Pre-Board Examination 1- 2019-20

Sub:Mathematics (041)

Class: XII Marks: 80

Date: Time: 3 hrs.

General Instructions::

- All Questions are compulsory.
- The question paper consists of 5 printed pages and there are 36 questions divided into 4 sections A, B, C and D.
- Section A comprises of 20 questions of 1 mark each, Section B comprises of 6 questions of 2 marks each, Section C comprises of 6 questions of 4 marks each and Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, internal choices have been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Draw a column on the right-hand side of the page and show all calculations neatly.
- Use of calculator is not permitted.

Section - A

Q1- Q10 are multiple choice type questions. Select the correct option.

1) If set A contains 5 elements and set B contains 6 elements, then the number of one-one and onto mappings from A to B is
(a) 720 (b) 120 (c) 0 (d) None of these

2) The range of principal value branch of $\boldsymbol{cot^{-1}}$is
(a)$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (b)$(0,\pi)$ (c) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ (d) $[0,\pi]$

3) If A is a non-singular square matrix of order 3 such that $|adj\ A| = 225$, then $|A'|$ is
(a) 25 (b) 15, -15 (c) -25 (d) None of these

4) A sphere of radius 100mm shrinks to radius 98mm, then the approximate decrease in its volume is

a)$12000\pi mm^3$ b) $800\pi mm^3$ c)$8000\pi mm^3$ $d)120\pi mm^3$

5) The sum of the order and degree of the differential equation

$$\mathbf{1} - \left(\frac{dy}{dx}\right)^{2} = \left(a\frac{d^2y}{dx^2}\right)^{1/3} \text{ is}$$

(a) 2 (b) 0 (c) 1 (d) 3

6) If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ *and* $\vec{c} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$ such that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$ then value of λ is

(a) 2 (b) 3 (c) -3 (d) -2

7) The sum of intercepts cut off by the plane $2x + y - z = 5$ on the co-ordinate axes is

(a)$\frac{5}{2}$ (b)5 (c) $-\frac{5}{2}$(d) -5

8) 10%of bulbs produced in a factory are red colored and 2% are red and defective. If one bulb is selected at random, then the probability of the bulb being defective if it is red is

(a)$\frac{1}{5}$ (b) 5 (c) $\frac{2}{5}$ (d) $\frac{1}{10}$

9) In a linear function $z = ax + by$,the variables x and y are called

(a) *objective function*(b) *decision variables* (c)*linear relation* (d)*constraints*

10) If A and B are independent events, and $P(A) = 0.40, P(B) = 0.30$then $P(\textit{neither } A \textit{ nor } B)$ is

(a)0.70 (b) 0.58 (c) 0.42 (d)0.12

11) The integral value of x if $[x \quad 4 \quad -1]\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix}[x \quad 4 \quad -1]' = 0$ *is* _____________.

OR

If $A = \textbf{diagonal}\,[1 \quad -2 \quad 5]$ *and* $B = \textbf{diagonal}\,[3 \quad 0 \quad -4]$then$3A - 2B$ is _____.

12) The slope of the tangent to the curve $x = 1 - a\sin\theta,\ y = b\cos^2\theta$ *at* $\frac{\pi}{2}$ is ____.

13) The area of the region bounded by the curve, $x = g(y), Y - \textbf{axis}$ *and the lines* $y = c,\ y = d$is given by ________________.

OR

The area of the region bounded by the two parabolas $\boldsymbol{y = x^2}$ *and* $\boldsymbol{y^2 = x}$ is

______.

14) The differential equation of the family of lines passing through origin is _____.

15) The projection of vector $\mathbf{2\hat{\imath} + 3\,\hat{\jmath} - \hat{k}}$ on the vector $\hat{\imath} + \hat{\jmath}$ is___________.

16) Find the derivative of $\sin(\sin x^2)$at $x = \sqrt{\frac{\pi}{2}}$.

OR

If $y = \tan^{-1}x$, find $\frac{d^2y}{dx^2}$in terms of y alone.

17) Evaluate: $\int \frac{dx}{x(x^5+3)}$.

