SOLUTION (SET-2)

Section - A

1. **(A)**

2. **(C)**

3. **(A)**

4. **(C)**

5. **(B)**

6. **(C)**

7. **(C)**

8. **(D)**

9. **(C)**

10. **(C)**

11. x

12. 1

13. neither symmetric nor skew-symmetric

14. an increasing function

OR

1

15. zero

OR

2

16. $k \neq 0$

17. $\frac{1}{2}[\ell n\, 17 - \ell n\, 5]$

18. $\log |x + \sin x| + C$

OR

$\frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C$

19. $\frac{x}{2} - x + 3\log|(x+1)| + C$

20. 2 and 4

Section - B

21. $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$

$\Rightarrow \quad \tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1}3x - \tan^{-1}x$

$\Rightarrow \quad \tan^{-1}\left(\frac{2x}{2-x^2}\right) = \tan^{-1}\left(\frac{2x}{1+3x^2}\right) \qquad \left(\because \tan^{-1}x \pm \tan^{-1}y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right)\right)$

$\Rightarrow \quad 2x(1 + 3x^2) = (2 - x^2)2x$

$\Rightarrow \quad 2x[4x^2 - 1] = 0$

$\Rightarrow \quad x = 0 \text{ and } x = \pm\frac{1}{2}$

OR

Given set A = {1, 2, 3, ...9}

Let (x, y) ∈ A × A which is related the (2, 5)

by given relation.

(x, y) R (2, 5) $\Rightarrow$ x + 5 = y + 2

$\Rightarrow$ 1 + 5 = 4 + 2, 2 + 5 = 5 + 2, 3 + 5 = 6 + 2, ...

Hence equivalence class of [(2, 5)] = {(1, 4), (2, 5), (3,6), (4,7), (5, 8), (6, 9)}.

22. Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$. Putting x = tanθ, then

$$y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$\Rightarrow \quad y = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$\Rightarrow \quad y = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{1}{2}\theta = \frac{1}{2}\tan^{-1}x$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{1+x^2}\right)$$

23. Given, $\frac{dx}{dt} = -5 \text{ cm / min}$

and $\frac{dy}{dt} = 4 \text{ cm / min}$

Area, A = xy

$\Rightarrow \frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$

= 8 × 4 + 6 × (–5)

= (32 – 30) cm^2/min

= 2 cm^2/min

24. We know that the four points A, B, C and D are coplanar if the three vectors $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, i.e., if $\left[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right] = 0$

Now $\overrightarrow{AB} = -(\hat{j}+\hat{k}) - (4\hat{i}+5\hat{j}+\hat{k}) = -4\hat{i} - 6\hat{j} - 2\hat{k}$

$\overrightarrow{AC} = (3\hat{i}+9\hat{j}+4\hat{k}) - (4\hat{i}+5\hat{j}+\hat{k}) = -\hat{i} + 4\hat{j} + 3\hat{k}$

and $\overrightarrow{AD} = 4(-\hat{i}+\hat{j}+\hat{k}) - (4\hat{i}+5\hat{j}+\hat{k}) = -8\hat{i} - \hat{j} + 3\hat{k}$

Thus $\left[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$

Hence A, B, C and D are coplanar.

OR

Let the required vector be $\vec{r}$

$\vec{r} = \lambda(2\vec{a} - \vec{b} + 3\vec{c})$

$= \lambda[2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}]$

$= \lambda[\hat{i} - 2\hat{j} + 2\hat{k}]$

As $|\vec{r}| = 6$

$\Rightarrow \sqrt{\lambda^2(1+4+4)} = 6 \Rightarrow |\lambda| \times 3 = 6$

$\Rightarrow \quad \lambda = \pm 2$

Hence, the required vector be $\pm 2(\hat{i} - 2\hat{j} + 2\hat{k})$

25. $\dfrac{x+2}{3} = \dfrac{y+1}{2} = \dfrac{z-3}{2} = \lambda$

Any point on the line will be

$(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$

Given that

$\sqrt{(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2} = 5$

$\Rightarrow \quad 9\lambda^2 - 18\lambda + 9 + 4\lambda^2 - 16\lambda + 16 + 4\lambda^2 = 25$

$\Rightarrow \quad 17\lambda^2 - 34\lambda = 0$

$\Rightarrow \quad \lambda^2 - 2\lambda = 0 \Rightarrow \lambda = 0, 2$

Hence the points are $(-2, -1, 3)$ or $(4, 3, 7)$

26. Let E : Obtaining sum 8 on die

F : Red die resulted in a number less than 4.

