Model examination 2017

Mathematics

1)

a) 18

b) 1+8 , 2+10, 3+12, 4+14, ------------

c) $X_n = 3n+6$ (3 marks)

2)

a) $X_{10} = 12+99 = 111$

b) 1111-12 = 1099

1099 is not exactly divisible by 11 hence 1111 is not a term of this sequence (2 marks)

3)

a) 6/2(first term + last term) = 66

3(6 + 6^{th} term) =66

6 + 6^{th} term = 22

6^{th} term = 22 -6 =16

b) 6^{th} term = 16

6 + 5d =16

5d = 10

d =2

c) 6 , 8 ,10 ,12, 14 ,16 (5marks)

4)

a) $X_n = 4n+2$

b) $S_n = n(2n+4) = 2n^2+4$

c) $2n^2+4n = 510$

$n^2+2n = 225$

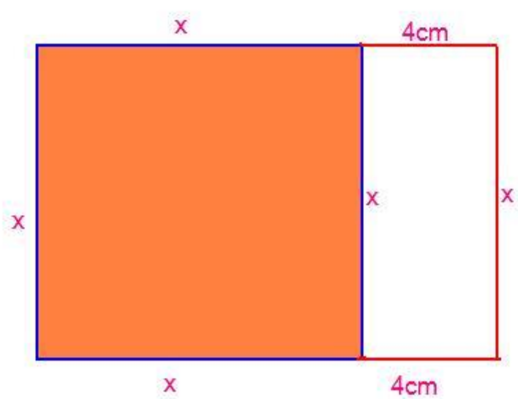
$(n+1)^2 = 16$

$n+1 = 16$ or $n+1 = -16$

Number of terms cannot be negative hence

$n = 15$

OR



One side of square = x

Then $x(x+4) = 396$

$x^2+4x = 396$

$(x+2)^2 = 400$

$x+2 = 20$ or $x+2 = -20$

$x = 18$ or $x = -22$

One side of square cannot be negative

Hence one side of the square = 18cm (4 marks)

5) Let the positive number be 'x'

Then $x+1 = x^2$

$x^2 - x - 1 = 0$

solving $x = \frac{1+\sqrt{5}}{2}$ or $x = \frac{1-\sqrt{5}}{2}$

Here the number is positive hence

Number $= \frac{1+\sqrt{5}}{2}$ (3marks)

6) $p(x) = x^3+x^2+x+1$

a) $p(1)=4$
b) $p(-1)=0$
c) Let the first degree polynomial $= ax+b$

   Then $p(x) = x^3+x^2+x+1-ax-b$

   $p(1)=0$ hence $4-a-b=0$ or $a+b=4$ ----(1)

   $p(-1)=0$ hence $0+a-b=0$ or $a-b=0$-----(2)

   solving (1) and (2) $a=b=2$

   hence first degree polynomial $= 2x+2$ (5 marks)

7)

a) 6 x 6 = 36
b) (1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5)

   There are 9 pairs
c) Probability of getting product as odd $= \frac{9}{36} = \frac{1}{4}$
d) Probability of getting product as even $= \frac{27}{36} = \frac{3}{4}$ (5 marks)

8)

Monthly Income(Rs)	Number of households
Up to 5000	4
Up to 6000	12
Up to 7000	16
Up to 8000	20
Up to 9000	23
Up to 10000	25

Income of the 13th worker is the median Income

Income of the 13th worker = 7000

Median Income = Rs.7000 (2marks)

9)

a) Height of the 21st child is the median height
b) 145-150
c) The difference in height and the corresponding difference in the number of children are proportional

Height(cm)	Number of children
Below 140	5
Below 145	13
Below 150	23
Below 155	32
Below 160	38
Below 165	41

x	140	145	150	155	160	165
y	5	13	23	32	38	41

Median of y = $\frac{41}{2} = 20.5$

When y increases 10 the corresponding increase in x is 5

Hence When y increases 1 the corresponding increase in y is $\frac{5}{10} = \frac{1}{2}$

Therefore when y increases 7.5 the corresponding increase in y is $\frac{1}{2}x7.5 = 3.75$

Median height = 145 + 3.75 = 148.75cm (5marks)

10) The coordinates of other two vertices are (2,1) and (5,3) (2amrks)

11)

- AP=4 and BP=2
- DQ=4 and CQ=2
- Coordinates of C is (7,10) (3 marks)

12)

$$\frac{difference\ of\ y\ coordinates}{difference\ of\ x\ coordinates} = \frac{6-4}{5-1} = \frac{2}{4} = \frac{1}{2}$$

a) If the x coordinate is 3 then the y coordinate is 5
b) If the y coordinate is 3 then the x coordinate is -1
c) The difference in y coordinate is half the difference in x coordinates
d) 2y-x=7 (5marks)

