SOLUTION (SET-1)

Section - A

1. (B)

2. (C)

3. (C)

4. (B)

5. (B)

6. (D)

7. (C)

8. (A)

9. (D)

10. (B)

11. $f^{-1}og^{-1}$

12. a = 8

13. Symmetric

14. y + x = 0

OR

$4\pi\ cm^2/sec$

15. $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$

OR

Proportional

16. |A| = 0

17. a = 2

18. $\frac{2}{3}x^{3/2}\log x - \frac{4}{9}x^{3/2} + C$

OR

sec x – cosec x + C

19. log |sin x + cos x| + C

20. $\frac{dy}{dx} + \frac{x-1}{y-2} = 0,$

Section - B

21. $\cos(\tan^{-1}x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$

$\Rightarrow \quad \cos\left(\tan^{-1}x\right) = \sin\left(\frac{\pi}{2} - \tan^{-1}\frac{3}{4}\right)$

or $\quad \cos\left(\tan^{-1}x\right) = \cos\left(\tan^{-1}\frac{3}{4}\right)$

$\Rightarrow \quad \tan^{-1}x = \tan^{-1}\frac{3}{4}$

or $\quad x = \frac{3}{4}$

OR

Let $x \in A$ which is related to 2 by given relation.

$\therefore \quad x R 2 \Rightarrow |x - 2|$ is a multiple of 5

$\Rightarrow |x - 2| = 0, 5, 10, 15$

$\Rightarrow x = 2, 7, 12$

Hence equivalence class of $[2] = \{2, 7, 12\}$.

22. Let $y = \sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$

$\Rightarrow \quad y = \sin^{-1}\left(\frac{2.(6)^x}{1+(6^x)^2}\right)$

Put $6^x = \tan\theta$; $\theta = \tan^{-1}6^x$

$\Rightarrow \quad y = \sin^{-1}\left(\frac{2.\tan\theta}{1+\tan^2\theta}\right)$

or $\quad y = \sin^{-1}(\sin 2\theta) \Rightarrow y = 2\theta$

or $\quad y = 2(\tan^{-1}6^x)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2}{1+(36)^x} \times 6^x \log_e 6$

23. $x^2 = 32y \Rightarrow \frac{dy}{dx} = \frac{x}{16}$ and $y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y}$

$\therefore \quad$ At $(16, 8)$; $\left(\frac{dy}{dx}\right)_{C_1} = 1, \left(\frac{dy}{dx}\right)_{C_2} = \frac{1}{4}$

So, required angle $= \tan^{-1}\left|\frac{1-\frac{1}{4}}{1+1\left(\frac{1}{4}\right)}\right| = \tan^{-1}\left(\frac{3}{5}\right)$

24. Let ABCD be the parallelogram with

$\Rightarrow \quad \overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\overrightarrow{AD} = \hat{i} - 2\hat{j} - 3\hat{k}$

Now, $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{AD}$

$= (2\hat{i} - 4\hat{j} + 5\hat{k}) + (\hat{i} - 2\hat{j} - 3\hat{k}) = 3\hat{i} - 6\hat{j} + 2\hat{k}$

Hence, a unit vector parallel to $\overrightarrow{AC} = \frac{1}{|\overrightarrow{AC}|}(\overrightarrow{AC})$

$= \frac{1}{\sqrt{3^2 + (-6)^2 + 2^2}}(3\hat{i} - 6\hat{j} + 2\hat{k})$

$= \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}) = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$

$\Rightarrow$ Area of parallelogram ABCD = $|\overrightarrow{AB} \times \overrightarrow{AD}|$

$= \text{magnitude of} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$

$= |\hat{i}(12 + 10) - \hat{j}(-6 - 5) + \hat{k}(-4 + 4)|$

$= |22\hat{i} + 11\hat{j}|$

$= \sqrt{(22)^2 + (11)^2 + 0^2} = \sqrt{(11)^2 + (2^2 \times 11^2)}$

$= 11\sqrt{5}$ square units.

