measures of the angles are given by the letters a, b, c . What is the relation between them?

Let's draw parallel lines as before.



From this figure, we see that

$$\angle CBY = \angle ACB = c^\circ$$



From the figure above,

$$\angle A + \angle ABY = 180^\circ$$

That is,

$$a + b + c = 180$$

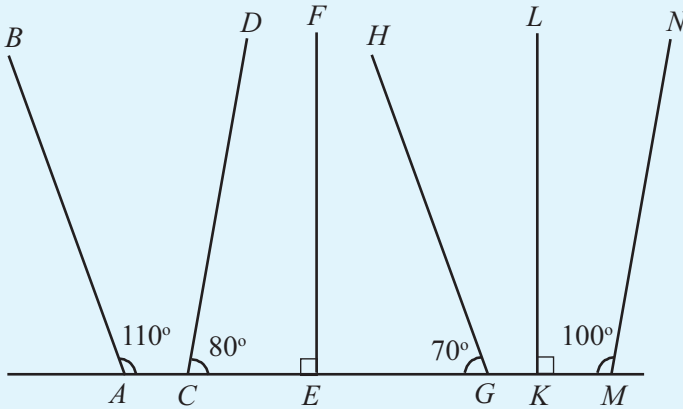
What do we get from this?

The sum of the angles of any triangle is 180°.

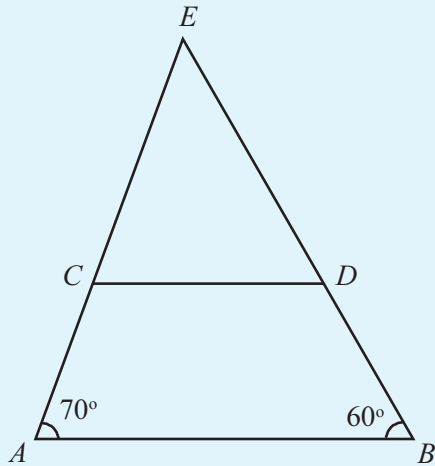


Let's do it!

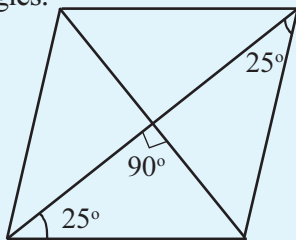
- Find out the pairs of parallel lines in the figure below:



- In the figure below, AB and CD are parallel. Compute all the angles in the figure.

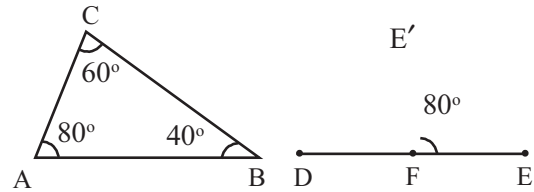


- In the figure below, a parallelogram is divided into four triangles by the diagonals. Calculate the angles of all these triangles.



Unchanging relation

In GeoGebra, use the polygon tool to draw triangle ABC . Using the **Angle** tool, we can get the measures of its angles.



Now draw a line DE and mark a point F on it. Select the **Angle with given size** tool and click on E and F in that order. In the dialog box, type α as the angle and click **OK**. We get a new Point E' . With the same tool, click on E' and F and type in β as the angles. We get a new point E'' . Click on E'' and F and type γ as the angle to get another point E''' . Join FE' and FE'' . In this picture, we have $\angle EFE' = \angle A$; $\angle E'FE'' = \angle B$; $\angle E''FE''' = \angle C$. Give the same colours to equal angles.

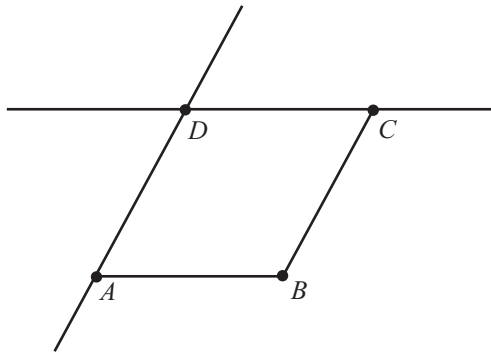
Use the **Move** tool to change the triangle's angles. The angles in the figure on the right also change. What remains unchanged?



Drawing parallelograms

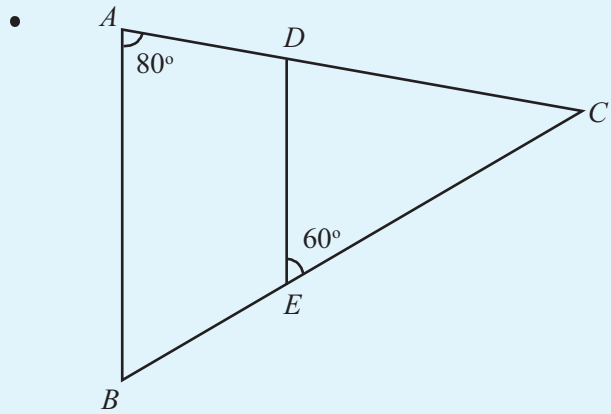
Let's draw a parallelogram using GeoGebra.

First draw two lines AB and BC . Use the **Parallel line** tool to draw the line through C parallel to AB and the line through A parallel to BC . Mark the point D where the lines meet. Use the **Polygon** tool to complete the parallelogram $ABCD$.



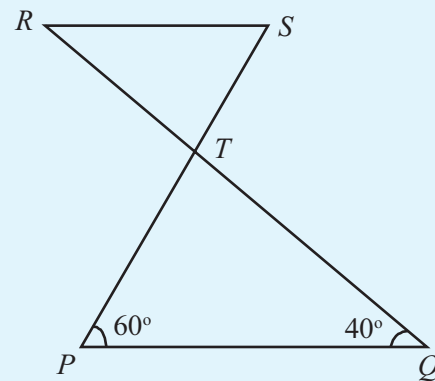
Lines sticking out may be deleted.

Now right click on AB and select **Trace on**. Do this for BC also. Select the **Move** tool, click within the parallelogram and drag upwards. What do you get?

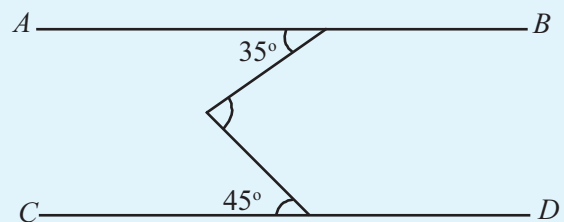


In the figure above, AB and DE are parallel. Compute the angles of both triangles.

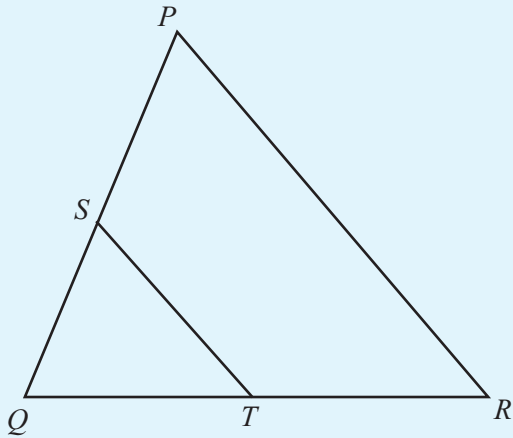
- In the figure below, PQ and RS are parallel. Calculate all other angles in the figure.



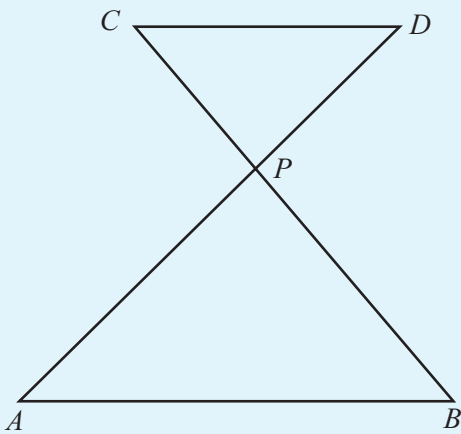
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In the figure, AB and CD are parallel. Compute the third angle.



In the figure, PR and ST are parallel. Is there any relation among the angles of the two triangles?



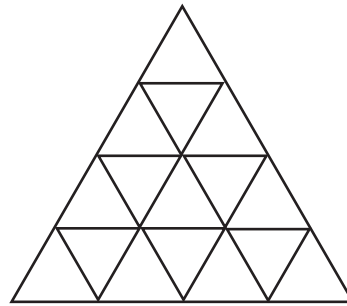
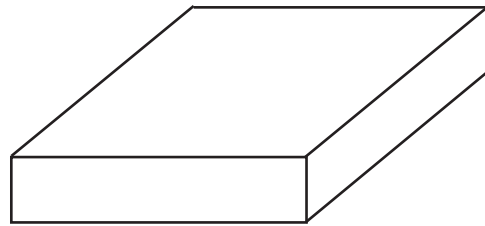
In the figure above, AB and CD are parallel. What is the relation between the angles of the small and large triangles?

- Draw a line AB and a line CD parallel to it. Draw a line EF cutting across these lines at the points M and N . Measure and write down one of the angles so made. Calculate the other angles. Write down the pairs of corresponding angles, alternate angles, co-interior angles and co-exterior angles.



Drawing pictures

Try to draw these pictures using GeoGebra.



Use the **Regular Polygon** tool to draw the large triangle.

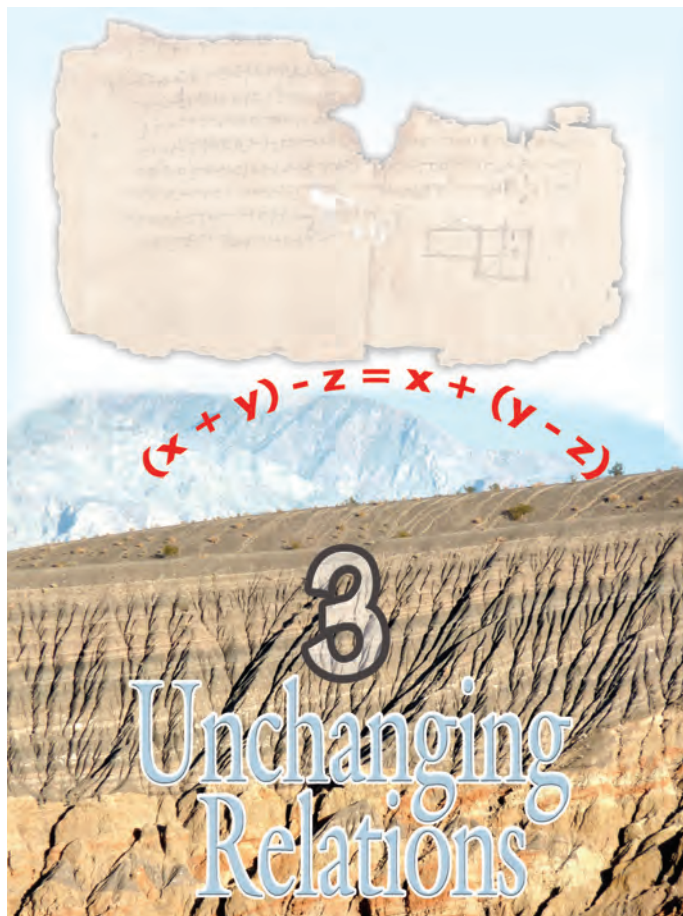
Looking back



Achievements	On my own	With teacher's help	Must improve
<ul style="list-style-type: none"> Explaining parallel lines as lines which are a constant distance apart. 			
<ul style="list-style-type: none"> Explaining parallel lines in terms of perpendicularity and slant. 			
<ul style="list-style-type: none"> Drawing parallel lines using different methods and proving that they are parallel. 			
<ul style="list-style-type: none"> Explaining parallel lines using models. 			
<ul style="list-style-type: none"> Given one angle made by a line cutting across a pair of parallel lines, computing the other angles and justifying the computation. 			
<ul style="list-style-type: none"> Using computer to describe various properties of parallel lines. 			
<ul style="list-style-type: none"> Explaining the classification of pairs of corresponding, alternate, co-interior and co-exterior angles. 			
<ul style="list-style-type: none"> Proving that the sum of the angles of a triangle is 180°. 			

3

Unchanging Relations



Measures and relations

We can draw squares of different sizes. The perimeter and area change with the length of sides. But in all the squares, the perimeter is always four times the length of side; and the area is the product of the length of side by itself.

There are many such situations where even though the measures change, certain relations between them do not change. For example, if we take several objects made of iron, their volume and weight will be different. But if we divide the weight by the volume, we get the same number, 7.8. This is called the density of iron. Similarly if we do this for objects made of copper, weight divided by volume gives 8.9. This is the density of copper.

In such contexts, the unchanging relation between measures is denoted using letters. For example, if we write the weight of an object made of iron as w and the volume as v , then we can write

$$w = 7.8v$$

If we take copper instead of iron, the relation will be

$$w = 8.9v$$

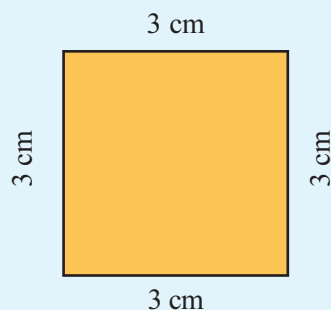
In general, if the weight of an object is w , its volume is v and if it is made of a material of density d , then their relation can be written as

$$w = dv$$

Measures and numbers

One side of a square is 3 centimeters.

What is its perimeter?



What about a square of side 5 centimeters?

The perimeter of any square is four times the length of a side, right? And don't you remember how we wrote it in a short form using letters?

If we write s for the length of a sides of a square and p for the perimeter, then we can write

$$p = 4 \times s$$

We also know that when we write such relation between numbers using letters, we don't write the multiplication symbol \times (and why). So, we write the relation between the length of side s and perimeter p of a square as

$$p = 4s$$

What about a rectangle, instead of a square?

If we know the lengths of two unequal sides, how do we find out the perimeter?

If we denote the length of each side as l and b and perimeter as p , how do we write the relation between p , l and b ?

How do we write the relation between the lengths of sides of a rectangle and its area using letters?

Number relations

Look at these sums:

$$1 + 2 = 3$$

$$2 + 3 = 5$$

$$3 + 4 = 7$$

We add two consecutive natural numbers.

Now look at these:

$$(2 \times 1) + 1 = 3$$

$$(2 \times 2) + 1 = 5$$

$$(2 \times 3) + 1 = 7$$

We double natural numbers and add 1.

How come we end up with the same numbers?

Let's take a natural number and do the first operation. For example, if we start with 7, the next natural number is 8; and the sum

$$7 + 8 = 15$$

Suppose we write the 8 here as $7 + 1$? We see that

$$7 + 7 + 1 = (2 \times 7) + 1 = 15$$

This we can do for any natural number instead of 7.

If we take any natural number and add the next natural number, or if we double the first number and add 1, we get the same number as the result.

Can we do this only with natural numbers?

For example let's take the fraction, half. There is no meaning in saying the next fraction; but we can say, the number got by adding one to it. That is, half and one make one and a half. The half we started with, together with the one and a half we got now makes two.

On the other hand, doubling half makes one; and one added to it makes 2. That is,

$$\frac{1}{2} + 1\frac{1}{2} = \frac{1}{2} + \left(\frac{1}{2} + 1\right) = \left(2 \times \frac{1}{2}\right) + 1$$

Measures and numbers

Numbers were invented by Man to indicate and compare various measurements. For example, instead of saying, “a large group of people”, if we say, “a group of hundred people”, one gets a clear picture. Or instead of saying “walked some distance”, we can be more precise and say, “walked two and a half kilometers”.

Length, weight and time are measured directly using instruments; area, volume and density are not directly measured, but are got by computations. For that we need operations with numbers. For example, to find volume of a rectangular block we must measure its length, width and height and calculate the product.

In course of time, men started thinking about operations with pure numbers without linking them to measurement. For example, after discovering that

to compute the perimeter of a rectangle, instead of measuring all four sides and adding, we need only measure two different sides and then double their sum

they went on to discover the number principle that

instead of multiplying two numbers separately by two and adding, we need only multiply their sum by two.

Much latter, we started writing this using letters as

$$2x + 2y = 2(x + y)$$

Number theory

We have seen that general properties of operations on numbers can be stated in shorthand, using letters.

For example, the fact that

zero added to any number gives the same number.