$\therefore$ E = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

$\Rightarrow$ n(E) = 5

F = {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)}

$\Rightarrow$ n(F) = 18

$\Rightarrow E \cap F = \{(5, 3), (6, 2)\}$

$\therefore n(E \cap F) = 2$

Hence, $P(F) = \dfrac{18}{36} = \dfrac{1}{2}$

and $P(E \cap F) = \dfrac{2}{36} = \dfrac{1}{18}$

$\therefore$ Required probability

= P(E/F)

$= \dfrac{P(E \cap F)}{P(F)} = \dfrac{1/18}{1/2} = \dfrac{1}{9}$

Section - C

27. $f : R_+ \to [4, \infty)$

Now, $f(x) = x^2 + 4$

$\Rightarrow$ f is invertible $\Rightarrow f$ is one-one and onto

For one-one : Let $x_1, x_2 \in R_+$

We have, $f(x_1) = f(x_2)$

$\Rightarrow$ $x_1^2 + 4 = x_2^2 + 4$

or $(x_1 + x_2)(x_1 - x_2) = 0$

$\Rightarrow$ $x_1 = x_2$ $\because$ $x_1 + x_2 \neq 0$

Now, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\Rightarrow$ f is one-one function

For onto : Let $y = f(x) : x = f^{-1}(y)$ ....(1)

$\Rightarrow$ $y = x^2 + 4$

or $x = \sqrt{y-4} \in R_+$ ....(2)

If $y - 4 \geq 0$,

then $y \geq 4$

$\Rightarrow$ Range of function = $[4, \infty)$

and Codomain = $[4, \infty)$

Hence, Range = Codomain

$\Rightarrow$ f is onto function

Now, f is one-one and onto $\Rightarrow f$ is invertible

From equations (1) and (2), we have

$f^{-1}(y) = \sqrt{y-4}$

28. $x = a\cos^3\theta$; $y = a\sin^3\theta$

$\Rightarrow \dfrac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta)$ ....(i)

and $\dfrac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$ ....(ii)

(ii) / (i) gives

$$\frac{dy}{dx} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\sec^2\theta \times \frac{d\theta}{dx}$$

or $\dfrac{d^2y}{dx^2} = -\sec^2\theta \times \dfrac{1}{-3a\cos^2\theta\sin\theta}$ [from eq. (i)]

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right) = \frac{1}{3a.\cos^4\theta.\sin\theta}$$

or $\left(\dfrac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{6}} = \dfrac{1}{3a \times \left(\dfrac{\sqrt{3}}{2}\right)^4 \times \dfrac{1}{2}} = \dfrac{32}{27a}$

OR

$(ax+b)e^{\frac{y}{x}} = x$

$\Rightarrow \quad e^{y/x} = \dfrac{x}{ax+b}$ ....(1)

or $\quad \dfrac{y}{x} = [\log x - \log(ax+b)]$ (Taking log both sides)

Differentiate w.r.t. x,

$$\frac{x.\frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{a}{ax+b}$$

$$\Rightarrow \quad \frac{x.\frac{dy}{dx} - y}{x^2} = \frac{ax+b-ax}{x(ax+b)}$$

or $\quad x\dfrac{dy}{dx} - y = \dfrac{bx}{(ax+b)}$

From equation (1), we have

$x\dfrac{dy}{dx} - y = be^{y/x}$ ....(2)

