FIRST PRE-BOARD EXAMINATION (2017-18)
CLASS: XII

Subject: MATHEMATICS **Date: 14.12.2017**

Time allowed: 3 Hours. Maximum Marks: 100

General instructions:

(1) **All** questions are **compulsory**.
(2) This question paper contains **29** questions.
(3) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(4) Question **5 - 12** in **Section B** are short-answer type questions carrying **2** mark each.
(5) Question **13 - 23** in **Section C** are long-answer-I type questions carrying **4** mark each.
(6) Question **24 - 29** in **Section D** are long-answer-II type questions carrying **6** mark each.

Section – A

Questions 1 to 4 carry 1 mark each.

1. If det(A) = 3, then write det(A^{-1}) where A^{-1} is the inverse of the matrix A.
2. Write a relation on the set A = {1, 2, 3} which is reflexive, symmetric but not transitive.
3. If $3\bar{i} + 4\bar{j} + 5\bar{k}$ and $\bar{j} - m\bar{k}$ are perpendicular to each other, write the value of m.
4. If the mappings f and g are given by f = {(a, b), (m, n), (p, q)} and g = {(2, a), (3, p)}, then write fog.

Section – B

Questions 5 to 12 carry 2 marks each.

5. Find the value of x, if $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 8 \end{bmatrix} = 0$.

6. Find the value of tan($\cos^{-1}$x) and hence find the value of $\tan\left(\cos^{-1}\frac{8}{17}\right)$.

7. The volume of a cube is increasing at the rate of 8 cm^3/ sec. How fast is its surface area increasing when the length of an edge is 12 cm?

8. Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x\,, x > 0.$

9. Prove that $\left(\overline{a} \times \overline{b}\right)^2 = (\overline{a})^2\left(\overline{b}\right)^2 - \left(\overline{a}.\overline{b}\right)^2.$

10. Evaluate $\int \frac{1}{\sqrt{x} + x\sqrt{x}}dx.$

11. A bag contains 4 red, 5 black and 6 green balls. Find the probability of drawing a red, a black and a green ball in succession in that order without replacement.

12. Write the order and degree of the differential equation
$\frac{d^3y}{dx^3} + 4x^2\frac{d^2y}{dx^2} - sin\left(\frac{dy}{dx}\right) = 6xy.$

Section – C

Questions 13 to 23 carry 4 marks each.

13. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 + cosA & 1 + cosB & 1 + cosC \\ cosA + \ cos^2A & cosB + \ cos^2B & cosC + \ cos^2C \end{vmatrix} = 0$, then show that the triangle ABC is an isosceles triangle.

14. Let a, b, c be distinct non-negative numbers. If the vectors $a\overline{i} + a\overline{j} + c\overline{k}, \overline{i} +$ $+\overline{k}$ and $c\overline{i} + c\overline{j} + b\overline{k}$ lie in a plane, then prove that c is the geometric mean of a and b.

15. Find the intervals in which the function f given by f(x) = sinx – cosx + 3, $0 \le x \le \pi$ is strictly increasing and strictly decreasing.

16. In a game, a man wins Rs. 5 for getting a number greater than 4 and loses Rs.1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number more than 4. Fine the expected value of the amount he wines/loses. Is it worth playing gambling?

17. Find the points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at tangents are (i) parallel to x-axis (ii) parallel to y-axis.

OR

If the sum of lengths of the hypotenuse and a side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between them is 60^0.

18. Evaluate $\int \frac{1}{\cos(x+a)\cos(x+b)} dx$.

19. If cosy = x cos (a + y), with $cosa \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{cos^2(a+y)}{sina}$.

20. Solve the differential equation $\frac{dy}{dx} = \tan(x-y)$.

OR

Find the particular solution of the differential equation dx + x dy = $e^{-y} sec^2 y$ dy, given $(0) = \frac{\pi}{4}$.

21. There are 10 coins in a bag in which four are biased. A coin chosen at random and tossed, it is found to show head. If the probability of getting head on biased coin is 2/3, then find the probability that one of the unbiased coin is tossed.

22. Check the differentiability of the function $(x) = \begin{cases} 1-x & , x < 1 \\ (1-x)(2-x), & 1 \le x \le 2 \\ 3-x & , x > 2 \end{cases}$, at x = 2.

OR

Discuss the continuity of the function $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & , x < 0 \\ 8 & , x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & , x > 0 \end{cases}$ at x = 0.

23. Find the foot of the perpendicular from the point (5, 6, 7) to the plane $\bar{r}.(\bar{i} + 2\bar{j} + 3\bar{k}) = 10$.

Section – D

Questions 24 to 29 carry 6 marks each.

24. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4, y = 4 and y = 0 into three equal parts.

25. Let Q be set of rational numbers and an operation $*$ on Q is defined by $a * b = a + b - ab$. Check whether $*$ is (i) a binary operation (ii) commutative and (iii) associative on Q. Also find the identity element in Q under $*$, if exists.

OR

Let A = R – { -1} and B = R – { 2} and $f: A \rightarrow B$ is defined by $f(x) = \frac{2x-3}{x+1}$. Then show that f is bijective and find its inverse .

26. Solve the system of simultaneous equations 2x + y – 3z + 5 = 0, 5x + 2y + z = 12 and x – 6y + 4z = 1, using inverse of a matrix.

OR

Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1$. Show that the equation

$$\begin{vmatrix} ax-by-c & bx+ay & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix} = 0$$ represents a straight line.

27. Evaluate $\int_0^4 (e^{2x} + x)dx$ as a limit of sums.

OR

Evaluate $\int_0^{\pi} \frac{x}{1+\sin\alpha \, \sin x}\, dx$.

28. Show the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the plane containing these lines.

29. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectively, find the number of tennis rackets and crickets bats that the factory must manufacture to earn the maximum profit. Make it as an L.P.P. and solve graphically.

...@...