Can be shortened to

$$x + 0 = x, \text{ for any number } x$$

Similarly, the fact that

To find the sum of two numbers, they can be added in any order

can be written

$$x + y = y + x, \text{ for all number } x \text{ and } y.$$

Simple ideas like these which are easily recognised need not be shortened in this manner.

But look at this:

Adding to a number, one more than itself, and adding one to double of the number give the same result.

It is much more convenient to shorten the elaborate statement as

$$x + (x + 1) = 2x + 1, \text{ for any number } x.$$

Here we must note an important thing. It is easy to remember such shorthand formulas, but to use them in time, we must know their meaning clearly.

Folding umbrellas are easy to carry; but if we don't know how to open them, we are sure to get wet.

This computation is right, whatever fraction we start with. So we can extend the fact given above.

If we add to any number, one more than itself or if we double the number and add one to it, we get the same number as the result.

This general property of numbers can be denoted in a shorter form using letters. For this, let's write x for the number we start with. One added to it is $x + 1$; and the sum of these is $x + (x + 1)$. Next, x doubled is written as $2x$; and one added to this is $2x + 1$. So the general property we have discovered can be written thus;

$$x + (x + 1) = 2x + 1, \text{ for every number } x.$$

This mathematical shorthand of writing number related facts using letters, is called *algebra*.

Let's look at a simple example. Suppose, to one number we add another and then subtract the added number. What do we have now? The first number back, right?

If we take the first number as x and the added number (which is later subtracted) as y , then we can write in algebra what happens as

$$(x + y) - y = x, \text{ for all numbers } x, y.$$

Note that this is a general principle, which applies to all numbers. Certain properties hold only for specific numbers; for example both $2 + 2$ and 2×2 give 4. But $x + x = x \times x$ is not a general principle (if we replace 3 for 2, it becomes incorrect).

Now do each of the following operations on several numbers and describe the results in a different form. Write each such relation in ordinary language. Then write it as an algebraic expression, using letters.

- Add to a number, two more than the number.
- Add one to a number and subtract two.
- From a number, subtract another and then add twice the subtracted number.
- Add to a number the double of itself.
- Add two consecutive natural numbers and find the number, one less than this.
- Add two consecutive odd numbers and subtract the even number in their middle.
- To a number, add another and subtract the first.
- To a number, add another and then add this sum to the first number.
- Subtract two times a number from five times the number.
- Add two times a number with three times the same number.

However we add

What is $38 + 25 + 75$?

We can add in the given order:

$$38 + 25 = 63$$

$$63 + 75 = 138$$

We can also add like this:

$$25 + 75 = 100$$

$$38 + 100 = 138$$

We don't need pen and paper to do it the second way.

Now try this:

$$29 + \frac{1}{3} + \frac{2}{3}$$

To do this easily, which two numbers would you add first?

What do we see from these two sums?

Two operations, one result

Adding one to twice a number is an arithmetical operation and the number got by doing it is the result. For example, doing this on 3 gives 7, and on 10 gives 21.

Adding one to a number and then adding the sum to the original number is another operation. For example, this operation on 4 gives $4 + (4 + 1) = 9$. These two operations done on the same number give the same result. It is this fact that we write in algebra as

$$x + (x + 1) = 2x + 1$$

In this, the short form $x + (x + 1)$ on the left side means, adding a number to one more than the number. The form $2x + 1$ on the right means, doubling a number and adding one to it. The equal sign says that these two operations lead to the same result.

Similarly, we can write the operation of doubling two numbers separately and adding as $2x + 2y$ in algebra. The algebraic form of the operations of doubling the sum of two numbers is $2(x + y)$. The general principle that these two operations on the same pairs of numbers give the same result is written

$$2x + 2y = 2(x + y) \text{ for all numbers } x, y.$$

Many general properties of numbers have a similar form, starting that apparently different operations give the same results.

Arithmetic and algebra

Studies on numbers are generally known as arithmetic; and stating them using letters is algebra.

In arithmetic, the operations of adding 3 and 7 is written $3 + 7$. The sum, or the result of the operation, is 10. And we write,

$$3 + 7 = 10$$

combining the operation and the result.

In algebra, the operation of adding two numbers can be written $x + y$. How do we write the sum? We cannot find it without knowing the actual numbers added. So we can only write $x + y$ for the sum also.

But the fact that

Any number added to itself is double the number

can be written in algebra as

$$x + x = 2x, \text{ for all number } x.$$

Note that this is not a general principle, but is the very definition of multiplication by 2.

What do you mean?

Algebra is easy?

Yes! Whenever I don't know a number, I can write x .



To find the sum of three numbers, either we can find the sum of the first two and add this to the third, or we can find the sum of the last two and add this to the first. This we can state in another form

Instead of adding to one number, two numbers one after another, we need only add their sum.

We can show the order of operations using brackets. For example, we can write the first sum like this:

$$(38 + 25) + 75 = 38 + (25 + 75)$$

And the second sum like this

$$\left(29 + \frac{1}{3}\right) + \frac{2}{3} = 29 + \left(\frac{1}{3} + \frac{2}{3}\right)$$

So we can write the general principle of adding three numbers using algebra:

$(x + y) + z = x + (y + z)$, for all numbers x, y, z .

Now suppose we went to calculate $36 + 25 + 64$.

Isn't it easier to add 36 and 64 first?

First write $25 + 64$ as $64 + 25$ and then write the full sum as $(36 + 64) + 25$.

That is, addition of numbers can be done in any order.

Now try to find these sums in your head:

- $49 + 125 + 75$
- $347 + 63 + 37$
- $88 + 72 + 12$
- $\frac{1}{4} + 1\frac{3}{4} + 2$
- $15.5 + 0.25 + 0.75$
- $8.2 + 3.6 + 6.4$

Addition and subtraction

We have seen the general principle of adding three numbers.

What if we subtract repeatedly, instead of adding?

Look at this problem:

Unni had 500 rupees with him and gave 100 rupees to Appu. Sometime later, Abu borrowed 50 rupees from him. Now how much money does Unni have?

After the loan to Appu, he had

$$500 - 100 = 400 \text{ rupees}$$

And after lending money to Abu, he had

$$400 - 50 = 350 \text{ rupees}$$

We can also think in a different way. The total money he lent is

$$100 + 50 = 150 \text{ rupees}$$

So what he finally has is

$$500 - 150 = 350 \text{ rupees}$$

In other words, whether we do $(500 - 100) - 50$ or $500 - (100 + 50)$, we get the same number as the result.

Similarly can you do this in your head?

$$218 - 20 - 80$$

How can we state what we have seen here as a general principle?

Instead of subtracting from one number, two numbers one after another, we need only subtract the sum of these two numbers.

And in algebra?

$$(x - y) - z = x - (y + z), \text{ for all numbers } x, y, z.$$

Instead of adding or subtracting two numbers in succession, suppose we add one number and subtract another?

Look at this problem.

There were 38 children when the class started. 5 came in late. Sometime later, 3 went to attend the Math Club meeting. How many are in the class now?

Let's do this in the order of events. When 5 more joined, there were

$$38 + 5 = 43$$

Difference of difference

Finding the sum of three numbers is very natural. So there is actually no need to keep in mind the algebraic expression

$$(x + y) + z = x + (y + z)$$

Its only use is that it sometimes make computation easier. For example, in calculating $29 + 37 + 63$, if we can quickly see $37 + 63 = 100$, then the total can be easily formed as 129. (To add in the given order may require pen and paper).

But we must be careful with subtraction.

The meaning of

$$(10 - 3) - 2$$

is, subtract 3 from 10 and then subtract 2 from the resulting 7; that is, the final result is 5.

What about this?

$$10 - (3 - 2)$$

First subtract 2 from 3 to get 1; then subtract this 1 from 10, to get 9.

In other words, these operations give different results. But the result of both $(10 - 3) - 2$ and $10 - (3 - 2)$ is 5. We must keep in mind the general result

$$(x - y) - z = x - (y + z)$$

and its meaning:

instead of subtracting two numbers one after another, we need only subtract their sum.

Theory and practice

To do $25 + 20 - 15$, we can first add and then subtract to get $45 - 15 = 30$; or we can first do the subtraction and then addition to get $25 + 5 = 30$.

But in $25 + 10 - 15$, it is easy to see that we cannot do the subtraction first.

In operations like this with actual numbers, it is easy to see that certain operations cannot be done. But when we write them in algebra, we must also state the conditions which make them true.

That is why in writing

$$(x + y) - z = x + (y - z)$$

we also write the condition $y > z$.

And when 3 children left, there were

$$43 - 3 = 40$$

If instead, we look at the events together, we can compute like this: 5 children came in and 3 others left. So the final increase in number is only

$$5 - 3 = 2$$

At the beginning there were 38 children.

So the total number now is

$$38 + 2 = 40$$

Thus, instead of adding one number and then subtracting another, we can subtract the second number from the first and add. For example,

$$(108 + 25) - 15 = 108 + (25 - 15) = 118$$

We should be a bit careful here. To compute like this, the number subtracted should be less than the number added. For example, look at this problem:

$$25 + 10 - 15$$

To do this, we cannot first subtract 15 from 10.

In algebra

$$(x + y) - z = x + (y - z), \text{ for all numbers } x, y, z \text{ with } y > z.$$

Using these ideas, try to do these problems without pen and paper:

- $(135 - 73) - 27$
- $(37 - 1\frac{1}{2}) - \frac{1}{2}$
- $(298 - 4.5) - 3.5$
- $(128 + 79) - 29$
- $(298 + 4.5) - 3.5$
- $(149 + 3\frac{1}{2}) - 2\frac{1}{2}$

Subtracting and adding

Look at this problem.

Gopu had 110 rupees in his savings box. He took out 15 rupees to buy a pen. He got a pen for 10 rupees. He

returned 5 rupees to the box. How much is in the box now?

Let's first compute in the order of what Gopu did. When he took out 15 rupees, the box had

$$110 - 15 = 95 \text{ rupees}$$

Since he put back 5 rupees, the box now has

$$95 + 5 = 100 \text{ rupees}$$

After all this have happened, we can also think like this: he took out 15 rupees and put back 5 rupees; so the actual decrease in the box is only

$$15 - 5 = 10 \text{ rupees}$$

So the box now has

$$110 - 10 = 100 \text{ rupees}$$

Writing the first computation as $(110 - 15) + 5$ and the second as $110 - (15 - 5)$, what we see is that

$$(110 - 15) + 5 = 110 - (15 - 5)$$

That is, instead of subtracting a number and then adding another, we need only subtract the difference of the second from the first. For example,

$$(29 - 17) + 7 = 29 - (17 - 7) = 19$$

Can we do this in all problems of subtracting and then adding? For example, can we rewrite

$$(29 - 7) + 17$$

in this manner?

So, the change of operation is written in algebra as

$$(x - y) + z = x - (y - z), \text{ for all numbers } x, y, z \text{ with } y > z$$

Now use this idea to calculate these without pen and paper.

- $(135 - 73) + 23$
- $(38 - 8\frac{1}{2}) + \frac{1}{2}$
- $(19 - 6.5) + 5.5$
- $135 - (35 - 18)$
- $4.2 - (3.2 - 2.3)$

Less and more

Look at these differences:

$$10 - 9 = 1$$

$$10 - 8 = 2$$

$$10 - 7 = 3$$

$$10 - 6 = 4$$

When we subtract less, we get more, right?

How much more?

When one is subtracted, we get one more; when two is subtracted, we get two more.

In general

When less is subtracted, the result is more; whatever less is subtracted, that much more is the result.

Let's write this in algebra. First note that if x and y are two numbers, then y subtracted from x is $x - y$.

Now taking another number z , the number $y - z$ is z less than y ; so $x - (y - z)$ is z more than $x - y$.

Thus,

$$x - (y - z) = (x - y) + z$$

No use weeping! If you lessen your laziness, you will get more.



Sum and difference

What happens when we add the sum and difference of two numbers?

Difference is the smaller number subtracted from the larger; sum is the small number added to the larger.

For example, taking the numbers as 7 and 3, the sum is $7 + 3$ and difference is $7 - 3$. Without reducing these to 10 and 4, if we write their sum, we get

$$(7 + 3) + (7 - 3)$$

Here the larger number 7 is added twice; the smaller 3 is added once and then subtracted.

So that net result is $7 + 7 = 14$.

In other words, changing the order of operations, we find

$$(7 + 3) + (7 - 3) = (7 + 7) + (3 - 3) = 14$$

It is this fact that we write as the general algebraic rule,

$$(x + y) + (x - y) = (x + x) + (y - y) = 2x$$

You mean this large mathbook became so small on translation?

Brother! That is algebraic shorthand for you!



Sums and differences

Athulya often teases her classmates with her new discoveries. Today she had a new trick. “Think of any two numbers and give me their sum and difference; I can tell you the numbers.”

“Sum is 10 and difference is 2”, Rahim started.

“Easy! numbers are 6 and 4”, Athulya said.

“Sum 16, difference 5”, mischievous Jessie challenged.

Athulya thought for a moment and said,

“Nice try! Numbers are $10\frac{1}{2}$ and $5\frac{1}{2}$ ”

How did Athulya find the numbers?

How do we find two numbers, using their sum and difference?

Let’s take the numbers as x and y . Then the sum is $x + y$. If we take the larger number as x , the difference is $x - y$. Using these, we have to find x and y .

To find x from $x + y$, we need only subtract y :

$$(x + y) - y = x$$

But we don’t know y .

What if we add x again?

$$(x + y) - y + x = x + x = 2x$$

Subtracting y and then adding x is the same as adding x and then subtracting y , right?

$$(x + y) + (x - y) = 2x$$

What does this mean?

Adding sum and difference gives twice the larger number.

For example, Rahim’s sum is 10 and difference is 2. Their sum is 12. This is twice his larger number. So the larger number is 6; and his smaller number is $10 - 6 = 4$.

Now let's look at what Jessy said: sum 16, difference 5; their sum is 21. So the larger number is half of this and so it is $10\frac{1}{2}$; the smaller is $16 - 10\frac{1}{2} = 5\frac{1}{2}$.

Do you get Athulya's trick?

We can see another thing here. Subtract the difference from the sum:

$$\begin{aligned}(x + y) - (x - y) &= (x + y) - x + y \\ &= x + y - x + y \\ &= x - x + y + y \\ &= 2y\end{aligned}$$

What does this mean?

Subtracting the difference from the sum gives twice the smaller number.

For example, in Rahim's case, sum is 10 and difference 2. So, double the smaller number is $10 - 2 = 8$, and so the smaller number is half of this, which is 4.

The sum and difference of some pairs of numbers are given below. Can you find the numbers?

- Sum 12, difference 8
- Sum 140, difference 80
- Sum 23, difference 11
- Sum 20, difference 5

Addition and multiplication

We have seen that twice a number added to thrice the number gives five times the number. (The last problem in the section **Number relations**)

What is the algebraic form of this statement?

$$2x + 3x = 5x, \text{ for every number } x.$$

Different ways

Look at this problem:

The price of a book and pen together is 16 rupees. The price of the book is 10 rupees more than that of the pen. What is the price of each?

If we forget about books and pens and look at the prices as mere numbers, the problem is like this:

The sum of two numbers is 16 and their difference is 10. What are the numbers?

Double the larger number is $16 + 10 = 26$; so the larger number is 13. And then the smaller number is $16 - 13 = 3$. Thus the price of the book is 13 rupees and the price of the pen is 3 rupees.

There is another way. Instead of a book and pen, suppose two books are bought. Since the price of the book is 10 rupees more, we would have to give 10 more rupees; that is, $16 + 10 = 26$ rupees.

This is the price of two books. So, the price of one book is 13 rupees.

Calendar math

Take any month's calendar and mark four numbers making a square:

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

In the picture, the sum of the chosen numbers is $8 + 9 + 15 + 16 = 48$. Divide this by 4 and subtract 4. We get the first number 8, right?

Why does this happen?

If we take the first number as x , the numbers marked are

x	$x + 1$
$x + 7$	$x + 8$

Their sum is

$$x + (x + 1) + (x + 7) + (x + 8) = 4x + 16$$

We can write this as

$$\begin{aligned} 4x + 16 &= (4 \times x) + (4 \times 4) \\ &= 4(x + 4) \end{aligned}$$

This is adding 4 to the first number and then multiplying by 4.

So to get back the first number we need to divide by 4 and then subtract 4.

This we can state in another manner.