OR

Let vector $\vec{c} = a\hat{i} + b\hat{j} + c\hat{k}$

Given $\vec{a} \times \vec{c} = \vec{b}$,

$$\Rightarrow \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix} = 0\hat{i} + \hat{j} - \hat{k}$$

$\Rightarrow \quad \hat{i}(c - b) - \hat{j}(c - a) + \hat{k}(b - a) = 0\hat{i} + \hat{j} - \hat{k}$

Equating the coefficient on both the sides, we get

$c - b = 0$ ....(i), $a - c = 1$ ....(ii), $b - a = -1$ ....(iii)

Now $\vec{a}.\vec{c} = 3$

$\Rightarrow \quad (\hat{i} + \hat{j} + \hat{k}).(a\hat{i} + b\hat{j} + c\hat{k}) = 3$

or $a + b + c = 3$ ....(iv)

On solving equations (i), (ii), (iii), (iv), we get

$a = \frac{5}{3}$, $b = c = \frac{2}{3}$

So the required vector $\vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

25. Given lines can be written as

$$\ell_1 \Rightarrow \frac{x-1}{-3} = \frac{y-2}{p/7} = \frac{z-3}{2}$$

$$\ell_2 \Rightarrow \frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

$\because$ ℓ_1 and ℓ_2 are perpendicular to each other (given)

So, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow (-3) \times \left(\frac{-3p}{7}\right) + \left(\frac{p}{7}\right) \times 1 + 2 \times (-5) = 0$

$\Rightarrow p = 7$

Now, equation of a line passing through a point (3, 2, –4) and parallel the line ℓ_1 is

$$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$$

26. Let events A and B are as defined below

A : Events of exactly 2 girls on the committee

B : Event of at least one girl on the committee

$\Rightarrow$ Now $P(B') = \frac{{}^8C_4}{{}^{12}C_4} = \frac{70}{495} = \frac{14}{99}$

or $P(B) = 1 - \frac{14}{99} = \frac{85}{99}$

and $P(A \cap B) = P\text{ (2 boys and 2 girls)} = \frac{{}^8C_2 . {}^4C_2}{{}^{12}C_4} = \frac{6 \times 28}{495} = \frac{56}{165}$

Hence $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{56}{165} \times \frac{99}{85} = \frac{168}{425}$

Section - C

27. $\because f(x) = \frac{4x+3}{3x+4}$

(i) For one-one function : we have $f(x_1) = f(x_2)\ \forall\ x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$

$\Rightarrow \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4}$

$\Rightarrow 12x_1x_2 + 16x_1 + 9x_2 + 12 = 12x_1x_2 + 9x_1 + 16x_2 + 12$

$\Rightarrow 7x_1 - 7x_2 = 0 \Rightarrow x_1 = x_2$

function is one-one.

(ii) For onto function : Let y = f(x)

$y = \frac{4x+3}{3x+4}$

$\Rightarrow 3xy + 4y = 4x + 3$

$\Rightarrow x(3y - 4) = 3 - 4y$

$\Rightarrow \quad x = \dfrac{3-4y}{3y-4} \in R - \left\{-\dfrac{4}{3}\right\}, \forall y \in R - \left\{\dfrac{4}{3}\right\}$

if $\quad 3y - 4 \neq 0$

$y \neq \dfrac{4}{3}$

$\Rightarrow$ Every element of co-domain has its preimages in domain.

So range of f(x) = co-domain of f(x)

function is onto

Hence, $f(x)$ is bijective since it it one-one and onto both.