13)

a) Length of the line = $\sqrt{20}$ or $2\sqrt{5}$

b) Coordinate of the midpoint = (2,2)

c) Let (x,y) be a point on the circle

$(x-2)^2+(y-2)^2=(\sqrt{5})^2$

$x^2-4x+4+y^2-4y+4=5$

$x^2+y^2-4x-4y+3=0$

d) Let the circle touches the x axis (x,0) then

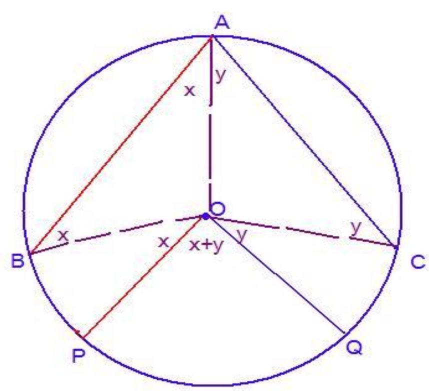
$x^2-4x+3=0$

$(x-3)(x-1)=0$

x = 3 or x =1

Circle touches the x axis at (3,0) and (1,0) (5marks)

14)



Let <ABO = x and <ACO = y then <BAC = (x+y)

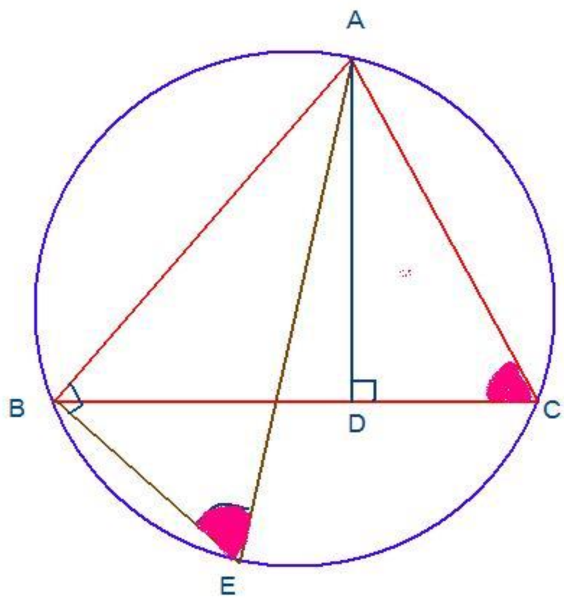
By central angle theorem if <BAC = (x+y) we have <BOC = 2(x+y)

AB//OP hence <BOP =x (alternate angles) similarly <COQ= y

Here <BOC = 2(x+y) hence <POQ=(x+y)

So we can seen that length of arc BC double the length of arc PQ (3marks)

15)



a) Consider Triangle ABE and ADC

Since AE is the diameter <ABE =90^0

<ADC =900 (given)

<AEB = ACB = x (angles in same part of the circle)

Hence <ABE =90^0=<ADC and <AEB = ACD = x

Therefore Triangle ABE and ADC are similar

b) Since ABE and ADC are similar

$$\frac{AD}{AB} = \frac{AC}{AE}$$

From this we have $AD = \frac{ABxAC}{AE}$

Area of triangle ABC = ½ x BC x AD

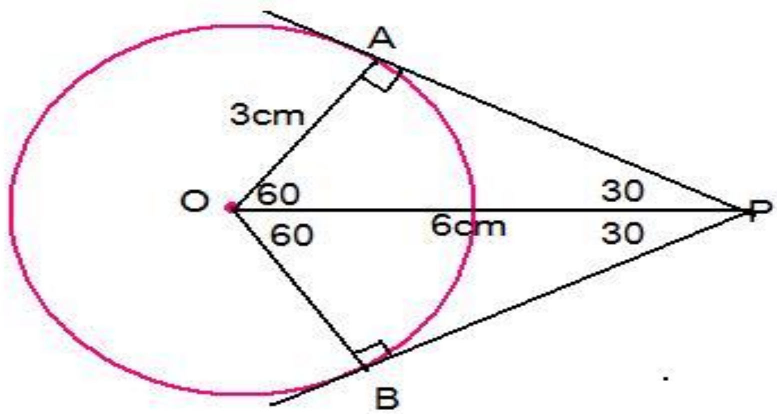
$$= \frac{1}{2} x \frac{ABxBCxCA}{AE}$$

Hence the proof (4marks)

16) construction (4marks)

17)

a) Construction
b) Angle between the tangents is 60^0



(3 marks)

18)

a) Measures of another two angles in triangle AQR = 70^0
b) <QPR = 70^0
c) Measures of another two angles in triangle PQR = 60^0 and 50^0 (4marks)