Instead of multiplying a number by 2 and 3 separately and adding, we need only multiply this number by 5.

For example,

$$(2 \times 16) + (3 \times 16) = 5 \times 16 = 80$$

In this, suppose we take other numbers instead of 2 and 3?

Look at this problem:

In a math conference, two rooms are used for discussion. In one room there are 40 people and in the other, 35. At tea-time each is to be given 2 biscuits. How many biscuits are needed?

In the first room, we need

$$40 \times 2 = 80$$

And in the second,

$$35 \times 2 = 70$$

So altogether, we need

$$80 + 70 = 150$$

We can also think like this: The total number of people is

$$40 + 35 = 75$$

So the number of biscuits needed is

$$75 \times 2 = 150$$

What do we see here? Instead of multiplying 2 by 40 and 35 separately and then adding, we need only multiply their sum 75 by 2.

This can be done in multiplication by fractions also. For example, if we add half of 4 and half of 6, we get $2 + 3 = 5$; half of the sum 10 is also 5.

What general principle do we see in all these?

Multiplying two numbers by a number separately and adding give the same result as multiplying their sum by the number.

How do we write this in algebra?

$$xz + yz = (x + y)z, \text{ for all numbers } x, y, z$$

What about subtraction?

Multiplying two numbers by a number separately and subtracting give the same result as multiplying their difference by the number.

In algebra,

$$xz - yz = (x - y)z, \text{ for all numbers } x, y, z$$

Now try these problems.

- $(63 \times 12) + (37 \times 12)$ • $(15 \times \frac{3}{4}) + (5 \times \frac{3}{4})$
- $(\frac{1}{3} \times 20) + (\frac{2}{3} \times 20)$ • $(65 \times 11) - (55 \times 11)$
- $(2\frac{1}{2} \times 23) - (1\frac{1}{2} \times 23)$ • $(13.5 \times 40) - (3.5 \times 40)$



Let's do it!

- From the square below, take any 9 numbers making a square. Find the relation between their sum and the number at the middle of the square. Justify this relation using algebra.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Now try this with 25 numbers in a square.

Another trick

Take any month's calendar and mark nine numbers in a square, instead of four.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

Their sum in the picture above is 144. And it is 9 times the middle number 16.

Mark other numbers like this and check whether this happens everytime.

To see why this is so, take the middle number as x . We can fill in the other numbers like this.

	$x - 7$	
$x - 1$	x	$x + 1$
	$x + 7$	

$x - 8$	$x - 7$	$x - 6$
$x - 1$	x	$x + 1$
$x + 6$	$x + 7$	$x + 8$

In this, if we match up pairs like $x - 8, x + 8$, we can see without any computation that the sum is $9x$. That is 9 times the middle number.

Why are you doing nothing when all others are working?

Don't you see? I am x and it is always free!



$x-8$	$x-7$	$x-6$
$x-1$	x	$x+1$
$x+6$	$x+7$	$x+8$

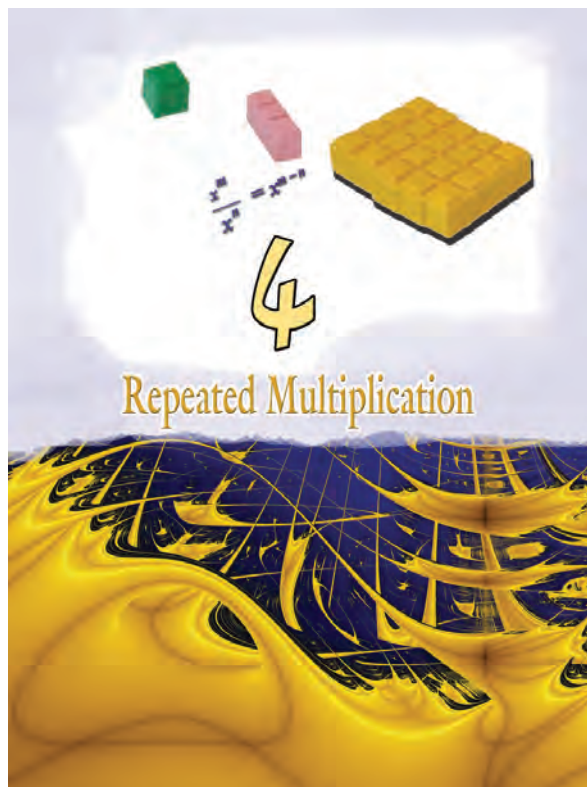
Looking back



Achievements	On my own	With teacher's help	Must improve
<ul style="list-style-type: none">• Finding general principles in arithmetical operations.			
<ul style="list-style-type: none">• Writing general principles in ordinary languages.			
<ul style="list-style-type: none">• Expressing relations between numbers and operations using letters.			
<ul style="list-style-type: none">• Using general principles to make computations easier.			

4

Repeated Multiplication



Power of products

This is an old story. A poor man went to a rich man for help. Said the rich man, "I can give you a thousand rupees a day for thirty days; or I can give you one paise the first day, two paise the third day and so on, doubling each day for thirty days. Choose which you want."

Which is the better deal?

Let's see.

The first offer will get the poor man 30000 rupees in 30 days.

What about the second?

$$1 + 2 + 4 + 8 + 16 + \dots$$

We must add the thirty numbers. Do you know how much it is? 1073741823 paise. That is more than one crore rupees!

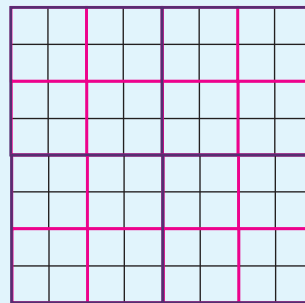
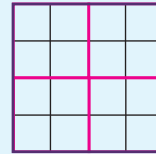
Yes! The same rich man who made a deal with you. I've lost my crores and you can crow over it!

You should've been more careful with your powers!



Multiply again and again

Look at these figures:



How many cells are there in the first square?

What about the second and third squares?

If we continue like this, how many cells would be there in the next square?

Let's look at the problem like this:

The first square has four cells. And the next one has four such squares.

So it has $4 \times 4 = 16$ cells.

The third square is made up of four squares like second one and so it has $16 \times 4 = 64$ cells.

What about the next?

$64 \times 4 = 256$ cells.

We can write like this also:

Number of cells

in the first square 4

in the second square 4×4

in the third square $4 \times 4 \times 4$

What about the 10th square?

$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$ cells.

Instead of writing in this elaborate manner, we write it shortly as 4^{10} . And read "4 raised to 10".

If we do the actual multiplication, we can see that it is

$$1048576.$$

Now we can easily say that the number of cells in these squares are $4, 4^2, 4^3, \dots$. So that the number of cells in the 20th square is 4^{20} , the number of cells in the 100th square is 4^{100} and so on. When it becomes difficult to compute the actual numbers, we can use a computer.

The numbers $4, 4^2, 4^3, 4^4, \dots$ which we saw here are called *powers* of 4.

4^2 is the second power of 4.

4^3 the third power of 4.

And so on. If need be, we can write 4 itself as 4^1 and call it the first power of 4.

In 4^3 , the number 3 is called the *exponent* (or *power*).

We also call the second power of a number, its *square* and the third power, its *cube*.

Exponentiation

Just as the name of repeated addition is called multiplication, the name for repeated multiplication is *exponentiation*.

Let's look at some more examples.

What are the powers of 3?

How do we compute $3^1, 3^2, 3^3, \dots$?

We can compute the powers one by one as

$$3^1 = 3$$

$$3^2 = 3 \times 3 = 9$$

$$3^3 = 3 \times 3 \times 3 = 9 \times 3 = 27$$

How do we compute 3^6 ?

Instead of computing powers one after another as above, let's see if there is another way. For example

$$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

Instead of multiplying one by one, let's take three at a time.

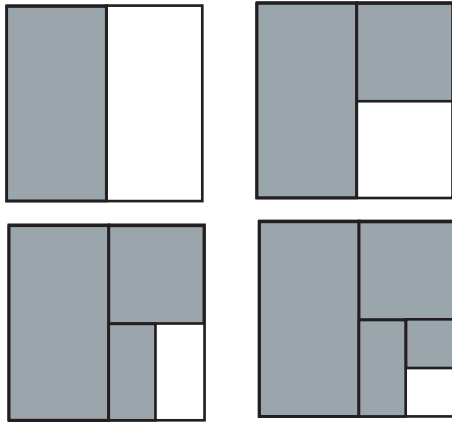
Exponentiation

We usually do the four operations of addition, subtraction, multiplication and division in arithmetic. Exponentiation is the fifth operation. Just as multiplication by natural numbers is repeated addition, exponentiation is repeated multiplication.

The first four operations are indicated by a symbol (+, -, ×, ÷) between the numbers operated on. But no such symbol is used to indicate exponentiation.

The notation is to write on the upper right of the number multiplied, the number showing how many times to multiply, in a smaller font. For example, $4 \times 4 \times 4 = 4^3$

Sum of powers



What part of each square is shaded?

$\frac{1}{2}$ in the first square.

In the second?

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

We can look at it this way.

The unshaded part is $\frac{1}{4}$.

So, the shaded part is

$$1 - \frac{1}{4} = \frac{3}{4}$$

What do we see here?

$$\frac{1}{2} + \frac{1}{4} = 1 - \frac{1}{4}$$

Similar reasoning gives from the third square,

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 1 - \frac{1}{8}$$

And from the fourth,

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 1 - \frac{1}{16}$$

We can continue this:

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} = 1 - \frac{1}{2^3}$$

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} = 1 - \frac{1}{2^4}$$

In general, the sum of the powers $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}$ and so on is equal to the last power subtracted from 1.

$$\begin{aligned} 3^6 &= (3 \times 3 \times 3) \times (3 \times 3 \times 3) \\ &= 27 \times 27 \\ &= 729 \end{aligned}$$

How about 2^9 ?

$$\begin{aligned} 2^9 &= (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \\ &= 16 \times 32 \\ &= 512 \end{aligned}$$

Can you compute this in any other way?

Now compute these powers:

- 2^6
- 3^8
- 4^4
- 2^9
- 10^6
- 1^{10}
- 100^4
- 0^{20}

Power of ten

What are the powers of 10?

They can be written 10, 10^2 , 10^3 , ...

How do we compute them?

$$10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1000$$

So what is 10^8 ?

Suppose we want to compute powers of 20.

How do we compute 20^4 ?

$$\begin{aligned} 20^4 &= 20 \times 20 \times 20 \times 20 \\ &= (2 \times 10) \times (2 \times 10) \times (2 \times 10) \times (2 \times 10) \\ &= (2 \times 2 \times 2 \times 2) \times (10 \times 10 \times 10 \times 10) \\ &= 16 \times 10000 = 160000 \end{aligned}$$

What about $2^4 \times 5^5$?

We can write it as $(2 \times 2 \times 2 \times 2) \times (5 \times 5 \times 5 \times 5 \times 5)$

Reordering the numbers, this becomes,

$$\begin{aligned} &(2 \times 5) \times (2 \times 5) \times (2 \times 5) \times (2 \times 5) \times 5 \\ &= 10 \times 10 \times 10 \times 10 \times 5 \\ &= 10^4 \times 5 = 50000 \end{aligned}$$

What is 100^3 ?

$$100^3 = 100 \times 100 \times 100$$

We can write this as $10 \times 10 \times 10 \times 10 \times 10 \times 10$

$$100^3 = 10^6 \\ = 1000000$$

Now try these problems.

- Write hundred, thousand, ten thousand, lakh, 10 lakh and crore as powers of 10.
- Compute the powers.
 - 30^4 ■ 50^5 ■ 200^3

Place value

How do we split 3675 according to place values?

$$(3 \times 1000) + (6 \times 100) + (7 \times 10) + 5$$

Using powers of 10, we can write this as

$$(3 \times 10^3) + (6 \times 10^2) + (7 \times 10) + 5$$

Can you split these numbers like this?

- 1221 • 60504 • 4321 • 732

What about decimals?

How do we split 362.574?

$$362.574 = (3 \times 100) + (6 \times 10) + 2 \\ + \left(5 \times \frac{1}{10}\right) + \left(7 \times \frac{1}{100}\right) + \left(4 \times \frac{1}{1000}\right)$$

We can write this as

$$(3 \times 10^2) + (6 \times 10) + 2 + \left(5 \times \frac{1}{10}\right) + \left(7 \times \frac{1}{10^2}\right) + \left(4 \times \frac{1}{10^3}\right)$$

Try to split these numbers like this

- 437.54 • 23.005 • 4567 • 201

Factorization

We can factorize any number as a product of prime numbers. For example, we can write 72 as

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

Using powers, this can be written

$$72 = 2^3 \times 3^2$$

Another sum

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 1 - \frac{1}{8}$$

If we multiply the numbers on either side by 8, we get

$$8 \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \right) = 8 \left(1 - \frac{1}{8} \right)$$

That is,

$$\left(8 \times \frac{1}{2}\right) + \left(8 \times \frac{1}{4}\right) + \left(8 \times \frac{1}{8}\right) = 8 - \left(8 \times \frac{1}{8}\right) \\ 4 + 2 + 1 = 8 - 1$$

Likewise, multiplying either side of

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 1 - \frac{1}{16}$$

by 16, we get

$$8 + 4 + 2 + 1 = 16 - 1$$

Rearranging the numbers, we get

$$1 + 2 + 4 = 8 - 1$$

$$1 + 2 + 4 + 8 = 16 - 1$$

That is,

$$2 + 4 = 8 - 2$$

$$2 + 4 + 8 = 16 - 2$$

Using powers, we can write this as

$$2 + 2^2 = 2^3 - 2$$

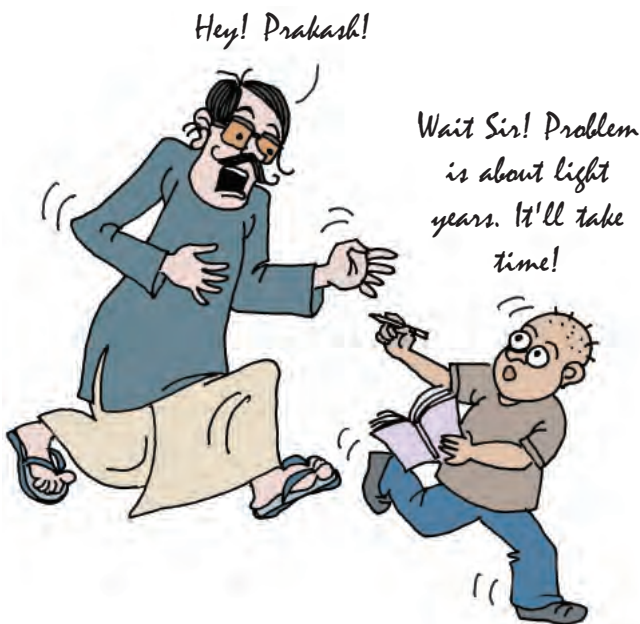
$$2 + 2^2 + 2^3 = 2^4 - 2$$

And we can continue.

In general, the sum of the powers $2, 2^2, 2^3$ and so on, is equal to 2 subtracted from the next power to the last.

Numbers in science

Science often requires large numbers. For example, the average distance between the Earth and Sun is 149000000 kilometres. This is written in scientific notation as 1.49×10^8 . Similarly, the distance travelled by light in one year is about 9.46×10^{17} kilometres. This distance is called a light year. Astronomical distances are often given in terms of light years. The star nearest to the Earth is the Sun. The next nearest star is Proxima Centauri, which is about 4.22 light years away, that is, about 3.99×10^{18} kilometres. We can describe this in another way. Light rays from this star take more than four years to reach the earth. Thus what we now see is how the star was four years ago. So even if the star dies now, we would go on seeing it for four more years!



How do we split 1000 like this?

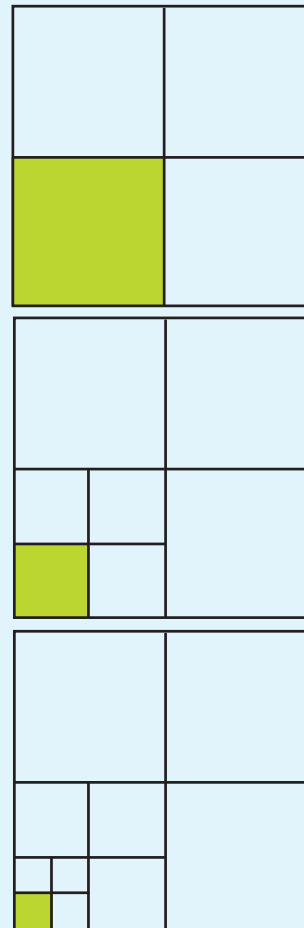
$$\begin{aligned}1000 &= 2 \times 2 \times 2 \times 5 \times 5 \times 5 \\ &= 2^3 \times 5^3\end{aligned}$$

Now try to write the numbers below as product of powers of prime numbers.