(iii) Inverse of f : Let y = f(x) $\Rightarrow \quad x = f^{-1}(y)$

$x = \dfrac{3-4y}{3y-4}$

$f^{-1}(y) = \dfrac{3-4y}{3y-4}$

$f^{-1}(x) = \dfrac{3-4x}{3x-4}$

(iv) $f^{-1}(0) = -\dfrac{3}{4}$

$f^{-1}(x) = 2$

$\dfrac{3-4x}{3x-4} = 2 \Rightarrow 3 - 4x = 6x - 8 \Rightarrow 10x = 11$

$\Rightarrow x = \dfrac{11}{10}$

28. $y = \sin^{-1}\left[\dfrac{6x - 4\sqrt{1-4x^2}}{5}\right]$

$\Rightarrow y = \sin^{-1}\left[\dfrac{3}{5}.2x - \dfrac{4}{5}\sqrt{1-(2x)^2}\right]$

put $\quad 2x = \sin\theta, \quad$ and $\quad \dfrac{3}{5} = \cos\phi$,

$\Rightarrow \quad \theta = \sin^{-1}2x \qquad \dfrac{4}{5} = \sin\phi$

$\Rightarrow \quad y = \sin^{-1}\left[\sin\theta\cos\phi - \cos\theta\sin\phi\right]$

or $\quad y = \sin^{-1}[\sin(\theta - \phi)]$

$\Rightarrow \quad y = \theta - \phi \qquad (\because \sin^{-1}(\sin x) = x)$

or $\quad y = \sin^{-1} 2x - \sin^{-1}\dfrac{4}{5}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-4x^2}}.2 - 0$

Differentiating w.r.t. x, we have

$\dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-4x^2}}$

OR

$x = a(\cos 2t + 2t\sin 2t)$

$\Rightarrow \quad \frac{dx}{dt} = a[-2\sin 2t + 2(2t\cos 2t + \sin 2t)]$

Differentiating w.r.t. t, we have

or $\quad \frac{dx}{dt} = 4at\cos 2t \qquad$ ....(1)

and $\quad y = a(\sin 2t - 2t\cos 2t)$

Differentiating w.r.t. t, we have

$\Rightarrow \quad \frac{dy}{dt} = a\left[2\cos 2t - 2\left(-2t\sin 2t + \cos 2t\right)\right]$

or $\quad \frac{dy}{dt} = 4at\sin 2t \qquad$ ....(2)

By chain rule,

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4at\sin 2t}{4at\cos 2t}$

$\Rightarrow \quad \frac{dy}{dx} = \tan 2t$

$\Rightarrow \quad \frac{d^2y}{dx^2} = 2\sec^2 2t \times \frac{dt}{dx}$

or $\quad \frac{d^2y}{dx^2} = 2\sec^2 2t \times \frac{1}{4at\cos 2t} \qquad$ (from eq (1))

or $\quad \frac{d^2y}{dx^2} = \frac{1}{2at} \times \sec^3 2t$

29. $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$

Let $P = \frac{1}{1+x^2}$ and $Q = \frac{e^{\tan^{-1}x}}{1+x^2}$

I.F. = $e^{\int Pdx} = e^{\int \frac{1}{1+x^2}} = e^{\tan^{-1}x}$

$\therefore$ Solution of D.E. is given as :

$y \times e^{\tan^{-1}x} = \int \frac{e^{\tan^{-1}x}}{1+x^2} \times e^{\tan^{-1}x} dx + C$

$\Rightarrow \quad ye^{\tan^{-1}x} = \int \frac{(e^{\tan^{-1}x})^2}{1+x^2} dx + C$

Put $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2}dx = dt$

$\Rightarrow \quad ye^{\tan^{-1}x} = \int e^{2t}dt$

$= \frac{e^{2t}}{2} + C$

$\Rightarrow \quad ye^{\tan^{-1}x} = \frac{e^{2\tan^{-1}x}}{2} + C$

30. Let $I = \int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$

$\Rightarrow \quad I = \int_0^{\pi/4} \frac{\sin x + \cos x}{4 - (\sin x - \cos x)^2}$