Differentiate w.r.t. x,

$$x.\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{be^{y/x}\left(x.\frac{dy}{dx} - y\right)}{x^2}$$

From equation (2), we have

$$x^3\left(\frac{d^2y}{dx^2}\right) = \left(x\frac{dy}{dx} - y\right)^2$$

29. Given differential equation is $x^2\dfrac{dy}{dx} = y^2 + 2xy$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y^2+2xy}{x^2}$$

$\because$ This is homogeneous differential equation, Put y = vx

$$\therefore \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v^2x^2 + 2x.vx}{x^2} \quad \Rightarrow v + x\frac{dv}{dx} = v^2 + 2v$$

$$\therefore \quad x\frac{dv}{dx} = v^2 + v \quad \Rightarrow \quad \frac{dv}{v^2+v} = \frac{dx}{x}$$

Now, $\displaystyle\int \frac{dv}{v(v+1)} = \int \frac{dx}{x}$

Let $\dfrac{1}{v(v+1)} = \dfrac{A}{v} + \dfrac{B}{v+1}$

$1 = A(v+1) + Bv$

Put v = 0, –1

$1 = A + 0 \quad \Rightarrow \quad A = 1$

$1 = 0 + B(-1) \quad \Rightarrow \quad B = -1$

$\therefore \quad \int \frac{dv}{v} + \int \frac{(-1)dv}{v+1} = \int \frac{dx}{x}$

$\log|v| - \log|v+1| = \log|x| + \log c$

$\log\left|\frac{v}{v+1}\right| = \log|cx|$

$\therefore \quad \frac{v}{v+1} = cx$

Putting the value of v, we get

$\frac{\frac{y}{x}}{\frac{y}{x}+1} = cx \Rightarrow \frac{y}{x+y} = cx$

$\therefore \quad y = cx(x + y)$

When x = 1, y = 1 $\quad \therefore \quad c = \frac{1}{2}$

$\therefore \quad y = \frac{1}{2}x(x + y)$

30. Let $I = \int_{-2}^{2} \frac{x^2}{1+5^x} dx$ ....(1)

Using property $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$, we have

$I = \int_{-2}^{2} \frac{x^2}{1+5^{-x}}$

$\Rightarrow \quad I = \int_{-2}^{2} \frac{x^2 5^x}{1+5^x} dx$ ....(2)

From equations (1) and (2), we have

$2I = \int_{-2}^{2} x^2 dx$

$\Rightarrow \quad 2I = 2\int_0^2 x^2 dx \quad [\because \; x^2 \text{ is even function}]$

$\therefore \quad I = \int_0^2 x^2 dx = \frac{1}{3}(x^3)_0^2 = \frac{1}{3}(2^3 - 0^3)$

or $\quad I = \frac{8}{3}$

31. Let X denotes number of red cards in a draw of two cards

$P(X=0) = P(\text{No red cards}) = \frac{^{26}c_2}{^{52}c_2} = \frac{26\times 25}{52\times 51} = \frac{25}{102}$

$P(X=1) = P\text{ (one red card and 1 non red card)} = \frac{^{26}C_1 \times {}^{26}C_1}{^{52}C_2} = \frac{26\times 26\times 2}{52\times 51} = \frac{26}{51}$

$P(X=2) = P\text{ (two red cards)} = \frac{^{26}C_2}{^{52}C_2} = \frac{25}{102}$

The probability distribution of X is

X	0	1	2
P(X)	25/102	26/51	25/102

$\Rightarrow$ Mean of X = E(X) = $\sum_{i=1}^{n} x_i p(x_1) = 0\times\frac{25}{102} + 1\times\frac{26}{51} + 2\times\frac{25}{102} = \frac{26}{51} + \frac{50}{102} = 1$

Now, $E(X^2) = \sum_{i=1}^{n} x_i^2 p(x_i) = 0^2\times\frac{25}{102} + 1^2\times\frac{26}{51} + 2^2\times\frac{25}{102} = \frac{26}{51} + \frac{50}{51} = \frac{76}{51}$