- 36
- 225
- 500
- 784
- 750
- 625
- 1024

Powers of a fraction

Look at these pictures:



What part of the first square is coloured?

And in the second?

$\frac{1}{4}$ of

That is,

$$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

In the third square, what part is coloured?

$$\frac{1}{4} \times \frac{1}{16} = \frac{1}{64}$$

This is the product of three $\frac{1}{4}$'s.

If we continue like this, what part of the next square should be coloured?

And in the fifth square?

We must multiply together five $\frac{1}{4}$'s.

We can shorten this as $\left(\frac{1}{4}\right)^5$.

$$\begin{aligned}\left(\frac{1}{4}\right)^5 &= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \\ &= \frac{1}{4 \times 4 \times 4 \times 4 \times 4} \\ &= \frac{1}{4^5} \\ &= \frac{1}{64 \times 16} \\ &= \frac{1}{1024}\end{aligned}$$

That is in the fifth square, only $\frac{1}{1024}$ of square need be coloured.

We can write the repeated multiplication of any fraction as a power like this. For example,

$$\begin{aligned}\left(\frac{3}{5}\right)^3 &= \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \\ &= \frac{3 \times 3 \times 3}{5 \times 5 \times 5} = \frac{3^3}{5^3} \\ &= \frac{27}{125}\end{aligned}$$

Let's examine one more example:

$$\begin{aligned}\left(2\frac{2}{5}\right)^3 &= \left(\frac{12}{5}\right)^3 \\ &= \frac{12}{5} \times \frac{12}{5} \times \frac{12}{5}\end{aligned}$$



Project

Last digit

The last digit of every power of 10 is 0. What is the last digit of a power of 5?

What about 6?

Look at the last digits of powers of 4; are they all the same?

What are the last digits?

Check the last digit of the powers of other single digit numbers.

One more question: What is the last digit of 2^{100} ?

Increase or decrease?

We have seen how the powers 2, 4, 8, 16, ... of 2 grow very fast. Do the powers of other numbers also increase like this?

What are the powers of $\frac{1}{2}$?

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$$

But these have become smaller and smaller, right?

And the powers of $\frac{2}{3}$?

And the powers of $\frac{3}{2}$?

What are the numbers whose powers become larger and larger? And what numbers have smaller and smaller powers?

What about the powers of 1?

$$= \frac{1728}{125} = 13 \frac{103}{125}$$

Compute the powers given below:

$$\bullet \left(\frac{2}{3}\right)^5 \quad \bullet \left(\frac{3}{5}\right)^4 \quad \bullet \left(\frac{1}{2}\right)^{10} \quad \bullet \left(2\frac{1}{2}\right)^3$$

Powers of decimal

What is $(1.2)^2$?

$$\begin{aligned}(1.2)^2 &= 1.2 \times 1.2 \\ &= 1.44\end{aligned}$$

Can you compute $(1.5)^3$ like this?

What is $(0.2)^4$?

We know that $2^4 = 16$

We can write 0.2 as $\frac{2}{10}$.

$$\begin{aligned}(0.2)^4 &= \left(\frac{2}{10}\right)^4 \\ &= \frac{2^4}{10^4} \\ &= \frac{16}{10000} \\ &= 0.0016\end{aligned}$$

Can't you do this without writing it out?

Can you compute $(0.3)^3$ in your head?

What is 3^3 ?

How many decimal places would $(0.3)^3$ have?

$12^3 = 1728$. Can you find out $(1.2)^3$ and $(0.12)^3$ using this?

Compute the powers given below:

$$\bullet (1.1)^3 \quad \bullet (0.02)^5 \quad \bullet (0.1)^6$$

Given that $16^3 = 4096$, can you compute these powers?

$$\bullet (1.6)^3 \quad \bullet (0.16)^3 \quad \bullet (0.016)^3$$

Multiplication rule

We know how to write the sum of two multiples of a number as a single multiple of the same number:

$$(3 \times 2) + (5 \times 2) = (3 + 5) \times 2 = 8 \times 2$$

Why is this true?

$$3 \times 2 = 2 + 2 + 2$$

$$5 \times 2 = 2 + 2 + 2 + 2 + 2$$

And so

$$\begin{aligned}(3 \times 2) + (5 \times 2) &= (2 + 2 + 2) + (2 + 2 + 2 + 2 + 2) \\ &= 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 \\ &= 8 \times 2\end{aligned}$$

Similarly, we can find the product of powers.

For example, $2^3 \times 2^5$

$$2^3 = 2 \times 2 \times 2$$

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$

Then,

$$\begin{aligned}2^3 \times 2^5 &= (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^8\end{aligned}$$

Suppose we multiply the third and fifth powers of some other number?

$$\begin{aligned}\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^5 &= \left(\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}\right) \times \left(\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}\right) \\ &= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \\ &= \left(\frac{2}{3}\right)^8\end{aligned}$$

What if we indicate the number as x ?

$$\begin{aligned}x^3 \times x^5 &= (x \times x \times x) \times (x \times x \times x \times x \times x) \\ &= x \times x \times x \times x \times x \times x \times x \times x \times x = x^8\end{aligned}$$

Multiples and powers

For any natural number n and for any number (natural number or fraction) x , the notation nx or $n \times x$ is used to denote the sum of x taken n times. And x^n denotes the product of x taken n times.

Look at the rule for adding multiples of a number by natural numbers, and multiplying powers of a number:

$$mx + nx = (m + n)x$$

$$x^m \times x^n = x^{m+n}$$

We can multiply a number by fraction also, but it is not repeated addition. For this multiplication, it is still true that $mx + nx = (m + n)x$, even when m and n are fractions. But if n is a fraction, the symbol x^n as yet has no meaning.

Multiples and powers of 2

All powers of 2 are even; but all even numbers are not powers of 2. For example, 6 is an even number, and not a power of 2. But

$$6 = 2 + 4 = 2^1 + 2^2$$

We can also write

$$10 = 2 + 8 = 2^1 + 2^3$$

$$12 = 4 + 8 = 2^2 + 2^3$$

$$14 = 2 + 4 + 8 = 2^1 + 2^2 + 2^3$$

Check whether other even numbers can also be written as sums of powers of 2.

For example, can we write 100 as a sum of the powers of 2?

If we look at the powers of 2 one by one, we see that $2^6 = 64$ is less than 100, while $2^7 = 128$ is more than 100. We can start by writing

$$100 = 2^6 + 36$$

And then we note that $2^5 = 32 < 36$ and $2^6 = 64 > 36$, so that we can write

$$36 = 2^5 + 4 = 2^5 + 2^2$$

Thus

$$100 = 2^6 + 2^5 + 2^2$$

Now try to write 150 as a sum of powers of 2, using this technique.

Now suppose we take some other powers:

$$\begin{aligned} x^2 \times x^4 &= (x \times x) \times (x \times x \times x \times x) \\ &= x \times x \times x \times x \times x \times x \\ &= x^6 \end{aligned}$$

Let's write the exponents also as m, n .

Then in general,

$$\begin{aligned} x^m \times x^n &= \underbrace{(x \times x \times x \times \dots \times x)}_{m \text{ times}} \times \underbrace{(x \times x \times x \times \dots \times x)}_{n \text{ times}} \\ &= \underbrace{(x \times x \times x \times \dots \times x)}_{m+n \text{ times}} \\ &= x^{m+n} \end{aligned}$$

What is the general principle here?

$x^m \times x^n = x^{m+n}$, for all numbers x and all natural numbers m, n

How do we say this in ordinary language?

There are two facts here

- (i) The product of two powers of the same number is a power of this number.
- (ii) The exponent of the product is the sum of the exponents of the factors.

Try the problems given below, using this idea.

- What power of 2 is got by multiplying 2^5 and 2^3 ?
- What is the number $10^2 \times 10^5$ in ordinary language?
- What power of 2 is twice 2^{10} ?
- What must be added to 2^{10} to get 2^{11} ?
- What must be added to 3^{10} to get 3^{11} ?
- The table below gives some powers of 2.

2^1	2	2^6	64	2^{11}	2048
2^2	4	2^7	128	2^{12}	4096
2^3	8	2^8	256	2^{13}	8192
2^4	16	2^9	512	2^{14}	16384
2^5	32	2^{10}	1024	2^{15}	32768

Using this, compute these products.

- 16×64
- 64×256
- 32×512
- 128×256

Division rule

Is there any trick to compute the quotient of two powers of the same number, as for product?

For example, what is $4^5 \div 4^2$?

Using the product rule,

$$4^5 = 4^2 \times 4^3$$

So what do we get on dividing 4^5 by 4^2 ?

$$4^5 \div 4^2 = 4^3$$

Likewise, how do we compute $5^7 \div 5^3$?

How do we write 5^7 as multiple of 5^3 ?

$$5^7 = 5^3 \times \dots\dots$$

From this we get,

$$5^7 \div 5^3 = \dots\dots$$

What about $8^{23} \div 8^{16}$?

To get 8^{23} , by what should we multiply 8^{16} ?

For this, we need only find what should be added to 16 to get 23.

$$23 - 16 = 7$$

So

$$8^{23} = 8^{16} \times 8^7$$

Now can't you find $8^{23} \div 8^{16}$?

This we can do for powers of fractions also.

For example, let's divide $\left(\frac{2}{3}\right)^{16}$ by $\left(\frac{2}{3}\right)^9$.

As before, if we write

$$\left(\frac{2}{3}\right)^{16} = \left(\frac{2}{3}\right)^9 \times \left(\frac{2}{3}\right)^7$$

Odd numbers and powers of 2

We saw that even numbers can be written as sums of powers of 2. Any odd number, except 1, is 1 added to an even number. So, we can write it as sums of powers of 2 and 1.

For example, to split 25 like this, we first write

$$25 = 24 + 1$$

Then we write 24 as sum of powers of 2 as before

$$24 = 16 + 8 = 2^4 + 2^3$$

Thus

$$25 = 2^4 + 2^3 + 1$$

In general, any natural number can be written as the sum of some of the numbers 1, 2, 2³,...

There is only one odd number which does not contain an even number.

And that is...



Subtraction and division

We have rules for subtracting multiples of a number, just as we have rules for adding such numbers. There is also a condition that we can only subtract the smaller from the larger.

That is

$$mx - nx = (m - n)x, \text{ for any number } x \\ \text{and for any natural numbers } m, n \text{ with} \\ m > n$$

What about powers?

$$\frac{x^m}{x^n} = x^{m-n}$$

Here, we must also state the conclusion

$$x \neq 0$$

As in the case of addition, the rule for subtraction of multiplies hold, even when m and n are fractions.

Then we can find

$$\left(\frac{2}{3}\right)^{16} \div \left(\frac{2}{3}\right)^9 = \left(\frac{2}{3}\right)^7$$

Now let's see in general what we get when we divide a power of a number by a smaller power.

Let's write the number as x . Since the operation is division, x should not be zero. Let's write the larger exponent, as m and the smaller exponent as n .

Now how do we compute $x^m \div x^n$?

What should be added to n to make it m ?

So,

$$x^m = x^n \times x^{m-n}$$

From this we get

$$\frac{x^m}{x^n} = x^{m-n}$$

That is,

$$\frac{x^m}{x^n} = x^{m-n}, \text{ for any non zero } x \text{ and any natural} \\ \text{numbers } m, n \text{ with } m > n$$

As in the case of multiplication, can you say this in ordinary language?

Not try these problems.

- What power of 2 we get on dividing 2^5 by 2^3 ?
- What is $10^9 \div 10^4$?
- What power of 2 is half of 2^{10} ?
- Look at the table of powers of 2 in page 58. Using it, can you compute these quotients?
 - $64 \div 16$ ■ $512 \div 32$
 - $1024 \div 128$ ■ $16384 \div 2048$
- What is $2^8 \times \frac{1}{2^3}$?
- By what should 7^6 be multiplied to get 7^2 ?

Another division

Look at the last but one problem above:

We have

$$2^8 \times \frac{1}{2^3} = 2^8 \div 2^3 = 2^5$$

From this we get,

$$2^5 \div 2^8 = \frac{1}{2^3}$$

Likewise, from the last problem, can you find $7^2 \div 7^6$?

We have

$$7^6 \times \frac{1}{7^4} = 7^2$$

and from this we get

$$7^2 \div 7^6 = \frac{1}{7^4}$$

In general,

$$\frac{x^m}{x^n} = \frac{1}{x^{n-m}}, \text{ for any non zero } x \text{ and any natural numbers } m, n \text{ with } m < n$$

Now try these problems:

- Simplify:
 - $\frac{2^5 \times 2^3}{2^4}$
 - $\frac{3^7}{3^2 \times 3^4}$
 - $\frac{5^2 \times 5^4}{5^5 \times 5^4}$
 - $\frac{8^2 \times 8^7}{8^6 \times 8^3}$
 - $\frac{4^3 \times 4^5}{4^2 \times 4^4}$
 - $\frac{10^4 \times 10^5}{10^6 \times 10^7}$
- What powers of $\frac{1}{5}$ do we get on dividing 5^6 by 5^{10} ?
- What is the decimal form of the number got, on dividing 10^8 by 10^{12} ?
- What natural number is got by dividing $\left(\frac{1}{2}\right)^5$ by $\left(\frac{1}{2}\right)^8$?
- By what natural number should $(0.25)^6$ be multiplied to get $(0.25)^4$?

Division and subtraction

When fractions are also taken into account, we can divide a small number by a large number also; the result would be a fraction. So we can consider division of a larger power by a smaller power also.

$$\frac{x^m}{x^n} = \frac{1}{x^{n-m}}, \text{ if } m < n$$

There is no analogous rule for multiplies. There is no way we can subtract a larger number from a smaller number as yet.

Pouch problem

100 one-rupee coins are to be put into several pouches, such that any amount upto 100 rupees can be taken without opening any pouch. Can we do it?

Put a single coin in the first pouch. 2 coins in the second, 4 in the third and so on, using powers of 2.

$$1 + 2 + 4 + 8 + 16 + 32 = 64 - 1 = 63$$

Put the remaining $100 - 63 = 37$ coins in a single pouch.

Now if the required amount is less than 63, we split it into powers of 2 (and 1 if needed) and take the corresponding pouches. For example,

$$35 = 32 + 2 + 1$$

so that we need only take the first, second and the sixth pouch.

For amounts larger than 63 rupees?

For example, to take 65 rupees, first take the last pouch of 37 rupees. We need $65 - 37 = 28$ rupees more. And this can be done by splitting

$$28 = 16 + 8 + 4$$

- Make a table of powers 3, and use it to find these products and quotients.

- 81×9
- 729×81
- $6561 \div 243$
- 243×81
- $2187 \div 9$
- $59049 \div 729$

Power of powers

Can we write 64 as the power of a number?

In what all ways can we do this?

$$2^6 = 64$$

$$4^3 = 64$$

$$8^2 = 64$$

$$64^1 = 64$$

Likewise, can we write 3^{12} as powers of other numbers?

$$\begin{aligned} 3^{12} &= 3^6 \times 3^6 \\ &= (729) \times (729) \\ &= 729^2 \end{aligned}$$

There is another way:

$$\begin{aligned} 3^{12} &= 3^8 \times 3^4 \\ &= (3^4 \times 3^4) \times 3^4 \\ &= 81 \times 81 \times 81 \\ &= 81^3 \end{aligned}$$

And one more:

$$\begin{aligned} 3^{12} &= 3^6 \times 3^6 \\ &= (3^3 \times 3^3) \times (3^3 \times 3^3) \\ &= 27 \times 27 \times 27 \times 27 \\ &= 27^4 \end{aligned}$$

Is there any other? Try!

In these, what does $3^6 \times 3^6$ mean?

It is the product of two 3^6 , right?

We can shorten it as $(3^6)^2$.

Thus

$$\begin{aligned}(3^6)^2 &= 3^6 \times 3^6 \\ &= 3^{6+6} \\ &= 3^{6 \times 2} \\ &= 3^{12}\end{aligned}$$

Similarly, we can write $3^4 \times 3^4 \times 3^4$ as $(3^4)^3$.