Put $\sin x - \cos x = t$

$\therefore \quad (\cos x + \sin x)dx = dt$

When $x = 0 \Rightarrow t = -1$, and

When $x = \frac{\pi}{4} \Rightarrow t = 0$

or $\quad I = \int_{-1}^{0} \frac{dt}{(2)^2 - t^2}$

$\Rightarrow \quad I = \frac{1}{4}\left[\log \frac{2+t}{2-t}\right]_{-1}^{0}$

or $\quad I = \frac{1}{4}\left[\log 1 - \log \frac{1}{3}\right]$

$\Rightarrow \quad I = -\frac{1}{4}\log\frac{1}{3} = \frac{1}{4}\log 3$

31. Let X = Sum of the numbers on the two drawn cards; then possible values of X are 4, 6, 8, 10 or 12.

$P(X = 4) = P((1, 3), (3, 1)) = \frac{2}{12}$

$P(X = 6) = P((1, 5), (5, 1)) = \frac{2}{12}$

$P(X = 8) = P((3, 5), (5, 3),(1, 7), (7, 1)) = \frac{4}{12}$

$P(X = 10) = P((3, 7), (7, 3)) = \frac{2}{12}$

$P(X = 12) = P((5, 7), (7, 5)) = \frac{2}{12}$

X	P(X)	X.P(X)	X^2.P(X)
4	$\frac{2}{12} = \frac{1}{6}$	$\frac{4}{6}$	$\frac{16}{6}$
6	$\frac{2}{12} = \frac{1}{6}$	$\frac{6}{6}$	$\frac{36}{6}$
8	$\frac{4}{12} = \frac{2}{6}$	$\frac{16}{6}$	$\frac{128}{6}$
10	$\frac{2}{12} = \frac{1}{6}$	$\frac{10}{6}$	$\frac{100}{6}$
12	$\frac{2}{12} = \frac{1}{6}$	$\frac{12}{6}$	$\frac{144}{6}$

Mean of X = $\Sigma X.P(X) = \frac{4}{6} + \frac{6}{6} + \frac{16}{6} + \frac{10}{6} + \frac{12}{6} = \frac{48}{6} = 8$

Variance of X $= \Sigma X^2.P(X) - [E(X)]^2 = \frac{424}{6} - \frac{384}{6} = \frac{40}{6} = \frac{20}{3}$

OR

Let E_1 = Bulb is manufactured by factory A

E_2 = Bulb is manufactured by factory B

E_3 = Bulb is manufactured by factory C

and E = Bulb is defective

Here $P(E_1) = 60\% = \frac{60}{100} = \frac{6}{10}$,

$P(E_2) = 30\% = \frac{30}{100} = \frac{3}{10}$,

$P(E_3) = 10\% = \frac{10}{100} = \frac{1}{10}$

$\therefore \quad P(E/E_1) = 1\% = \frac{1}{100}$, $P(E/E_2) = 2\% = \frac{2}{100}$

and $P(E/E_3) = 3\% = \frac{3}{100}$

By Baye's Theorem,

$$P(E_1/E) = \frac{P(E_1)P(E/E_1)}{P(E_1)P(E/E_1)+P(E_2)P(E/E_2)+P(E_3)P(E/E_3)}$$

$$= \frac{\frac{6}{10}\times\frac{1}{100}}{\frac{6}{10}\times\frac{1}{100}+\frac{3}{10}\times\frac{2}{100}+\frac{1}{10}\times\frac{3}{100}} = \frac{6}{6+6+3} = \frac{6}{15} = \frac{2}{5}$$

32. Let merchant stock of desktop model be x units and that of portable model be y units.

Total profit to maximize Z = 4500x + 5000y

subject to constraints

$x + y \le 250$

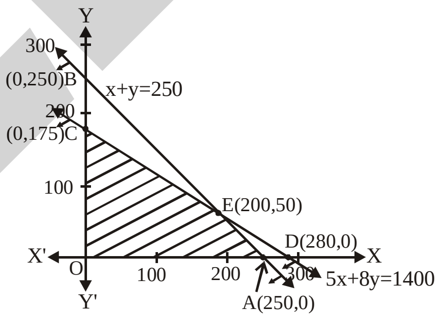
$25,000\,x + 40,000\,y \le 70,00,000$

$\Rightarrow \quad 5x + 8y \le 1400$

$x \ge 0,\ y \ge 0$

Now, plot the straight lines on the graph

and find the corner points of feasible region.



