MODEL EXAMINATION, JANUARY-2020
SUBJECT : MATHEMATICS

Class : XII **Time Allowed: 3 hours** **Maximum Marks: 80**

SET : A

General Instructions:

(i) *All questions are compulsory.*
(ii) *The question paper consists of 36 questions divided into four sections A, B, C and D.*
(iii) *Section A comprises of 20 questions of one mark each.*
(iv) *Section B comprises of 6 questions of two marks each.*
(v) *Section C comprises of 6 questions of four marks each.*
(vi) *Section D comprises of 4 questions of six marks each.*
(vii) *There is no overall choice. However internal choice has been provided in 3 questions of one mark each, 2 questions of 2 marks each, 2 questions of 4 marks each and 2 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.*
(viii) *Use of calculators is not permitted.*

SECTION : A

Q 1 to Q10 are multiple choice type questions. Select the correct option.

1. The number of all possible matrices of order 2x3 with each entry 0 or 1 is
a) 64 b) 12 c) 36 d) 6

2. If A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$, then $|3AB|$ is equal to
a) –9 b) –27 c) –81 d) 81

3. If $(2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}) \times (\hat{\imath} + p\hat{\jmath} + q\hat{k}) = \overrightarrow{O}$, then the values of p and q are?
a) p = 6, q = 27 b) p = 3, q = $\frac{27}{2}$ c) p = 6, q = $\frac{27}{2}$ d) p = 3, q = 27

4. If P(A) = 0.6, P(B) = 0.5 and P(A/B) = 0.3, then the value of $P(A \cup B)$ is ?
a) 1.1 b) 0.1 c) 0.95 d) 0.59

5. The corner points of the feasible region determined by the following system of linear inequalities $2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0, 0), (5,0), (3, 4) and (0,5). Condition on p and q so that the maximum of $Z = px + qy$, p and q>0 occurs at both (3, 4) and (0, 5) is
a) p = q b) p = 2q, c) p = 3q d) q = 3p

6. $sin^{-1}\left(sin\left(\frac{7\pi}{4}\right)\right)$ is

a) $\frac{7\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{-3\pi}{4}$ d) $\frac{-\pi}{4}$

7. Two cards are drawn from a pack of 52 cards. The probability that first is heart card and second is red card is

a) $\frac{25}{102}$ b) $\frac{25}{204}$ c) $\frac{1}{169}$ d) $\frac{25}{169}$

8. $\int \frac{dx}{\sqrt{1+4x^2}}$ is

a) $\frac{1}{2}\, tan^{-1}2x$ + c b) $\frac{1}{2}$log $|2x + \sqrt{1+4x^2}|$ +c

c) $\frac{x}{2}\sqrt{1+4x^2} + \frac{1}{2}$ log $|2x + \sqrt{1+4x^2}|$ + c d) $sin^{-1}2x$+ c

9. What is the distance (in units) between the two planes
$2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$?

a) 2 b) 4 c) 8 d) $\frac{2}{\sqrt{29}}$

10. If the lines $\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$ are perpendicular, the value of λ is

a) -2 b) 2 c) $\frac{1}{2}$ d) $\frac{-1}{2}$

II. (Q 11 – Q15) Fill in the blanks:

11. If f:R→R be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then fof (x) is ------------------------.

12. If $f(x) = \begin{cases} \frac{kx}{|x|} \text{ if } x < 0 \\ 3 \text{ if } x \geq 0 \end{cases}$ is continuous at x = 0, then the value of k is --------.

13. If tangent to the curve $y_2 + 3x - 7 = 0$ at the point (h, k) is parallel to line $x - y = 9$, then the value of k is -----------------.

OR

The approximate change in the value of $\frac{1}{x^2}$, when x changes from 2 to 2.002 is -----------.

14. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, then the value of x + y is ----------------.

15. The magnitude of projection of $(2\hat{\imath} - \hat{\jmath} + \hat{k})$ on $(i + 2\hat{\jmath} + 2\hat{k})$ is ----------.

OR

$\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if θ = -----------------.

III. **(Q 16 - Q20), Answer the following questions.**

16. If A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ is written as P + Q, where P is a symmetric matrix and Q is a skew symmetric matrix, then write the matrix Q.

17. Evaluate

$$\int_0^{\frac{\pi}{2}} cos2x\, dx$$

18. Evaluate

$$\int \frac{(x-3)}{(x-1)^3} e^x \mathrm{d}x$$

OR

Evaluate

$$\int \frac{x^3}{\sqrt{1-x^8}} \mathrm{d}x$$

19. Find $\int_{-1}^{1} sin^5 x\ cos^4 x\ dx.$

20. Write the general solution of the differential equation $x^5 \frac{dy}{dx} = -y^5$.

SECTION : B

21. Show that $sin^{-1}\frac{3}{5} - sin^{-1}\frac{8}{17} = cos^{-1}\frac{84}{85}$.

OR

Let R be the relation in the set Z of integers given by R = {(a,b) : 2 divides a – b}. Show that R is transitive? Write the equivalence class [1].

22. If $(x - \mathrm{y}).e^{\frac{x}{x-y}}$ = a, prove that $\frac{dy}{dx} + x = 2\mathrm{y}$.

23. Find the point on the curve $y_2 = 8x + 8$ for which the abscissa and ordinate change at the same rate.

24. For any three non-zero vectors $\vec{a}, \vec{b}, \vec{c}$, prove that

$[\vec{a} + \overrightarrow{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$.

OR

Let $\vec{a}, \vec{b}, \vec{c}$, be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.

25. Find the distance between the lines l_1 and l_2 given by

$\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

$\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

26. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event `the sum of numbers on the dice is 4.'

SECTION : C

27. Consider f: $R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$.

28. If $y = (x + \sqrt{1 + x^2})^n$, then show that $(1+x^2)\frac{d^2y}{dx^2} + x.\frac{dy}{dx} = n^2y$.

OR

If $x.\cos(a+y) = \cos y$ with $\cos a \neq \pm 1$, then show that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence show that $\sin a.\frac{d^2y}{dx^2} + \sin 2(a+y).\frac{dy}{dx} = 0$.

29. Find the particular solution, satisfying the given condition, for the following differential equation:

$\frac{dy}{dx} - \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right) = 0$; y = 0 when x = 1.

30. Evaluate :

$$\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)\,dx\,.$$

31. Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6 and 7. Let X denote the larger of the two numbers obtained. Find the probability distribution of X. Also find mean of the distribution.

OR

Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

32. There are two types of fertilisers F1 and F2. F1 consists of 10% nitrogen and 6% phosphoric acid and F2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F1 costs ₹6/kg and F2 cost ₹5/kg, then graphically determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost.

SECTION : D

33. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

$2y-3z = 1$

$x - y + 2z = 1$

$3x - 2y+4z = 2$

OR

Using the properties of determinants, prove that

$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)\,(x-y)(y-z)\,(z-x).$

34. Find the area of the region bounded by the curves $y=\sqrt{5-x^2}$ and $y = |x-1|$, using the method of integration.

35. Show that semi vertical angle of right circular cone of given surface area and maximum volume is $sin^{-1}\left(\frac{1}{3}\right)$.

OR

A metal box with square base and vertical sides is to contain 1024 cm^3. The material for the top and bottom costs ₹$5/cm^2$ and the material for the sides costs ₹$2.50/cm^2$. Find the least cost of the box.

36. Find the coordinates of the foot of the perpendicular `Q' drawn from P(3, 2, 1) to the plane $2x - y + z + 1 = 0$. Also, find the distance PQ and the image of the point P treating this plane as a mirror.

THE END