So,

$$\begin{aligned}(3^4)^3 &= 3^4 \times 3^4 \times 3^4 \\ &= 3^{4+4+4} \\ &= 3^{4 \times 3} \\ &= 3^{12}\end{aligned}$$

In the same way, we can write

$$\begin{aligned}(4^2)^3 &= 4^2 \times 4^2 \times 4^2 \\ &= 4^{2 \times 3} \\ &= 4^6 \\ (5^4)^6 &= 5^{4 \times 6} \\ &= 5^{24}\end{aligned}$$

and so on.

Now let's look at such powers of fractions.

What does $\left(\left(\frac{2}{3}\right)^2\right)^3$ mean?

$$\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^2$$

That is,

$$\left(\frac{2}{3}\right)^{2+2+2} = \left(\frac{2}{3}\right)^{3 \times 2} = \left(\frac{2}{3}\right)^6$$

In general, for any number x and for any natural numbers m, n

$$\begin{aligned}(x^m)^n &= \underbrace{x^m \times x^m \times \dots \times x^m}_{n \text{ times}} \\ &= x^{\overbrace{m+m+\dots+m}^{n \text{ times}}} \\ &= x^{nm} \\ &= x^{mn}\end{aligned}$$



Project

Some natural numbers can be written as sums of consecutive natural numbers.

For example,

$$3 = 1 + 2$$

$$7 = 3 + 4$$

$$15 = 1 + 2 + 3 + 4 + 5 = 7 + 8$$

But some natural numbers cannot be split like this. For example, 4 cannot be written in this form.

Can you find the specialty of those numbers which cannot be written as sums of consecutive natural numbers?

Perfect numbers

The factors of 6 are 1, 2, 3, 6.

The sum of the factors other than 6 itself is

$$1 + 2 + 3 = 6$$

Now let's look at 28:

$$28 = 2^2 \times 7$$

And so its factors can be listed as

$$\begin{array}{ccc} 1 & 2 & 2^2 \\ 7 & 2 \times 7 & 2^2 \times 7 \end{array}$$

And the sum of factors other than 28 itself is

$$1 + 2 + 2^2 + 7 + (2 \times 7) = 7 + 7 + 14 = 28$$

Next let's look at the factors of 496

$$2^4 \times 31 = 16 \times 31 = 496$$

Since 31 is a prime number, the factors are

$$\begin{array}{cccccc} 1 & 2 & 2^2 & 2^3 & 2^4 & \\ 31 & 2 \times 31 & 2^2 \times 31 & 2^3 \times 31 & 2^4 \times 31 & \end{array}$$

In this, the sum of the numbers in the first row is

$$1 + 2 + 2^2 + 2^3 + 2^4 = 2^5 - 1 = 31$$

(see the section, **Another sum**)

And the sum of the numbers, except $2^4 \times 31$ in the second row is

$$\begin{aligned} (1 + 2 + 2^2 + 2^3) \times 31 &= (2^4 - 1) \times 31 \\ &= (2^4 \times 31) - 31 \end{aligned}$$

So the sum of the factors except 496 itself is

$$31 + (2^4 \times 31) - 31 = 2^4 \times 31 = 496$$

Numbers like 6, 28 and 496 are called perfect numbers.

That is,

$$(x^m)^n = x^{mn}, \text{ for any number } x \text{ and any natural numbers } m, n$$

Now can you write these as a single power?

$$\begin{array}{ll} \bullet (4^2)^3 & \bullet (3^3)^2 \times 9^4 \\ \bullet \left(\left(\frac{1}{2} \right)^3 \right)^4 & \bullet (2^3)^4 \times 2^6 \end{array}$$

Write each of these as powers of different numbers.

$$\bullet 3^8 \quad \bullet 4^6 \quad \bullet 2^{15} \quad \bullet 5^{12}$$

Factors

What are the factors of 32?

$$1, 2, 4, 8, 16, 32$$

Except 1, all these are powers of 2. So we can write the factors of 32 as

$$1, 2, 2^2, 2^3, 2^4, 2^5$$

What about the factors of 81?

$$81 = 3^4$$

And so the factors are

$$1, 3, 3^2, 3^3, 3^4$$

Now let's find out the factors of 72.

$$72 = 2^3 \times 3^2$$

Let's look at a definite scheme to write all factors.

First we write 1 and all factors which are power of 2:

$$1, 2, 2^2, 2^3$$

Multiplying these by 3 gives four more factors:

$$3, 2 \times 3, 2^2 \times 3, 2^3 \times 3$$

Multiplying the first four factors by 3^2 instead, we get another four factors:

$$3^2, 2 \times 3^2, 2^2 \times 3^2, 2^3 \times 3^2$$

Are there any more?

How about writing down the factors of 200?

$$200 = 8 \times 25 = 2^3 \times 5^2$$

We can list the factors in order as below.

1	2	2^2	2^3
5	2×5	$2^2 \times 5$	$2^3 \times 5$
5^2	2×5^2	$2^2 \times 5^2$	$2^3 \times 5^2$

What about the factors of 240?

$$240 = 16 \times 15 = 2^4 \times 3 \times 5$$

We can list the factors like this:

1	2	2^2	2^3	2^4
3	2×3	$2^2 \times 3$	$2^3 \times 3$	$2^4 \times 3$
5	2×5	$2^2 \times 5$	$2^3 \times 5$	$2^4 \times 5$
3×5	$2 \times 3 \times 5$	$2^2 \times 3 \times 5$	$2^3 \times 3 \times 5$	$2^4 \times 3 \times 5$

Now list the factors of the numbers below like this

- 64
- 125
- 48
- 45
- 105



Let's do it!

- If $2^x = 128$, what is 2^{x+1} ?
- If $3^x = 729$, what is 3^{x-1} ?
- In $3^x, 3^{x+1}, 3^{x-1}, 3^x + 1$ which is an even number?
- If 6^{10} is computed, what would be the digit in one's place?
- If $5^6 \times \frac{1}{5^x} = \frac{1}{5^{10}}$, then what is x ?
- Simplify

$$\bullet \frac{3^5 \times 3^6}{3^4 \times 3^4} \quad \bullet \frac{4^7 \times 4^8}{4^2 \times (4^3)^5} \quad \bullet \frac{(6^4)^2 \times (6^5)^3}{(6^2)^2 \times (6^4)^5}$$



Projects

$32 = 2^5$	number of factors	6
$81 = 3^4$	number of factors	5
$72 = 2^3 \times 3^2$	number of factors	12

Write some more numbers as products of powers of primes and also the number of factors they have.

How did you calculate the number of factors?

Is there any relation between the numbers in exponents and the number of factors?

Looking back



Achievements	On my own	With teacher's help	Must improve
<ul style="list-style-type: none">Describing exponentiation as the operation of repeated multiplication.			
<ul style="list-style-type: none">Justifying the rules of exponentiation.			
<ul style="list-style-type: none">Using the rules of exponentiation to solve problems.			
<ul style="list-style-type: none">Describing the positional system of notation using exponentiation.			
<ul style="list-style-type: none">Logically justifying number relations associated with powers.			

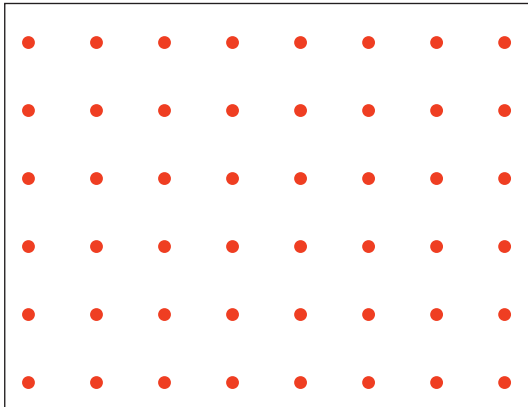
5

Area of a Triangle

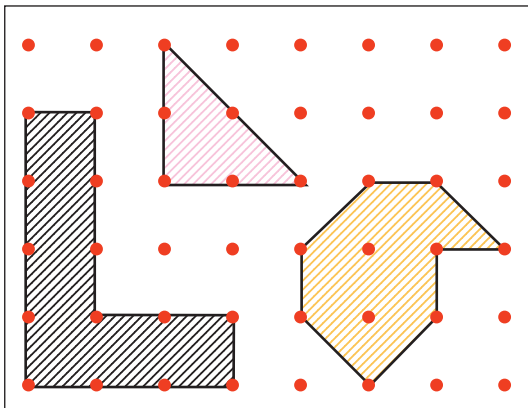


Dot Math

In the picture below, the horizontal and vertical dots are one centimetre apart.



What are the areas of the coloured figures below?



In the above picture, draw other figures by joining dots and find out their areas.

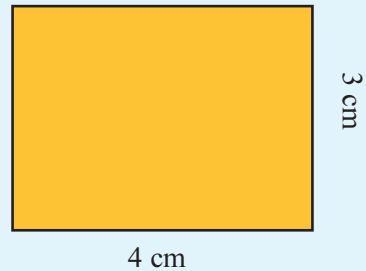


This can be done using the **Grid** option in GeoGebra. Select the **Polygon** tool and click at various intersections of the grid lines to draw different figures.

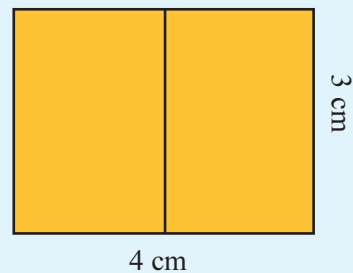
Calculate the area of these. To check the answers, select the **Area** tool and click within each figure.

Halving

Cut out a paper rectangle 4 centimetres wide and 3 centimetres high.



Draw a line down the middle as below:



Now we have two rectangles. What is the area of each?

To see that it's half the big rectangle, we need only fold it across.

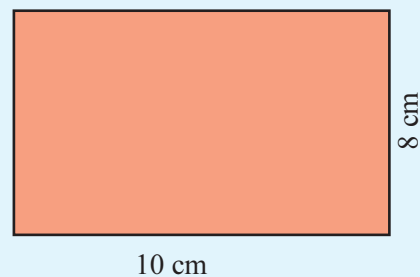
So the area of a small rectangle is half the area of the large rectangle. That is,

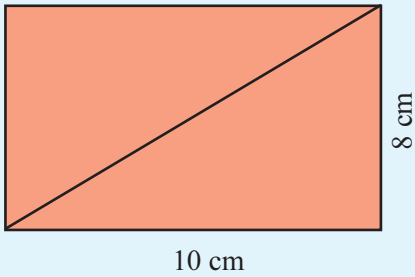
$$\frac{1}{2} \times 12 = 6 \text{ square centimetres}$$

Can you halve the area in any other way?

Another half

Cut out a paper rectangle 10 centimetres wide and 8 centimetres high.





Draw a line from corner to corner as shown.

The rectangle is split into two triangles.

Are their areas equal?

Can we fold and check as before?

How about cutting them out?

Then put one on top of the other.

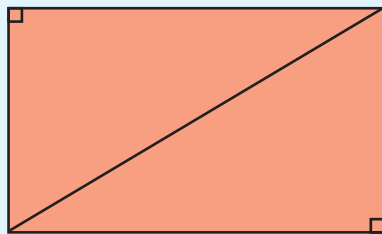
So, what is the area of each triangle?

The area of a triangle is half the area of the rectangle.

That is,

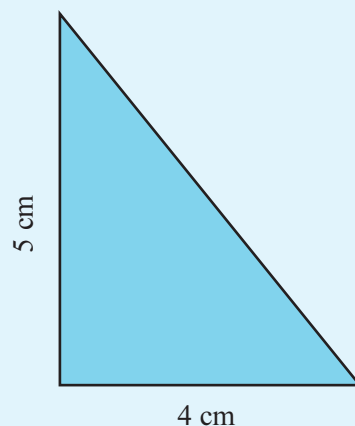
$$\frac{1}{2} \times 10 \times 8 = 40 \text{ sq. cm.}$$

Do you notice anything about the angles of these triangles?



A triangle with a right angle at one corner is called a *right angled triangle*.

What is the area of the right angled triangle shown on the right?

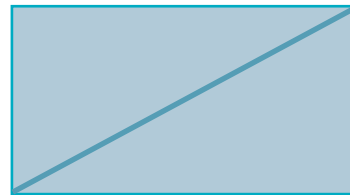


Different halves

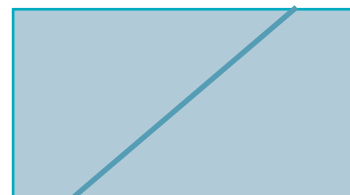
A rectangle can be halved by cutting vertically or horizontally along the middle:



We can also cut corner to corner to make two triangles of half the area.



What happens when we draw a slanted line through the middle?

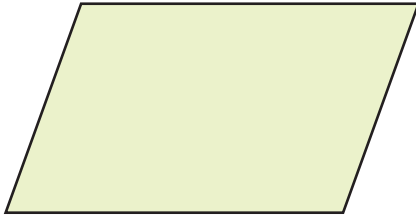


Didn't we get two quadrilaterals of half the area?

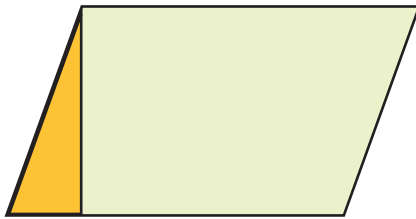
Such a quadrilateral with only one pair of parallel sides is called a *trapezium*.

Parallelogram and rectangle

How do we compute the area of this parallelogram?



Cut out a right angled triangle from the left as shown below.

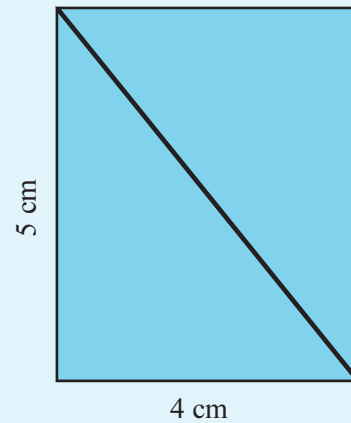


And place it on the right like this:



Now we get a rectangle of the same area.

Cut out two right angled triangles like this and place them together as shown below.



What is the area of this rectangle?

The area of the right angled triangle is half of this, right?

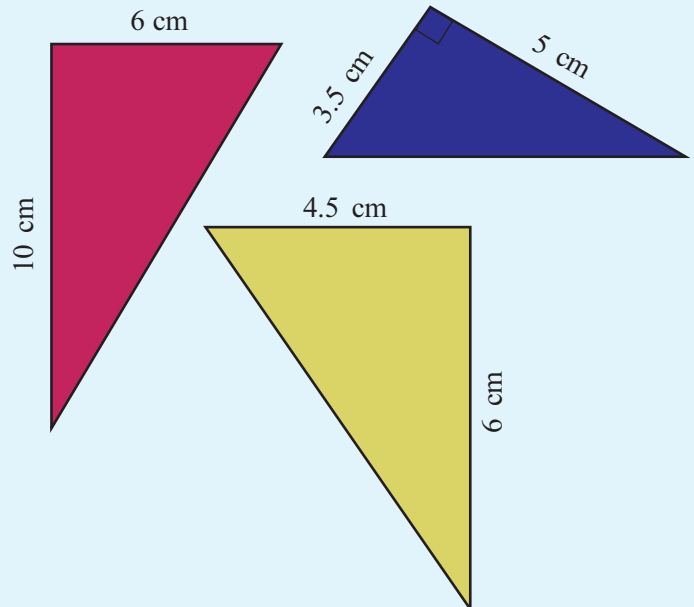
$$\begin{aligned}\text{Area of the triangle} &= \frac{1}{2} \times 4 \times 5 \\ &= 10 \text{ sq. cm.}\end{aligned}$$

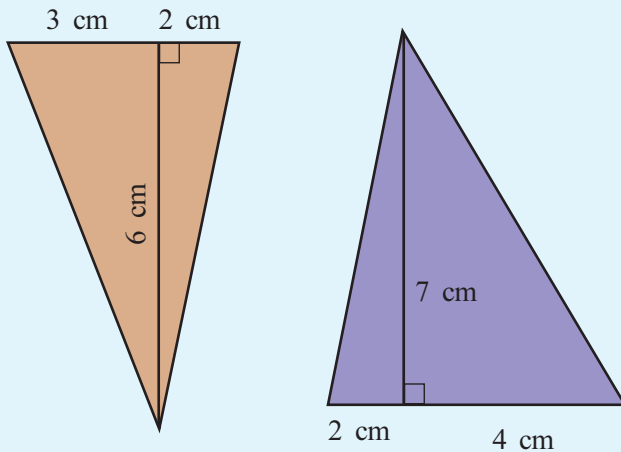
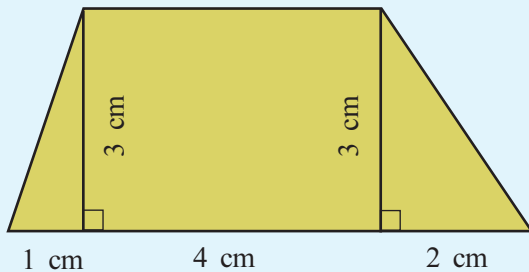
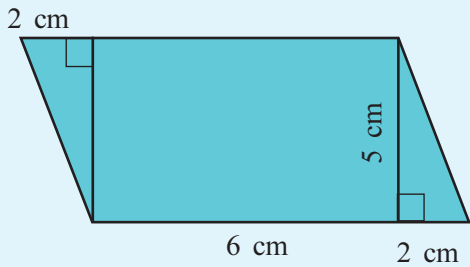
In this, 4 and 5 are the lengths of the perpendicular sides of the right angled triangle.