x + y = 250

5x + 8y = 1400

OAEC is feasible region

Corner Points	Z = 4500x+5000y
O (0,0)	0
A(250,0)	11,25,000
E(200,50)	11,50,000
C(0,175)	8,75,000

Clearly maximum profit is 11,50,000 at (200,50)

i.e. when 200 desktop models and 50 portable models are in stock.

Section - D

33. $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$\Rightarrow \quad A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$

and $\quad A^3 = A^2.A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$

Now $\quad A^3 - 6A^2 + 7A + kI = 0$

$\Rightarrow \quad \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + k\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

or $\quad \begin{bmatrix} -2+k & 0 & 0 \\ 0 & -2+k & 0 \\ 0 & 0 & -2+k \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \quad -2 + k = 0$

or $\quad k = 2$

OR

Given, $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$\therefore \quad$ A is invertible if $|A| \neq 0$

Now, $|A| = 2x[-4 + 4] + 3 \times [-6 + 4] + 5 \times [3 - 2]$

$= 2 \times 0 + 3 \times (-2) + 5 \times 1 = -1 \neq 0$

$\Rightarrow \quad adj(A) = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T$

$= \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

and $\quad A^{-1} = \dfrac{adj(A)}{|A|}$

$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

Now, given equations are

$2x - 3y + 5z = 11$

$3x + 2y - 4y = -5$

and $x + y - 2z = -3$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

or $AX = B$

$\Rightarrow$ $X = A^{-1}B$

$$\text{or} \quad X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} 0 \times 11 + (1) \times (-5) + (-2) \times (-3) \\ (-2) \times 11 + 9 \times (-5) + (-23) \times (-3) \\ (-1) \times 11 + 5 \times (-5) + (-13) \times (-3) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow$ $x = 1, y = 2, z = 3$

34. $y^2 \le 4x,\ 4x^2 + 4y^2 \le 9$

or $y^2 = 4x\ \&\ 4x^2 + 4y^2 = 9 \Rightarrow x^2 + y^2 = (3/2)^2$

For intersection points, we have

$4x^2 + 4(4x) = 9$

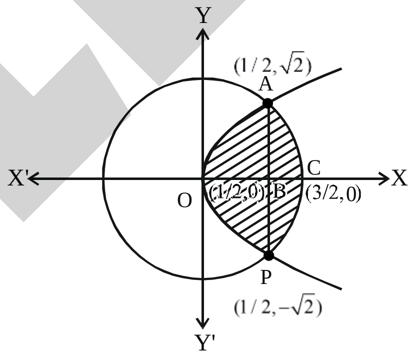
$\Rightarrow$ $4x^2 + 16x - 9 = 0$

or $4x^2 + 18x - 2x - 9 = 0$

$\Rightarrow$ $2x(2x + 9) - 1(2x + 9) = 0$

or $2x - 1 = 0$ or $2x + 9 = 0$

$\Rightarrow$ $x = 1/2$ or $x = -9/2$



$$\text{Total area} = 2\left[2\int_0^{1/2}\sqrt{x}.dx + \int_{1/2}^{3/2}\sqrt{\left(\frac{3}{2}\right)^2 - x^2}\,dx\right]$$

$$= 2.\left[\frac{2(x^{3/2})_0^{1/2}}{3/2} + \left(\frac{x}{2}\sqrt{\left(\frac{3}{2}\right)^2 - x^2} + \frac{9}{4 \times 2}\sin^{-1}\frac{2x}{3}\right)_{1/2}^{3/2}\right]$$