18) Write the Integrating Factor of the differential equation:

$$y + \frac{d}{dx}(xy) = x(\sin x + \log x).$$

19) Find the direction cosines of the line passing through the 2 points $(-2, 4, -5)$ and $(1, 2, 3)$.

20) Evaluate:$\int \frac{(x^2+\sin^2 x)\sec^2 x\, dx}{1+x^2}$.

Section - B

21) Evaluate: $\sin\left(2\cot^{-1}\left(-\frac{5}{12}\right)\right)$.

OR

Simplify: $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, $\quad$ $if\,\frac{a}{b}\tan x > -1$.

22) Find the inverse of the matrix $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$, if it exists using elementary transformation.

23) Find the co-ordinates of the foot of perpendicular drawn from the origin to the plane, $\mathbf{2x - 3y + 4z - 6 = 0}$.

24) Find the value of $\boldsymbol{\lambda}$ so that the four points with position vectors $-\hat{\jmath} + \hat{k}$, $2\hat{\imath} - \hat{\jmath} - \hat{k}$, $\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $3\hat{\jmath} + 3\hat{k}$ are coplanar.

OR

If $\vec{a} = 2\hat{\imath} - \hat{\jmath} - 2\hat{k}$ and $\vec{b} = 7\hat{\imath} + 2\,\hat{\jmath} - 3\hat{k}$ then express $\vec{b} = \vec{c} + \vec{d}$ where $\vec{c}$ is parallel to $\vec{a}$ *and* $\vec{d}$ is perpendicular to $\vec{a}$

25) Find the shortest distance between the lines whose vector equation are $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$.

26) A and B throw a pair of dice alternately A wins the game if he gets a total of 7and B wins the game if he gets a total of 10. If A starts the game , then find the probability that B wins.

Section – C

27) Let N *be the set of all Natural numbers*.R be the relation defined on N x $N by$ $(a, b)R(c, d) \Leftrightarrow ad = bc$.Check whether R is an EQUIVALENCE Relation . Also, find the equivalence Class $[(2, 6)]$.

28) Let $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & if\ x < 0 \\ a, & if\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & if\ x > 0 \end{cases}$. Determine the value of "a", given that the function is continuous at $x = 0$.

29) Evaluate: $\int \frac{(2x-3)dx}{(x^2-1)(2x+3)}$

OR

Evaluate: $\int_1^3 (|x-1| + |x-2| + |x-3|)dx$

30) Find the distance of the point A (2,12,5) from the point of intersection of line $\vec{r} = 2\hat{\imath} - 4\hat{\jmath} + 2\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{\imath} - 2\hat{\jmath} + \hat{k}) = 0$ (4)

31) For candidates A, B and C, the chances of being selected as the manager of a firm are $4:1:2$ respectively. The respective probabilities for them to introduce a radical change in marketing strategy are $0.3, 0.8$ *and* 0.5respectively. If the change does take place, find the probability that it is due to the appointment of B. (4)

OR

Out of a group of 30 men 20 always speak the truth. Two men are selected at random from the group. Find the probability distribution of the number of selected men who speak the truth. Find the mean of the distribution.

32) If a young man drives his scooter at a speed of 25 km/hr, he has to spend₹2 per km on petrol. If he drives at a speed of 40km/hour, it produces more air pollution and increases his expenditure on petrol to ₹5kmper hour. He has a maximum of ₹ 100 to spend on petrol and travel at a maximum distance in one hour time with less pollution. Express this problem as an LPP and solve it graphically. (4)

Section - D

33) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations: (6)

$\boldsymbol{x + 2y + z = 4}$,

$\boldsymbol{-x + y + z = 0}$,

$\boldsymbol{x - 3y + z = 2}$.

OR

Prove that $\begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix} = (ab + bc + ca)^3$, using properties of determinants.

34) Find the area of the circle $\boldsymbol{4x^2 + 4y^2 = 9}$, which is the interior to the parabola $\boldsymbol{x^2 = 4y}$. (6)

35) Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (6)

OR

Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

36) Solve the differential equation: $\boldsymbol{(1 + y^2)(1 + logx)\, dx + xdy = 0}$, given that when $x = 1, y = 1$. (6)

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