Thus we have the method to compute the area of a right angled triangle.

The area of a right angled triangle is half the product of the perpendicular sides.

Now calculate the area of the figures shown below:

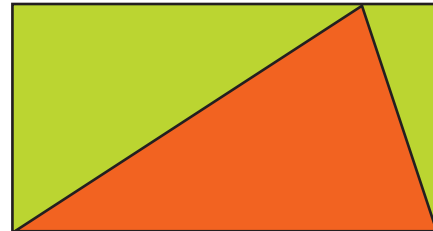




- The area of a right angled triangle is 96 square centimetres. One of the perpendicular sides is 16 centimetres long. What is the length of the other?
- The perpendicular sides of a right angled triangle are 12 and 15 centimetres long. Another right angled triangle of the same area has one of the perpendicular sides 18 centimetres long. What is the length of the other?

Rectangle and triangle

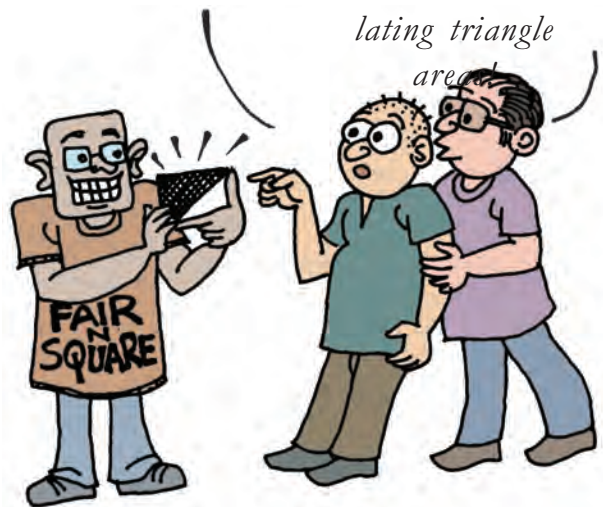
In the picture below, what part of the rectangle is the red triangle?



The answer is given in the next page. But think a bit before turning to it.

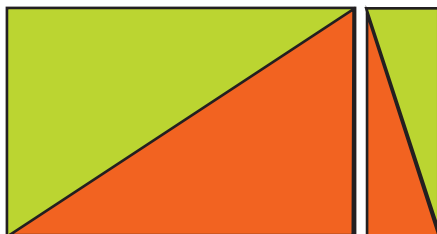
He looks a real blockhead!

Not when it comes to calculating triangle area!



Rectangle and triangle

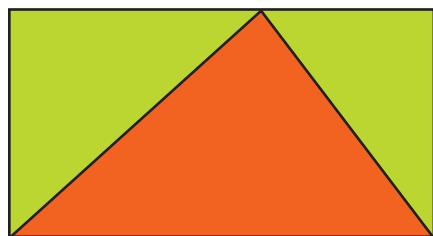
How about cutting the rectangle into two smaller rectangles?



The area of the red triangle in each of these pieces is half the area of that piece. So the sum of their areas is equal to half the area of the original rectangle.

The original red triangle is made up of these two smaller red triangles.

Thus the area of the original red triangle is half the area of the original rectangle.



What about the red triangle above?

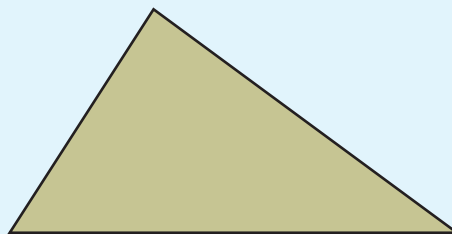


Draw a rectangle in GeoGebra and mark a point on the top side. Select the **Polygon** tool and draw a triangle as in the problem. Colour it red. Select the **Area** tool and find the area of the triangle.

Drag the point on top and see what happens to the area.

Other triangles

Look at this triangle:

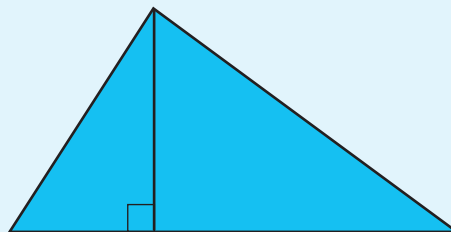


No angle of it is right.

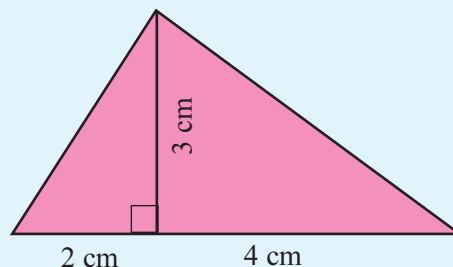
How do we calculate the area?

Can we cut it into two right angled triangles?

Look at the earlier problems you have done.



So to compute the area, what all lengths are to be measured?

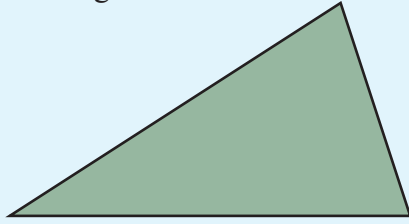


$$\begin{aligned}\text{Area} &= \left(\frac{1}{2} \times 2 \times 3\right) + \left(\frac{1}{2} \times 4 \times 3\right) \\ &= 3 + 6 \\ &= 9 \text{ sq. cm.}\end{aligned}$$

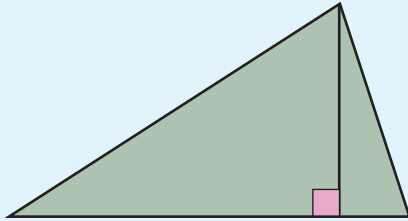
We can calculate the area of any triangle like this.

What is the general method to calculate the area of a triangle?

Look at this triangle:

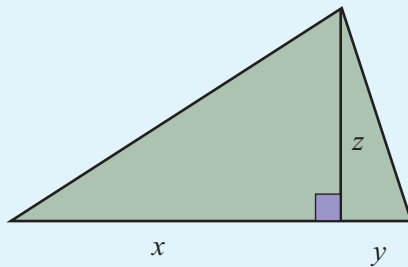


To find the area, first draw a perpendicular from the top corner to divide it into two right angled triangles.



Now some lengths are to be measured.

Let's write letters for these for the time being.

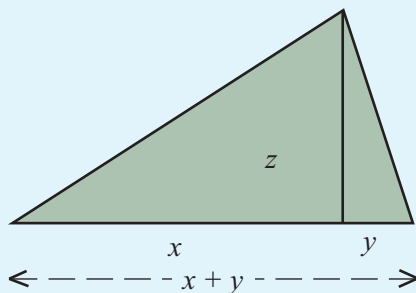


So, how do we write the area?

The sum of the areas of two right angled triangles is

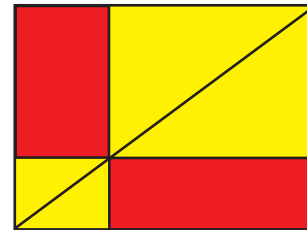
$$\begin{aligned} & \left(\frac{1}{2} \times x \times z \right) + \left(\frac{1}{2} \times y \times z \right) \\ &= \frac{1}{2} xz + \frac{1}{2} yz \\ &= \frac{1}{2} (x + y) z \end{aligned}$$

In this, $x + y$ is the length of the bottom side.



Rectangles within rectangle

Look at this rectangle:

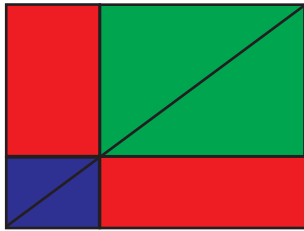


Do you see any relation between the areas of the red rectangles?

Think for a moment before turning the page for the answer.

Rectangles within rectangle

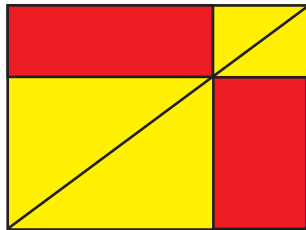
The diagonal of the large rectangle divides it into two right angled triangles of equal area. Each of these triangles is made up of the red rectangle within it and two small right angled triangles.



The areas of right angled triangles of the same colour are equal.

So, the area of the red rectangles are also equal.

What if we draw rectangle through some other point on the diagonal?

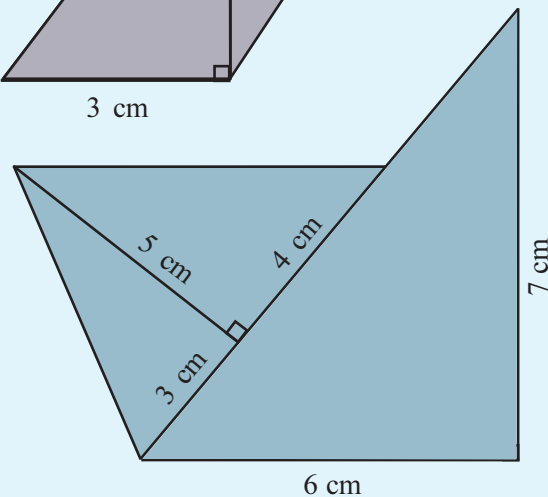
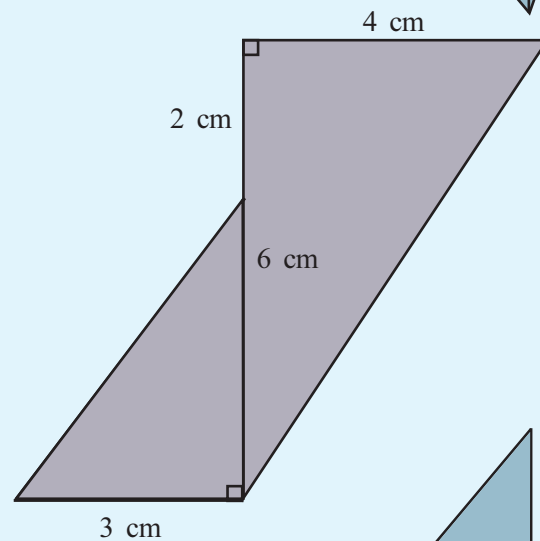
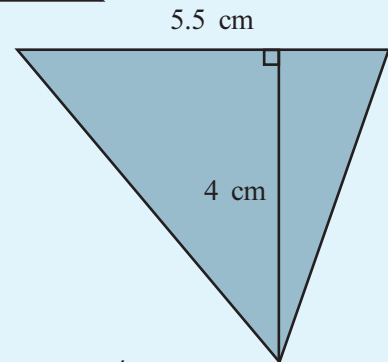
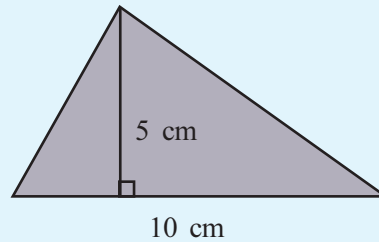


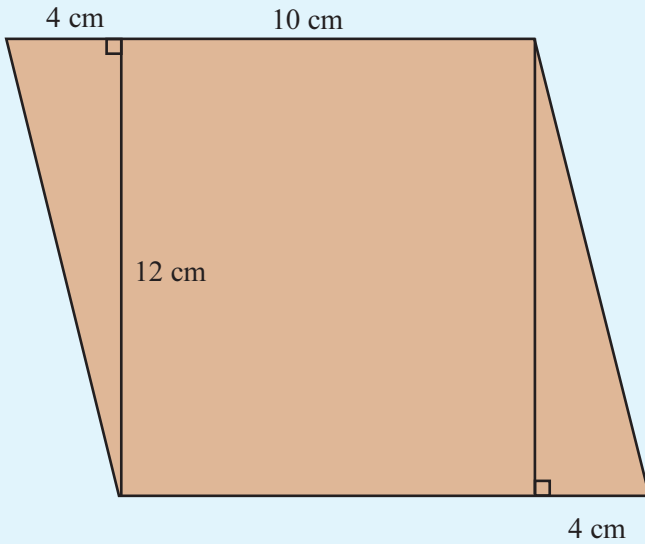
In GeoGebra, draw a pair of horizontal lines, 8 units apart. On the bottom line, mark two points D and F 4 units apart. Mark a point B on the top line and draw $\triangle DFB$ using the **Polygon** tool. What is the area of this triangle? Check the answer using the **Area** tool. Now drag B and see what happens to the area.

So, how do we compute the area of a triangle?

The area of a triangle is half the product of one side with the perpendicular from the opposite side.

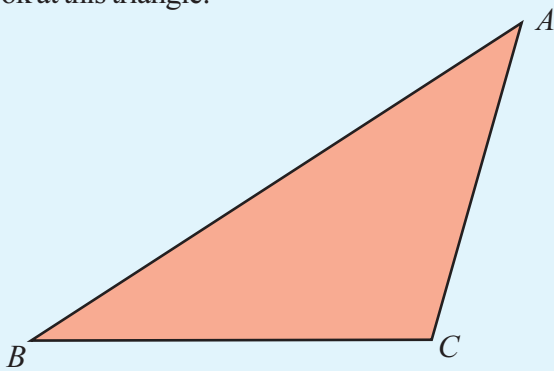
Now compute the area of these figures:





Another triangle

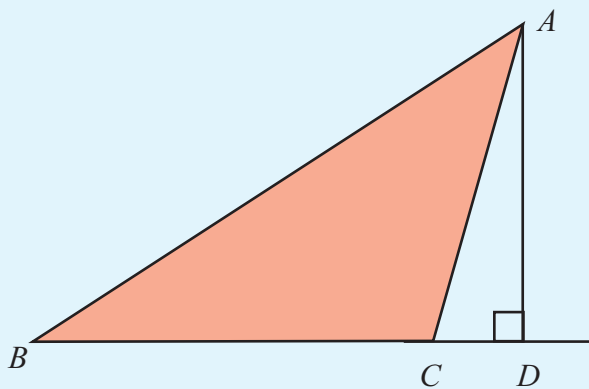
Look at this triangle:



How do we compute its area?

How do we draw a perpendicular from A to BC ?

How about extending BC to the right?

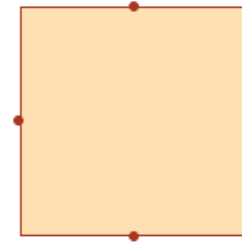


Now how do we calculate the area of $\triangle ABC$?

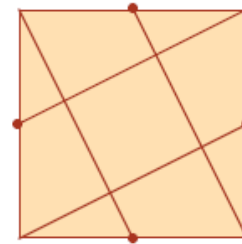
We get $\triangle ABC$ by removing $\triangle ACD$ from $\triangle ABD$.

Square division

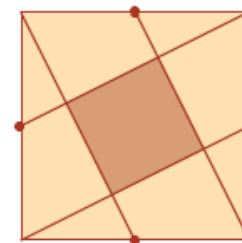
Draw a square and mark the mid points of its sides.