$\Rightarrow$ Variance of X = $E(X^2) - [E(X)]^2 = \frac{76}{51} - 1 = \frac{25}{51}$

OR

Let E_1 = Two headed Coin

E_2 = Biased coin that comes up heads (75%)

E_3 = Biased coin that comes up tails (40 %)

and E = Head comes up

We have $P(E_1) = P(E_2) = P(E_3) = 1/3$

and $P(E/E_1) = 1$; $P(E/E_2) = 3/4$; $P(E/E_3) = 1 - \frac{2}{5} = \frac{3}{5}$

By Baye's Theorem,

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\times P(E/E_1)}{P(E_1)\times P(E/E_1) + P(E_2)\times P(E/E_2) + P(E_3)\times P(E/E_3)}$$

$$= \frac{1/3 \times 1}{(1/3\times 1) + (1/3\times 3/4) + (1/3\times 3/5)} = \frac{20}{47}$$

32. Let quantity of food A = x units and quantity of food B = y units

We make the following table from the given data :

Types	Quantity	Vitamins	Minerals	Calories	Cost
A	x	200	1	40	5
B	y	100	2	40	4

Required L.P.P. is :

Minimize Z = 5x + 4y

subject to constraints

$200x + 100y \geq 4000$

$x + 2y \geq 50$

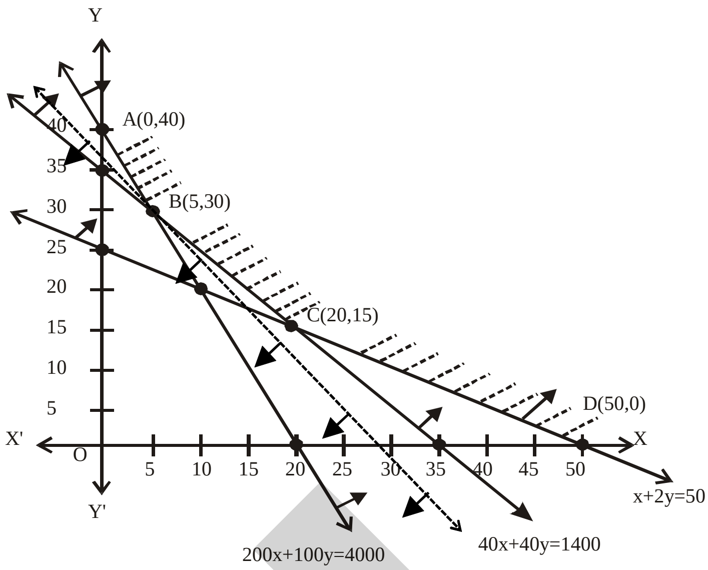
$40x + 40y \geq 1400$

$x, y \geq 0$

Now plot the straight lines on the graph and find the corner points of feasible region.

$\therefore$ Corner points of feasible region are A(0, 40), B(5, 30), C(20, 15), D(50, 0)

Now evaluate Z at the corner points



Corner Points	$Z = 5x + 4y$
A(0,40)	160
B(5,30)	145
C(20,15)	160
D(50,0)	250

$\because$ $5x + 4y < 145$ has no points in common with the feasible region;

Thus, the minimum value of Z is 145 attained at the point (5, 30)

$\therefore$ Least cost is Rs.145 at x = 5, y = 30

Section - D

33. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$\therefore$ $|A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2) = 0 - 6 + 5 = -1 \neq 0$, A is non-singular matrix so A^{-1} exist.

Now, $A_{11} = 0, A_{12} = 2, A_{13} = 1$

$A_{21} = -1, A_{22} = -9, A_{23} = -5$

$A_{31} = 2, A_{22} = 23, A_{33} = 13$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj}A) = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \quad ......(1)$$

Now, the given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

The solution of the system of equations is given by $X = A^{-1}B$.

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \quad \text{[Using (1)]}$$

$$= \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, x = 1, y = 2, and z = 3.