19) a =8cm , e= 5cm we have lateral edge = 3cm

If the lateral edge = 3cm and half of the base edge is 4cm height is negative it is not possible hence we cannot make such a square pyramid (3marks)

20)

a)

$$\frac{x}{360} = \frac{r}{l}$$

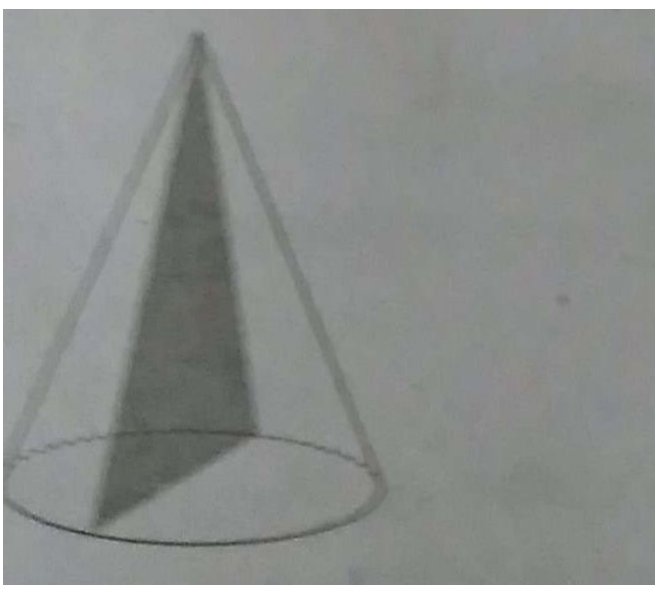
$$\frac{180}{360} = \frac{r}{l}$$

$$\frac{1}{2} = \frac{r}{l}$$

l = 2r

Ratio of base radius and slant height = 1:2

b) See the figure below



The sides of the triangles are base diameter and slant heights

Here slant height = 2r = diameter

Hence all sides of the triangle are equal and it is an equilateral triangle

21)

a) $3\pi r^2 = 120\ cm^2$

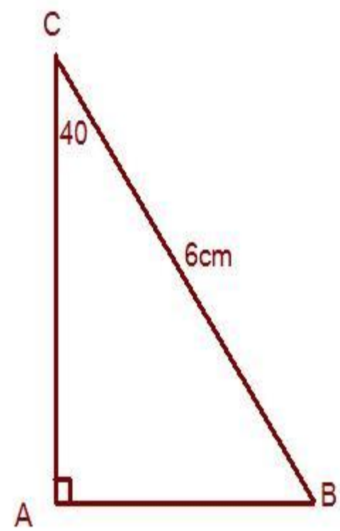
$\pi r^2 = 40\ cm^2$

Base area of the hemisphere = $40\ cm^2$

b) TSA of the sphere = $4\ \pi r^2 = 4 \times 40 = 160\ cm^2$ (3 marks)

22)

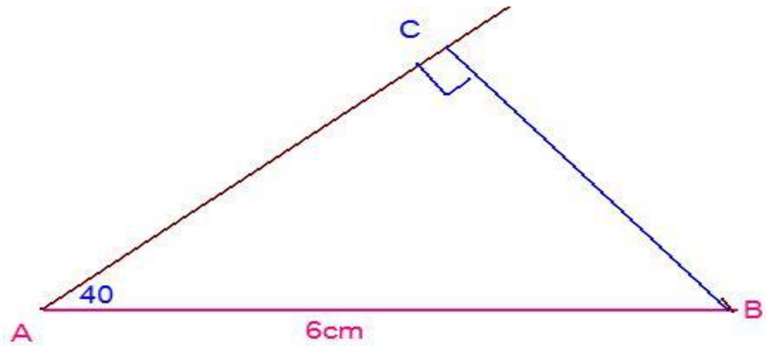
a)



If angles of a triangle are $30^0, 60^0, 90^0$ sides are in the ratio $1 : \sqrt{3} : 2$

If one angle is 30^0 and the hypotenuse is 6cm the side opposite to 30^0 angle is 3cm hence when the angle is 40^0 the side opposite to 40^0 angle is greater than 3cm

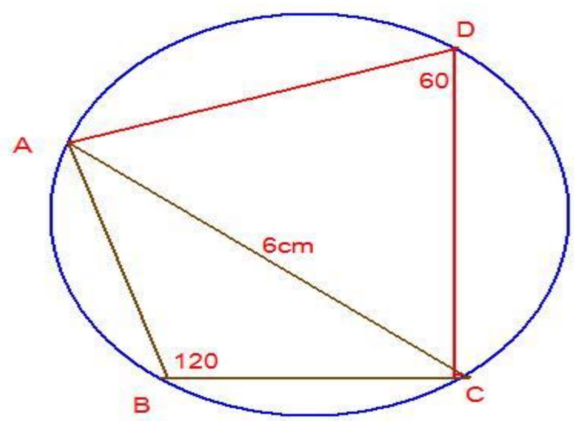
b) See the figure below



The minimum distance from B to the line drawn through A is the perpendicular distance . From the above problem we can see that when the angle is 40^0 the side opposite to 40^0 angle is greater than 3cm