Join these to the corners of the square as shown below:



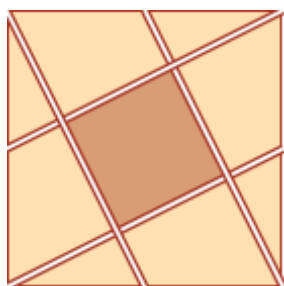
We get a square at the centre:



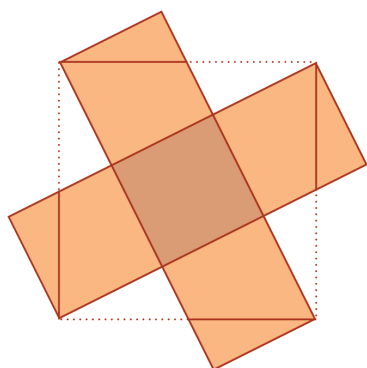
What part of the original square is this?

Square division

Cut out a figure like this in paper.



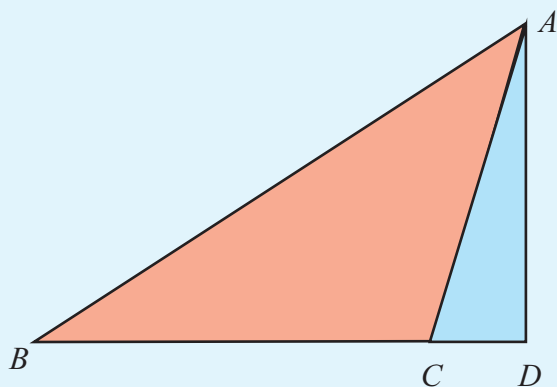
Rearrange the triangular pieces like this:



Now we get five squares of equal size.

So, the small square is $\frac{1}{5}$ of the original square.

$\triangle ABD$ is a right angled triangle.



$$\text{Area of } \triangle ABD = \frac{1}{2} \times BD \times AD$$

$\triangle ACD$ also is a right angled triangle.

$$\text{Area of } \triangle ACD = \frac{1}{2} \times CD \times AD$$

Now we can find the area of $\triangle ABC$

$$\text{Area of } \triangle ABC = \text{Area of } \triangle ABD - \text{Area of } \triangle ACD$$

$$= \frac{1}{2} \times BD \times AD - \frac{1}{2} \times CD \times AD$$

$$= \frac{1}{2} \times (BD - CD) \times AD$$

From the picture,

$$BD - CD = BC$$

Thus we have

$$\text{Area of } \triangle ABC = \frac{1}{2} \times (BD - CD) \times AD$$

$$= \frac{1}{2} \times BC \times AD$$

Measure BC , AD and compute the area.

Here AD is the height measured from BC .

So for triangles of this kind also, area is half the product of a side and the height from it.

Look at this triangle:

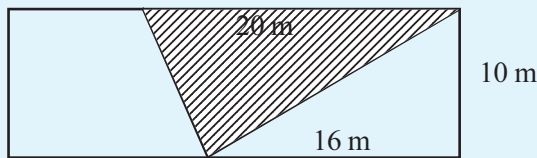


Compute the area of the triangle by measuring out the needed lengths.



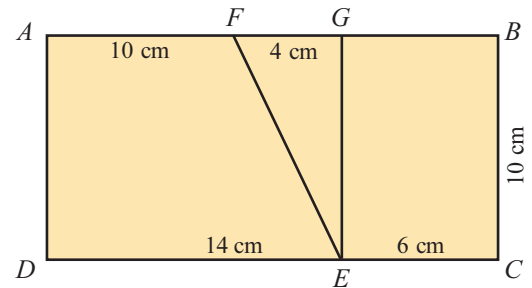
Let's do it!

- A rectangular plot is 30 metres long and 10 metres wide. Within it, a triangular part is marked for planting plantain.



- What is the area of this part?
- What is the area of the triangular part to the right of the area for plantain?
- What is area of the trapezium to the left of the plantain area?
- In $\triangle ABC$, the angle at B is right. Its area is 48 square centimetres and the length of BC is 8 centimetres. The side of BC is extended by 6 centimetres to D . What is the area of $\triangle ADC$?

Trapezium



In the figure $ABCD$ is a rectangle and EFG is a right angled triangle.

What are the areas of the trapeziums $AFED$, and $ECBF$?

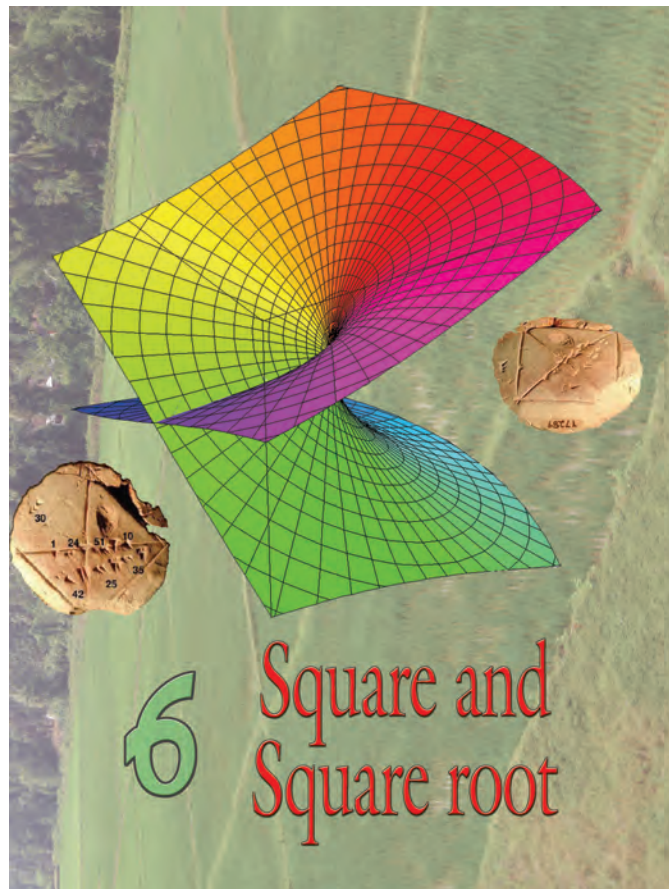
Looking back



Achievements	On my own	With teacher's help	Must improve
<ul style="list-style-type: none">Explaining the methods to compute the area of a right angled triangle.			
<ul style="list-style-type: none">Explaining how the area of any triangle can be computed by splitting into right angled triangles.			
<ul style="list-style-type: none">Solving problems on computation of triangular areas.			

6

Square and Square Root



Triangular numbers

See the dots arranged in triangles:



How many dots are there in each?

1, 3, 6

How many dots would be there in the next triangle?

Such numbers as 1, 3, 6, 10, ... are called triangular numbers.

The first triangular number is 1.

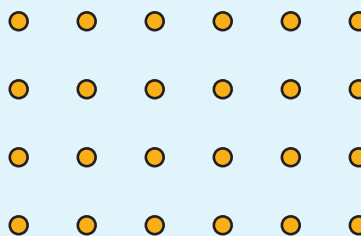
The second is $1 + 2 = 3$.

The third is $1 + 2 + 3 = 6$.

What is the tenth triangular number?

Rows and columns

Look this picture:



Dots in rows and columns make a rectangle.

How many dots in all?

Did you count the dots one by one?

Can you make other rectangles with 24 dots?

Is any one of these a square?

How many more dots do we need to make a square?

Can you remove some dots and make a square? How many?

Numbers which can be arranged in squares are called square numbers.

Do you see anything special about the number of dots making a square?

Squares

What are the ways in which we can write 36 as the product of two numbers?

$$2 \times 18, \quad 3 \times 12, \quad 4 \times 9$$

We can also write

$$36 = 6 \times 6$$

And we have seen that it can also be shortened as $36 = 6^2$.

36 is 6 multiplied by 6 itself; that is, the second power of 6.

There is another name for this:

36 is the square of 6.

Then what is the square of 5?

What is the square of $\frac{1}{2}$?

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Perfect squares

1, 4, 9, 16, ... are the squares of the natural numbers.

They are called perfect squares.

What is the perfect square after 16?

Why is 20 not a perfect square?

Let us look at the succession of perfect squares in another way.

To reach 4 from 1, we must add 3.

To reach 9 from 4?

We can state these as

$$4 - 1 = 3$$

$$9 - 4 = 5$$

$$16 - 9 = 7$$

All these differences are odd numbers, right?

So, the difference of two consecutive perfect squares is an odd number.

Let's write this as,

$$4 = 1 + 3$$

$$9 = 4 + 5 = 1 + 3 + 5$$

$$16 = 9 + 7 = 1 + 3 + 5 + 7$$

What do we see here?

When we add consecutive odd numbers starting from 1, we get the perfect squares.

This can be seen from these pictures also.



Can you write down the squares of natural numbers upto 20, by adding odd numbers? You can proceed like this

$$1^2 = 1$$

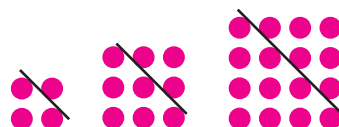
$$2^2 = 1 + 3 = 4$$

$$3^2 = 4 + 5 = 9$$

$$4^2 = 9 + 7 = 16$$

Squares and triangles

Look at these pictures:



Each square is divided into two triangles.

Let's translate this into numbers:

$$4 = 1 + 3$$

$$9 = 3 + 6$$

$$16 = 6 + 10$$

Check whether the same pattern continues.

What do we see?

All perfect squares after 1 are the sums of two consecutive triangular numbers.

What is the sum of the seventh and eighth triangular numbers?

Increase and decrease

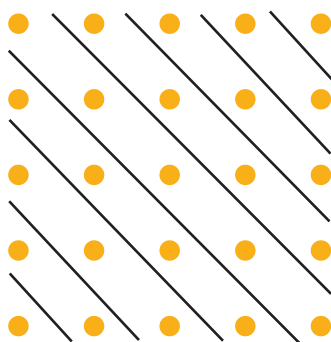
Look at this number pattern:

$$1 = 1$$

$$4 = 1 + 2 + 1$$

$$9 = 1 + 2 + 3 + 2 + 1$$

$$16 = 1 + 2 + 3 + 4 + 3 + 2 + 1$$



Can you split some more perfect squares like this?

What is the relation between the number of consecutive odd numbers from 1 and their sum?

What is the sum of 30 consecutive odd numbers starting from 1?

Tricks with ten

The square of 10 is 100. What is the square of 100?

In the square of 1000, how many zeros are there after 1?

What about the square of 10000?

What happens to the number of zeros on squaring?

So how do we spot the perfect squares among 10, 100, 1000, 10000 and so on?

Is one lakh a perfect square?

What about ten lakhs (million)?

Now find out the squares of 20, 200 and 2000.

Is 400000000 a perfect square?

What if we put in one more zero?

Now some problems. Do them all in your head.

- Find out the squares of these numbers.
 - 30 ■ 400 ■ 7000 ■ 6×10^{25}
- Find out the perfect squares among these numbers.
 - 2500 ■ 36000 ■ 1500
 - 9×10^7 ■ 16×10^{24}

Next square

What is the square of 21?

Wait a bit before you start multiplying.

The square of 20 is 400, isn't it? So to get the square of 21, we need only add an odd number.

Which odd number?

Let's start from the beginning. We can write

$$2^2 = 1^2 + 3 = 1^2 + (1 + 2)$$

$$3^2 = 2^2 + 5 = 2^2 + (2 + 3)$$

$$4^2 = 3^2 + 7 = 3^2 + (3 + 4)$$

$$5^2 = 4^2 + 9 = 4^2 + (4 + 5)$$

and so on. Continuing like this, how do we write 21^2 ?

$$21^2 = 20^2 + (20 + 21)$$

That is,

$$21^2 = 400 + 41 = 441$$

Now we can continue as before with

$$22^2 = 441 + 43 = 484$$

and so on.

How do we find out the square of 101?

$$100^2 = 10000$$

What more should we add?

$$100 + 101 = 201$$

So,

$$101^2 = 10000 + 201 = 10201$$

- Find out the squares of these numbers using the above idea.
 - 51 ■ 61 ■ 121 ■ 1001
- Compute the squares of natural numbers from 90 to 100.

Fraction squares

A fraction multiplied by itself is also a square.

What is the square of $\frac{3}{4}$?

$$\left(\frac{3}{4}\right)^2 = \frac{3}{4} \times \frac{3}{4} = \frac{3 \times 3}{4 \times 4} = \frac{9}{16}$$

That is,

$$\left(\frac{3}{4}\right)^2 = \frac{9}{16} = \frac{3^2}{4^2}$$

So to square a fraction, we need only square the numerator and denominator separately.

Square difference

We have seen that

$$2^2 = 1^2 + (1 + 2)$$

$$3^2 = 2^2 + (2 + 3)$$

$$4^2 = 3^2 + (3 + 4)$$

and so on.

We can write these in another manner also:

$$2^2 - 1^2 = 1 + 2$$

$$3^2 - 2^2 = 2 + 3$$

$$4^2 - 3^2 = 3 + 4$$

In general, the difference of the squares of two consecutive natural numbers is their sum.

Now look at these:

$$3^2 - 1^2 = 9 - 1 = 8$$

$$4^2 - 2^2 = 16 - 4 = 12$$

$$5^2 - 3^2 = 25 - 9 = 16$$

What is the relation between the difference of the squares of alternative natural numbers and their sum?





Project

Last digit

Look at the last digit of squares of natural numbers from 1 to 10:

1, 4, 9, 6, 5, 6, 9, 4, 1, 0

Now, look at the last digits of squares of natural numbers from 11 to 20.

Do we have the same pattern?

Let's look at another thing: Does any perfect square end in 2?

Which are the digits which do not occur at the end of perfect squares?

Is 2637 then a perfect square?

To decide that a number is not a perfect square, we need only look at the last digit.

Can we decide that a number is a perfect square from its last digit alone?

Now do these problems without pen and paper.

- Find out the squares of these numbers.

$$\blacksquare \frac{2}{3} \quad \blacksquare \frac{1}{5} \quad \blacksquare \frac{7}{3} \quad \blacksquare 1\frac{1}{2}$$

- Which of the fractions below are squares?

$$\blacksquare \frac{4}{15} \quad \blacksquare \frac{8}{9} \quad \blacksquare \frac{16}{25} \quad \blacksquare 2\frac{1}{4}$$
$$\blacksquare 4\frac{1}{9} \quad \blacksquare \frac{8}{18}$$

Decimal squares

What is the square of 0.5?

We know that $5^2 = 25$. How many decimal places would be there in the product 0.5×0.5 ?

Why?

$$0.5 = \frac{5}{10}, \text{ right?}$$

Can you find out the square of 0.05?

You have computed the squares of many natural numbers.

Using that table, can you find out the square of 0.15?

Do these problems also in your head.

- Find out the squares of these numbers.

$$\blacksquare 1.2 \quad \blacksquare 0.12 \quad \blacksquare 0.013$$

- Which of the following numbers are squares?

$$\blacksquare 2.5 \quad \blacksquare 0.25 \quad \blacksquare 0.0016$$

$$\blacksquare 14.4 \quad \blacksquare 1.44$$

Square product

What is $5^2 \times 4^2$?

$$5^2 \times 4^2 = 25 \times 16 = \dots\dots\dots$$

There is an easier way:

$$\begin{aligned} 5^2 \times 4^2 &= 5 \times 5 \times 4 \times 4 \\ &= (5 \times 4) \times (5 \times 4) \\ &= 20 \times 20 \\ &= 400 \end{aligned}$$

Can you find out the products below like this, without pen and paper?

■ $5^2 \times 8^2$ ■ $2.5^2 \times 4^2$ ■ $(1.5)^2 \times (0.2)^2$

What general rule did we use in all these?

The product of the squares of two numbers is equal to the square of their product.

How do we say this in algebra?

$x^2y^2 = (xy)^2$, for any numbers x, y

What about for three numbers?

Square factors

How do we write 30 as a product of prime numbers?

$30 = 2 \times 3 \times 5$

So how do we factorize 900?

$900 = 30^2 = (2 \times 3 \times 5)^2 = 2^2 \times 3^2 \times 5^2$

Similarly, using the facts that $24 = 2^3 \times 3$ and $24^2 = 576$, we get

$576 = 24^2 = (2^3 \times 3)^2 = (2^3)^2 \times 3^2 = 2^6 \times 3^2$

Can you write each number below and its square as a product of prime powers?

- 35 • 45 • 72
- 36 • 49

Did you note any peculiarity of the exponents of the factors of the squares?