OR

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

In order to use elementary row transformation, we may write A = IA

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_3$, $R_1 \to R_1 - 3R_3$

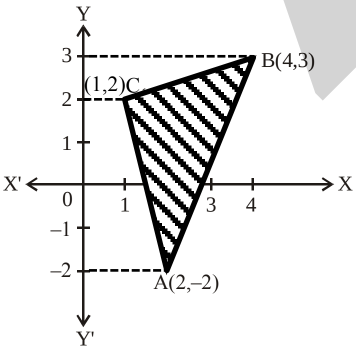
$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -3 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

34.



Line AB is : $y = \frac{5}{2}x - 7$; $x = \frac{2}{5}(y + 7)$, line BC is $y = \frac{1}{3}(x + 5) \Rightarrow x = 3y - 5$

Line AC is ; $y = -4x + 6$; $x = \frac{y - 6}{-4}$

Required area = $\left[\int_{-2}^{3}(\text{lineAB})dy\right]-\left[\int_{2}^{3}(\text{lineBC})dy+\int_{-2}^{2}(\text{lineAC})dy\right]$

$\Rightarrow \quad \left[\frac{2}{5}\int_{-2}^{3}(y+7)dy\right]-\left[\int_{2}^{3}(3y-5)dy-\frac{1}{4}\int_{-2}^{2}(y-6)dy\right]$

$=\frac{2}{5}\left[\left(\frac{y^2}{2}+7y\right)_{-2}^{3}\right]-\left[\left(\frac{3y^2}{2}-5y\right)_{2}^{3}-\frac{1}{4}\left(\frac{y^2}{2}-6y\right)_{-2}^{2}\right]$

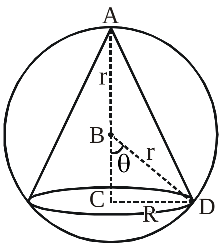
$=\frac{2}{5}\left[\left(\frac{9}{2}+21\right)-(2-14)\right]-\left[\left\{\left(\frac{27}{2}-15\right)-(6-10)\right\}-\frac{1}{4}\{(2-12)-(2+12)\}\right]$

$=\frac{2}{5}\left[\frac{9}{2}+33\right]-\left[\left(\frac{27}{2}-11\right)-\frac{1}{4}(-24)\right]$

$=\left(\frac{2}{5}\times\frac{75}{2}\right)-\left(\frac{5}{2}+6\right)$

$=15-\frac{17}{2}=\frac{13}{2}$ square units.

35. Let R and h be the radius and height of the cone respectively and r be the radius of sphere.



The volume (V) of the cone is given by

$V=\frac{1}{3}\pi R^2 h$

In Δ BCD

$r^2=(h-r)^2+R^2$

$r^2=h^2+r^2-2hr+R^2$

$R^2=2hr-h^2$ ...(i)

Now Volume of cone

$$V=\frac{\pi h}{3}(2hr-h^2)\Rightarrow\frac{\pi}{3}(2h^2r-h^3)$$

diff. w.r.t. h

$$\Rightarrow \frac{dV}{dh}=\frac{\pi}{3}(4hr-3h^2)$$

for maxima and minima $\frac{dV}{dh}=0$

$$\Rightarrow h=\frac{4r}{3}$$

and $\frac{d^2V}{dh^2}=\frac{\pi}{3}(4r-6h)$

$$\frac{d^2V}{dh^2} < 0 \text{ at } h = \frac{4r}{3}$$

V is max at $h = \frac{4r}{3}$

$\Rightarrow$ Maximum volume of cone = $\frac{1}{3}\pi R^2 h$

$$= \frac{\pi}{3}(2hr - h^2)h \qquad \text{(from eq (i))}$$

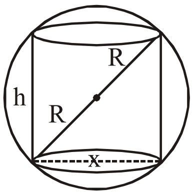
$$= \frac{\pi}{3}(2h^2r - h^3)$$

Putting the value of h = 4r/3

$$\Rightarrow \frac{\pi}{3}\left[2r\left(\frac{4r}{3}\right)^2 - \left(\frac{4r}{3}\right)^3\right] = \frac{4}{3}\pi r^3\left[\frac{8}{27}\right]$$

$$\Rightarrow \frac{8}{27} \text{ (Vol. of Sphere)}$$

OR



Given; Radius of the sphere = R

Let 'h' be the height and 'x' be the diameter of the base of inscribed cylinder.

