Series: SOS	Code No. 65 / 1

Roll No. | | | | | | | |

Candidates must write the code on the title page of the answer-book

Please check that this question paper contains 4 printed pages.

Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

Please check that this question paper contains 29 questions.

Please write down the serial number of the question before attempting it.

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First Pre-Board Examination, 2017-2018
Mathematics

Grade: 12 **Time allowed: 3 hours**

Date: 00.00.0000 **Maximum Marks: 100**

General Instructions.

1. **All questions are compulsory**
2. **This question paper contains 29 questions**
3. **Questions 1 – 4 in Section A are very short answer type questions carrying 1 mark each**
4. **Questions 5 – 12 in Section B are short answer type questions carrying 2 marks each**
5. **Questions 13 – 23 in Section C are long answer I type questions carrying 4 marks each**
6. **Questions 24 – 29 in Section D are long answer II type questions carrying 6 marks each**
7. **Read questions carefully before answering them.**
8. **Write neatly. Rough work may be done neatly in a working column or on the last page of your main sheet**
9. **Diagrams (if any) should be drawn neatly with pencil and labeled properly.**
10. **Write question numbers correctly.**

SECTION A

1) Let f : $\{1,3,4\} \rightarrow \{1,2,5\}$ and g :$\{1,2,5\} \rightarrow \{1,3\}$ be given by f = { (1,2),(3,5),(4,1)} and g = {(1,3), (2,3), (5,1)} write down g o f (3) (1)

2) If A and B are matrices of order 3 such that $|A|$ = -1 and $|B|$= 3 , then find $|3AB|$. (1)

3) Give an example of vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = |\vec{b}|$ but $\vec{a} \neq \vec{b}$ (1)

4) Let * be a binary operation on the set all real numbers R defined by a*b = $\mathrm{a} + \mathrm{b} + a^2 b$ for a , b ∈ R. Find 2 * 6. (1)

SECTION B

5) If 4 $\cos^{-1} x + \sin^{-1} x = \pi$ then find the value of x. (2)

6) Find the inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$. Hence find the matrix satisfying the matrix equation

P$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ = $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ (2)

7) Prove that $\tan^{-1} \frac{1-\sqrt{2}}{1-2\sqrt{2}} + \tan^{-1} \frac{1+\sqrt{2}}{1+2\sqrt{2}} = \frac{\pi}{4}$. (2)

8) Use differentiation to approximate $\sqrt{36.6}$. (2)

9) Evaluate: $\int \frac{e^x\sqrt{1-\sin 2x}}{1-\cos 2x} dx$ (2)

10) Show that y = $a\cos x$ + $b\sin x$ is the solution of the differential equation $\frac{d^2y}{dx^2}$ + y = 0 (2)

11) Find the projection vector of $2\hat{i} - \hat{j} + \hat{k}$ on $\hat{i} - 2\hat{j} + \hat{k}$. (2)

12) If A and B are two events such that P(A) = $\frac{7}{13}$ and P(B) = $\frac{9}{13}$ and P$(B|A)$ = $\frac{4}{7}$, then

find P$(A|B)$ (2)

SECTION C

13) If Δ = $\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix}$ = a then find the value of $\begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix}$ (4)

14) Consider f(x) = $\begin{cases} kx+1 \; if \; x \le 5 \\ 3x-5 \; if \; x > 5 \end{cases}$ find the value of k, if f(x) is continuous at x= 5

OR

Find the relationship between a and b, if the function given by f(x) = $\begin{cases} ax^2+b \,, if \; x < 1 \\ 2x+1 \,, if \; x \ge 1 \end{cases}$

is differentiable at x = 1. (4)

15) If y = $\sin^{-1} x$, then prove that (1-x^2)y'' – x y' = 0 (4)

16) Find the intervals in which the function f(x) = 3x^4 - 4 x^3 - 12 x^2 +5 is strictly increasing or strictly decreasing.

OR

Find the equation of tangent to the curve y = $\sqrt{3x-2}$, which is parallel to the line 4x -2y +5 = 0 (4)

17) The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (marginal revenue). If the total revenue received from the sale of x units of a product is given by R(x) = $3x^2$ +36x +5, find the marginal revenue when x =5, and write which value does the question indicate. (4)

18) Find $\int \frac{\mathbf{cosecx}}{\mathbf{1+secx}} dx$. (4)

19) Find the particular solution for the differential equation: $\frac{dy}{dx} + \frac{y}{x} = x^2$, y (1) = 1

OR

Show that the differential equation $x\frac{dy}{dx} sin\frac{y}{x} + x - y \; sin\frac{y}{x} = 0$ is homogeneous. Also find the general solution of the given differential equation. (4)

20) Prove that $[\mathbf{a}+\mathbf{b}\ \mathbf{b}+\mathbf{c}\ \mathbf{c}+\mathbf{a}] = \mathbf{2}\ [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ (4)

21) Find the equation of the line which intersects $\vec{r}$ =(-2$\hat{i}$ +3 $\hat{j}$ - $\hat{k}$)+ λ($\hat{i}$ +2 $\hat{j}$ +4$\hat{k}$) and $\vec{r}$ =($\hat{i}$ +2 $\hat{j}$ +3$\hat{k}$)+ μ(2$\hat{i}$ +3 $\hat{j}$ +4$\hat{k}$) and passing through the point (1,1,1). (4)

22) Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II. (4)

23) Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of kings. (4)

SECTION D

24) If the function f:R→ R be defined by f(x) = 2x - 3 and g : R→ R by g(x) = x^3 + 5 , then find the value of $(\mathrm{f\ o\ g})^{-1}(\mathrm{x})$ and hence find$(\mathrm{f\ o\ g})^{-1}(23)$

OR

Let * be a binary operation on Q defined by a*b =$\frac{\mathrm{ab}}{6}$ for a , b ∈ Q. Find identity and inverse and hence find inverse of 9 with respect to * (6)

25. If A = $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$,then find A^{-1} and hence solve the following system of equations

x + 2y +z = 8

2x + y – z = 1

x – y + z = 2

OR

If A = $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$,then find A^{-1} using elementary row transformations and hence solve the following matrix equation XA = $\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$ (6)

26) Using Integration, find the area in the first quadrant bounded by the curve y = x$|x|$, the circle x^2 + y^2 = 2 and the y –axis. (6)

27) Evaluate $\int_{\frac{-\pi}{6}}^{\frac{\pi}{6}} \frac{\mathbf{x+7}}{\mathbf{2-\cos 2x}}$dx

OR

Evaluate $\int_0^2 (\boldsymbol{x^2} + \mathbf{1})$dx as the limit of a sum (6)

28) Find the distance of the point (-2 , 3 , -4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3Z+4}{5}$ measured parallel to the plane 4x + 12y -3z +1 = 0 (6)

29. A company produces two types of cricket balls A and B. The production time of one ball of type B is double the type A. The company has the time to produce a maximum of 2000 balls per day. The supply of raw materials is sufficient for the production of 1500 balls per day. The company wants to make maximum profit by making profit of Rs.3 from a ball of type A and Rs.5 from type B.

(i) Write the objective function.

(ii) Write the constraints.

(iii) How many balls should be produced in each type per day in order to get maximum profit? (6)