Hence we cannot construct a triangle if the side opposite to 40^0 angle is 3cm

OR

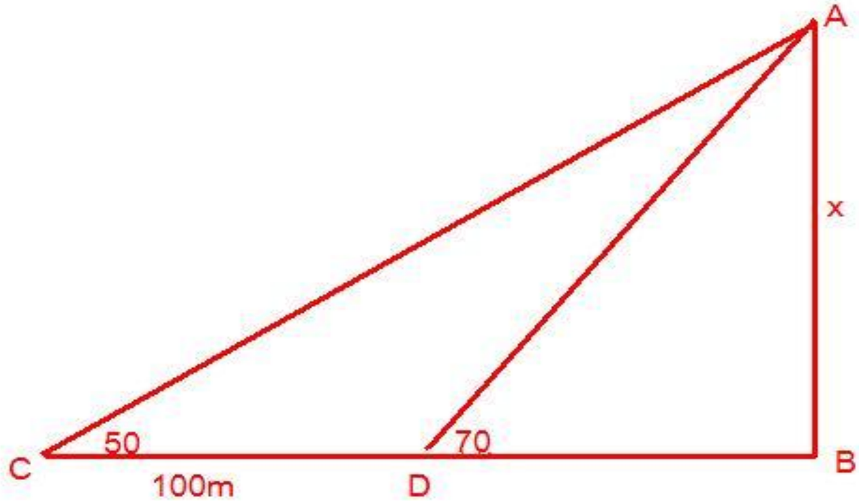


$$\frac{d}{sinD} = 2R$$

$$\frac{6}{sin60} = 2R$$

2R = $4\sqrt{3}$cm and radius = $2\sqrt{3}$ cm (3 marks)

23)



Let the height of the hill be AB

From triangle ABD , tan 70 = $\frac{AB}{BD}$ or AB = 2.8BD ---------- (1)

From triangle ABC , tan 50 = $\frac{AB}{100+BD} = \frac{2.8BD}{100+BD}$

$1.2 = \frac{2.8BD}{100+BD}$

120 + 1.2BD = 2.8BD

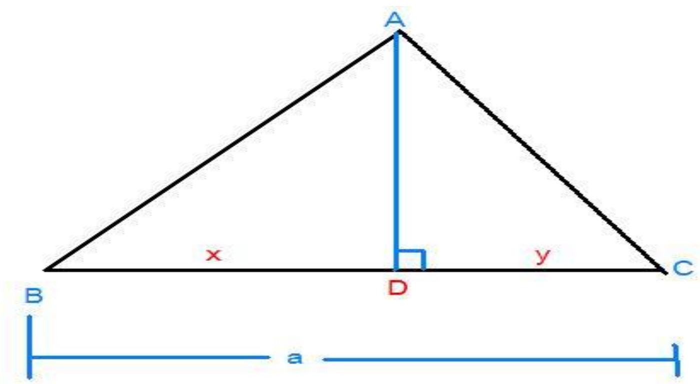
120 = 1.6 BD

BD = $\frac{120}{1.6} = 75$

Height of hill (AB) = 2.8BD = 2.8 x 75 = 210m

OR

24)



Let AD = h, BD =x and CD = y

From triangle ABD , tanB = $\frac{h}{a-x}$ and x = a– $\frac{h}{tanB}$

From triangle ACD , tanc = $\frac{h}{a-y}$ and y = a– $\frac{h}{tanC}$

x+y = a– $\frac{h}{tanB}$+ a– $\frac{h}{tanC}$= $\frac{a\ tanB-h}{tanB} + \frac{a\ tanC-h}{tanC}$

a = $\frac{a\ tanBtanC-h\ tanC+a\ tanB\ tanC-h\ tanB}{\tan B\ tanC}$

a $\tan B\ \ tan\ C$ = $a\ tanBtanC - h\ tanC + a\ tanB\ tanC - h\ tanB$

$h\ tanC$ + h tanB = $a\ tanB\ tanC$

h (tanB + tan C) = $a\ tanB\ tanC$ and hence h = $\frac{a \tan B \tan C}{\tan B tanC}$

Area of triangle =½ ah = ½ x a x $\frac{a \tan B \tan C}{\tan B tanC}$ = = $\frac{a\hat{}2 \tan B \tan C}{2 \tan B tanC}$

Hence the proof

(4marks)