Then, $h^2 + x^2 = (2R)^2$

$\Rightarrow h^2 + x^2 = 4R^2$ ..... (1)

$\therefore$ Volume of the cylinder (V)

$\Rightarrow \pi \times [\text{radius}]^2 \times (\text{height})$

$\Rightarrow V = \pi \times \left(\frac{x}{2}\right)^2 \times h$

$= \frac{1}{4}\pi x^2 h$ .... (2)

Putting the value of 'x^2' from (1) into (2); we get;

$$V = \frac{1}{4}\pi h(4R^2 - h^2) = \pi R^2 h - \frac{1}{4}\pi h^3$$

On differentiating w.r.t 'h'; we get,

$$\frac{dV}{dh} = \pi R^2 - \frac{3}{4}\pi h^2$$

for maxima and minima

Putting $\frac{dV}{dh} = 0; \Rightarrow R^2 = \frac{3}{4}h^2$

or $h = \frac{2R}{\sqrt{3}}$

Also; $\frac{d^2V}{dh^2} = \frac{-3}{4} \times 2\pi h$

At $h = \frac{2R}{\sqrt{3}}; \frac{d^2V}{dh^2} = \frac{-3}{4} \times 2\pi \times \left(\frac{2R}{\sqrt{3}}\right)$

$= -\sqrt{3}\pi R < 0$

$\Rightarrow$ V is maximum at $h = \frac{2R}{\sqrt{3}}$

$\therefore$ Maximum volume at $h = \frac{2R}{\sqrt{3}}$ is :

$$V = \frac{1}{4}\pi\left(\frac{2R}{\sqrt{3}}\right)\left(4R^2 - \frac{4R^2}{3}\right)$$

$$= \frac{\pi R}{2\sqrt{3}}\left(\frac{8R^2}{3}\right)$$

$$= \frac{4\pi R^2}{3\sqrt{3}} \text{ sq.units}$$

Thus, volume of the cylinder is maximum when $h = \frac{2R}{\sqrt{3}}$

36. Equation of line passing through two given points (3, –4, –5) and (2, –3, 1) is given by :

$\Rightarrow \quad \frac{x-3}{2-3} = \frac{y-(-4)}{-3-(-4)} = \frac{z-(-5)}{1-(-5)}$

$\Rightarrow \quad \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ ....(1)

Now, equation of the plane passing through the points (1, 2, 3); (4, 2, –3) and (0, 4, 3) is given by :

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ 4-1 & 2-2 & -3-3 \\ 0-1 & 4-2 & 3-3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0$$

$(x-1)(0+12) - (y-2)(0-6) + (z-3)(6) = 0$

$\Rightarrow \quad 12x - 12 + 6y - 12 + 6z - 18 = 0$

$\Rightarrow \quad 12x + 6y + 6z - 42 = 0$

$\Rightarrow \quad 2x + y + z - 7 = 0$ ....(2)

from (1), we get

$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$ (say)

$\Rightarrow$ any point on the line is given by

$x = -\lambda + 3; y = \lambda - 4; z = 6\lambda - 5$

This must satisfy the equation of the plane from equation

$\Rightarrow \quad 2(-\lambda + 3) + \lambda - 4 + 6\lambda - 5 - 7 = 0$

$\Rightarrow \quad -2\lambda + \lambda + 6\lambda + 6 - 4 - 12 = 0$

$\Rightarrow \quad 5\lambda - 10 = 0$

$\Rightarrow \quad \lambda = 2$

$\Rightarrow$ The point is given by

$x = -2 + 3; y = 2 - 4; z = 12 - 5$

$x = 1; \quad y = -2; \quad z = 7$