Reverse computation

We have to draw a square; and its area must be 9 square centimetres.

How do we do it?

The area of a square is the square of the side.

Rectangle and square

Look at this picture.

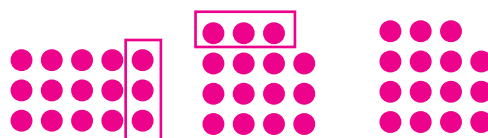


Dots in a rectangle.

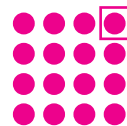
Can you rearrange the dots to make another rectangle?

Can you rearrange the dots to make a square?

Start like this:



How many more are needed to make a square?



How many dots were there in the original rectangle? How many in this square?

What do we see here?

$4^2 = (3 \times 5) + 1$

Can we do this for all rectangular arrangements?

The numbers here are 3, 4, 5.

So, for this trick to work, what should be the relation between the number of dots in each row and column of the rectangle?

We can write this in numbers as

$2^2 = (1 \times 3) + 1$

$3^2 = (2 \times 4) + 1$

$4^2 = (3 \times 5) + 1$

Try to continue this.

Square root of a perfect square

784 is a perfect square. What is its square root?

784 is between the perfect squares 400 and 900; and we know that their square roots are 20 and 30. So $\sqrt{784}$ is between 20 and 30. Since last digit of 784 is 4, its square root should have 2 or 8 as the last digit. So $\sqrt{784}$ is either 22 or 28.

784 is near to 900 than 400. So $\sqrt{784}$ must be 28. Now calculate 28^2 and check.

Given that 1369, 2116, 2209 are perfect squares, find their square roots like this.

So if the area is to be 9 square centimetres, what should be the side?

To draw a square of area 169 square centimetres, what should be the length of a side?

For that, we must find out which number squared gives 169. Looking up our table of squares, we find $13^2 = 169$. So we must draw a square of side 13 centimetres.

Here, given a number we found out which number it is the square of. This operation is called extracting the *square root*.

That is, instead of saying the square of 13 is 169, we can say in reverse that the square root of 169 is 13.

Just as we write

$$13^2 = 169$$

as shorthand for statement "the square of 13 is 169", we write the statement "the square root of 169 is 13" in shorthand form as

$$\sqrt{169} = 13$$

(the extraction of square root is indicated by the symbol $\sqrt{\quad}$).

Similarly, the fact that the square of 5 is 25 can also be stated, the square root of 25 is 5. In short hand form,

$$5^2 = 25$$

$$\sqrt{25} = 5$$

In general

For numbers x and y , if $x^2 = y$, then $\sqrt{y} = x$

Now find out the square root of these numbers:

- 100
- 256
- $\frac{1}{4}$
- $\frac{16}{25}$
- 1.44
- 0.01

Square root factors

How do we find the square root of 1225?

Since a product of squares is the square of the product, we need only write 1225 as a product of squares.

First factorize 1225 into primes:

$$1225 = 5^2 \times 7^2$$

And we can write

$$5^2 \times 7^2 = (5 \times 7)^2 = 35^2$$

So, $1225 = 35^2$

From this, we get $\sqrt{1225} = 35$

Let's take another example. What is the $\sqrt{3969}$?

As before, we first factorize 3969 into primes.

$$\begin{aligned} 3969 &= 3^2 \times 3^2 \times 7^2 \\ &= (3 \times 3 \times 7)^2 \end{aligned}$$

From this, we get $\sqrt{3969} = 3 \times 3 \times 7 = 63$

Now compute the square roots of these.

- 256
- 2025
- 441
- 9216
- 1089
- 15625
- 1936
- 3025
- 12544



Let's do it!

- The area of a square plot is 1024 square metres. What is the length of its sides?
- In a hall, 625 chairs are arranged in rows and columns, with the number of rows equal to the number of columns. The chairs in one row and one column are removed. How many chairs remain?
- The sum of a certain number of consecutive odd numbers, starting with 1, is 5184. How many odd numbers are added?
- The sum of two consecutive natural numbers and the square of the first is 5329. What are the numbers?



Project

Digit sum

16 is a perfect square and the sum of its digits is 7.

The next perfect square 25 also has digit sum 7.

The digit sum of 36 is 9.

The sum of the digits of the next perfect square 49 is 13. If we add the digits again, the sum is 4. Find the sum of the digit sums (reduced to a single digit number) of perfect squares starting from 1.

Do you see any pattern?

Is 3324 a perfect square?

Looking back



Achievements	On my own	With teacher's help	Must improve
<ul style="list-style-type: none">Describing squares and perfect squares with examples.			
<ul style="list-style-type: none">Explaining squares and perfect squares with examples.			
<ul style="list-style-type: none">Explaining the peculiarities of squares logically.			
<ul style="list-style-type: none">Describing methods to compute the square root of a perfect square.			
<ul style="list-style-type: none">Explaining the peculiarities of square roots with examples.			
<ul style="list-style-type: none">Solving practical problems using square and square root.			

7

Speed Math

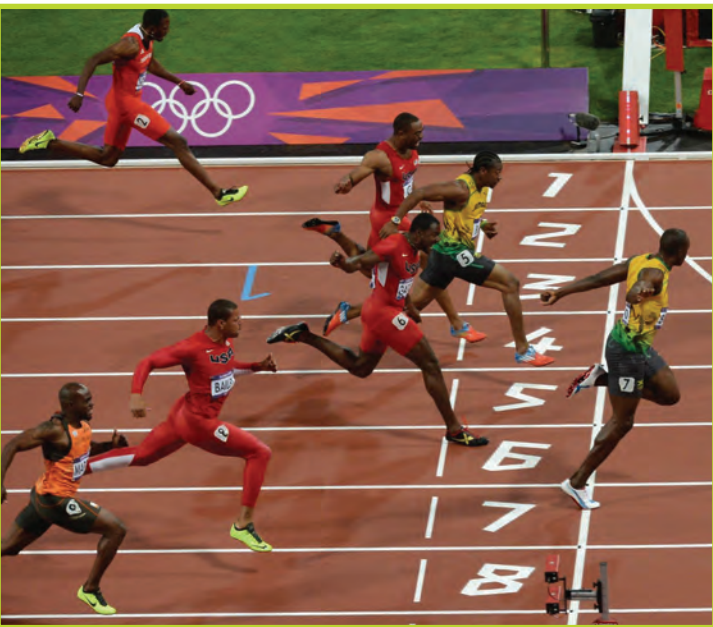


Olympics 2012

Look at the table showing the first five in men's 100 metres sprint in the London Olympics

Sl No.	Name	Time (in seconds)
1.	Usain Bolt	9.63
2.	Yohan Blake	9.75
3.	Justin Gatlin	9.79
4.	Tyson Gay	9.80
5.	Ryan Baily	9.88

How much time do you take to run 100 metres?



Who's the fastest?

"We have to find the fastest runner in the school. How do we do it?" The teacher asked.

"Let everyone run 100 metres", Raji said.

Reghu had another idea: "How about everyone running for 1 minute?"

They all went to the ground.

And all ran 100 metres.

These were the best four:

	Name	Time
1.	Shyam	16 second
2.	Joy	18 second
3.	Reghu	18 second
4.	Musthafa	17 second

Who won the race?

Is it easy to conduct a race as Reghu suggested?

Sports meet

Reghu and friends went in a bus to the sports meet at Kozhikode. They started at 7 in the morning and reached there at 10, travelling 150 kilometres. Would the speed have been the same during the entire trip?

Could have been 40 kilometres the first hour, 60 kilometres the next and 50 kilometres the last hour.

It's in such instances that we calculated the average, remember?

Here, the distance travelled is 150 kilometres.

And how much time did it take?

So, we can say they travelled $\frac{150}{3} = 50$ kilometres in one hour, on average.

We can put it this way: the average speed of the bus was 50 kilometres per hour,

This we write 50 km/h.

Average speed

Celina and Beena went to Kozhikode for the State Arts Festival. Celina travelled in a jeep, covering 90 kilometres in 2 hours; Beena came in a car, travelling 150 kilometres in 3 hours. Which vehicle is faster?

The jeep travelled 90 kilometres, right?

And took 2 hours for the trip.

So what is its average speed?

$$\frac{90}{2} = 45 \text{ km/h}$$

The average speed of the car can also be computed like this.

Which is greater?

Now try to solve these problems:

- Sudheer travelled in a train, covering 240 kilometres in 3 hours to reach Thiruvananthapuram. Ramesh came in another train, travelling 120 kilometres in 2 hours. Which train is faster? How much faster?
- A train took 4 hours and 30 minutes to cover 360 kilometres. What is its average speed?

*I can get us
there in no time
brother!*

*Be your age and
drive at an
average speed!*



Rotating earth

Do we ever remain still? The Earth which carries us all, turns all the time, spinning on its own and going round the Sun. It spins at about 1700 km/h and revolves round the Sun at about 100000 km/h.

My head starts spinning everytime I think about earth's spin

And to think we go up and down everyday...



Let's look at another problem.

If the average speed of a bus is 52 km/h, how much does it travel in 6 hours?

Since it travels 52 kilometres each hour on average, in 6 hours it travels

$$52 \times 6 = 312 \text{ km}$$

How long does it take to travel 520 kilometres at this speed?

- The table below gives some details of Joy's trip. Fill in the missing entries:

Vehicle	Distance	Time	Average speed
Train	4 hour	60 km/h
Car	120 km	2 hour
Aeroplane	5040 km	840 km/h

- Shyama's exam starts at 2 o'clock. To reach the place, she has to travel 50 kilometres by bus and 175 kilometres by train. Average speed of bus journey is 20 km/h and average speed of train journey is 50 km/h. To reach there 1 hour before the exam, when should Shyama leave home?

Saving time

A bus leaves Ernakulam at 6 in the morning and reaches Thiruvananthapuram at 12 noon, running 40 km/h on average. To reach one hour earlier, by how much should the average speed be increased?

What is the total distance covered?

If the journey time is to be reduced by 1 hour, what should be the time for the trip?

What should be the average speed to reach an hour earlier?

Railway station

Abu boarded a bus at 7 in the morning. Usually the bus travels 30 km/h on average, to reach the railway station at 11. But since it was raining, the bus could run only 20 km/h on average. Abu got down from the bus at 9, caught a car and reached the station at 11 itself. What was the average speed of the car?

What is the distance to the railway station?

How far did the bus travel in the first 2 hours?

So how many kilometres did the car travel?

How much time did it take?

Now can't you find its average speed?

Average of speeds and average speed

A vehicle did the first 120 kilometres of a trip at an average speed of 30 km/h and the next 120 kilometres at 20 km/h. What is the average speed of the entire trip?

The average of the two speeds is

$$\frac{30 + 20}{2} = 25 \text{ km/h}$$

Is this what we want?

What is the correct reasoning?

To compute the average speed, we must divide the total distance by the total time. Time to travel 120 kilometres at an average speed of 30 km/h is

$$\frac{120}{30} = 4 \text{ hours}$$

Time to travel 120 kilometres at an average speed of 20 km/h is

$$\frac{120}{20} = 6 \text{ hours}$$

Total time of travel $4 + 6 = 10$ hours

Total distance travelled = 240 km

$$\text{Average speed} = 24 \text{ km/h}$$



Value of time

The smallest unit of time we usually use is a second. Sometimes we need to use smaller units such as a microsecond or a nanosecond. A microsecond is a millionth of a second. A nanosecond is a thousandth of a microsecond.

Do you know by what part of a second did P.T. Usha lose an Olympic medal?



Look at the average speeds of some animals:

Sl.No.	Animal	km/h
1	Cheetah	112
2	Horse	70
3	Fox	65
4	Lion	80
5	Elephant	40
6	Zebra	64



Train and bus

Rahim travelled 350 kilometres in a train and 150 kilometres in a bus. The average speed of the train was 70 km/h. The bus trip lasted 5 hours. What is the average speed of the entire trip?

To Ratnagiri

Ratnagiri is 360 kilometres from Pavizhamala. Gopika's family travelled in a car, at an average speed of 60 km/h. They managed only 40 km/h during the return trip. What is the average speed of the entire trip?

Suppose we take the distance as 180 kilometres in this problem.

Does it change the average speed for the entire journey?

Unknown distance

Babu went to Mananthavady by bus to see his friend. The average speed of the bus was 40 km/h. He came back in a car, at an average speed of 60 km/h. What is the average speed of the entire trip?

To find this, we have to divide the total distance by the total time. But we don't know the distance.

We have seen in an earlier problem the average speed would be the same, whatever be the total distance.

Let's take the distance one way as 120 kilometres.

Total distance is then 240 kilometres

$$\text{Time for the onward trip} = \frac{120}{40} = 3 \text{ hours}$$

$$\text{Time for the return trip} = \quad = 2 \text{ hours}$$

$$\text{So, the average speed of the entire trip} = 48 \text{ km/h}$$

What if we take the one way distance as 240 kilometres?

Can't we find the average speed of this trip?

Johny's journey

- Johny went to his uncle's house, riding his bicycle at 15 km/h and came back at 10 km/h. What's his average speed for the whole journey?

In seconds

A vehicle travels at an average speed of 72 km/h. How much will it move in one second?

One hour means 60 minutes; and a kilometre is 1000 metres.

Thus in 60 minutes, this vehicle will travel 72000 metres.

So, in one minute, it would move $\frac{72000}{60} = 1200$ metres.

Which means that in one second it would move $\frac{1200}{60} = 20$ meters.

We can say that the average speed of the car is 20 meters per second, written 20 m/s.

Compute the speed per hour of a vehicle doing 15 m/s.

Now try these problems.

- A train travels at an average speed of 36 km/h. How far will it go in 3 minutes?
- A 180 metres long train takes 9 seconds to pass a vertical pole. What is the average speed of the train in kilometre per hour?



Let's do it!

- A car goes an average speed of 36 km/h for 15 minutes and 60 km/h for the next 15 minutes. How much distance did it cover?
- Ramu and Salim travelled to Thiruvananthapuram from the same place, each in his own car. Ramu did the

Over speeding

How much does a car doing 90 km/h move in a minute?

$$\frac{90}{60} = \frac{3}{2} = 1 \frac{1}{2} \text{ km}$$

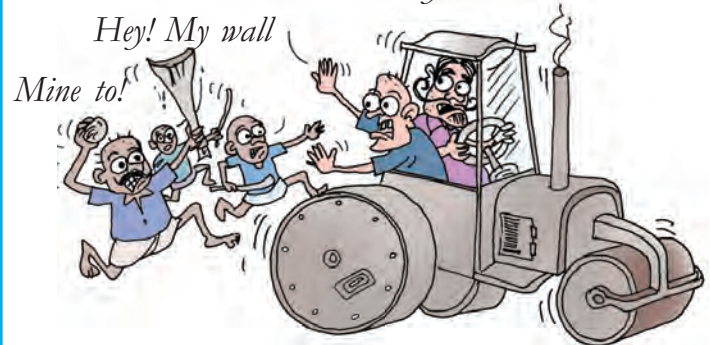
And in one second?

$1 \frac{1}{2}$ kilometers means 1500 metres.

$$\frac{1500}{60} = \frac{75}{3} = 25 \text{ metres}$$

So what if the driver is one second late in applying the brakes? The vehicle would have moved 25 metres ahead.

I told you to go at an average speed. Now you have an enraged mob behind you.



Road accidents

We read about traffic accidents every day in the papers. The main causes are speeding and carelessness. How many lives are lost on our roads everyday!

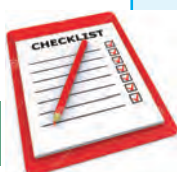
There is a law that heavy vehicles should install a "speed-lock". With this device, they cannot go beyond a certain speed.

If each of us obey the traffic rules, we can reduce the number of accidents.

onward journey at an average speed of 120 km/h and the return at 50 km/h. Salim did both trips at 60 km/h. Who made the total journey in less time?

- Two trains travelling along parallel tracks in the same direction do 50 km/h and 100 km/h on average. The faster train starts 2 hours later than the slower. After how many kilometres will the faster train catch up?
- A 125 metre long train travels at 90 km/h. How much time would it take to cross a 175 metre long bridge?

Looking back



Achievements	On my own	With teacher's help	Must improve
<ul style="list-style-type: none">• Using the idea of average speed to solve real life problems.			
<ul style="list-style-type: none">• Knowing the relations between distance, time and speed.			
<ul style="list-style-type: none">• Solving problems by using different units, depending on context.			