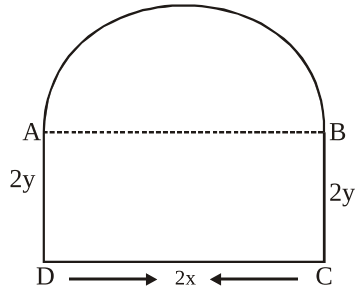
$$= 2.\left[\left(\frac{4}{3} \times \frac{1}{2\sqrt{2}}\right) + \left(\frac{9}{8} \times \frac{\pi}{2}\right) - \left(\frac{\sqrt{2}}{4} + \frac{9}{8}\sin^{-1}\frac{1}{3}\right)\right]$$

$$= \frac{4}{3\sqrt{2}} + \frac{9\pi}{8} - \frac{\sqrt{2}}{2} - \frac{9}{4}\sin^{-1}\frac{1}{3}$$

$$= \frac{\sqrt{2}}{6} + \frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right) = \left[\frac{\sqrt{2}}{6} + \frac{9}{4}\cos^{-1}\frac{1}{3}\right] \text{sq. units}$$

35. Let ABCD be a rectangle and let the semi-circle be described on side AB as diameter.

Let AB = 2x and AD = 2y. Let P be the perimeter and A be the area of the figure.



Then,

$P = 2x + 4y + \pi x$ (Given) ....(i)

and, $A = (2x)(2y) + \frac{\pi x^2}{2}$

$\Rightarrow A = 4xy + \frac{\pi x^2}{2}$

$\Rightarrow A = x(P - 2x - \pi x) + \frac{\pi x^2}{2}$ [Using (i)]

$\Rightarrow A = Px - 2x^2 - \pi x^2 + \frac{\pi x^2}{2}$

$\Rightarrow A = Px - 2x^2 - \frac{\pi x^2}{2}$

$\Rightarrow \frac{dA}{dx} = P - 4x - \pi x$ and $\frac{d^2A}{dx^2} = -4 - \pi$

For A to be maximum or minimum , we must have

$$\frac{dA}{dx} = 0 \quad \Rightarrow \quad P - 4x - \pi x = 0 \quad \Rightarrow \quad x = \frac{P}{\pi + 4}$$

Clearly, $\frac{d^2A}{dx^2} = -4 - \pi < 0$ for all values of x.

Thus, A is maximum when $x = \frac{P}{\pi + 4}$

Putting $x = \frac{P}{\pi + 4}$ in (i), we get $y = \frac{P}{2(\pi + 4)}$

So, dimensions of the figure are $2x = \frac{2P}{\pi + 4}$ and $2y = \frac{P}{\pi + 4}$

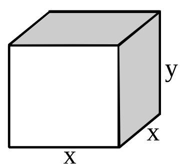
OR

Let each side of the square base be x and height by y, then

c^2 = total surface area of the box = $4xy + x^2$

Let V be the corresponding volume of the box, then

Volume of open box = Area of base × height



$V = x^2y = x^2\left(\frac{c^2 - x^2}{4x}\right)$

or $V = \frac{1}{4}(c^2x - x^3)$, $0 < x < c$

$\Rightarrow \frac{dV}{dx} = \frac{1}{4}(c^2 - 3x^2)$

and $\frac{d^2V}{dx^2} = \frac{1}{4} \times (-6x) = -\frac{3}{2}x$

For maximum value of volume V,

$\frac{dV}{dx} = \frac{1}{4}(c^2 - 3x^2) = 0$

$\Rightarrow c^2 - 3x^2 = 0 \Rightarrow x = \frac{c}{\sqrt{3}}$

$\therefore \quad \left(\frac{d^2v}{dx^2}\right)_{x=\frac{c}{\sqrt{3}}} = -\frac{3}{2} \times \frac{c}{\sqrt{3}} < 0$

Therefore, V is maximum at $x = \frac{c}{\sqrt{3}}$ and maximum volume will be

$$V = \frac{1}{4}\left(c^2 \times \frac{c}{\sqrt{3}} - \frac{c^3}{3\sqrt{3}}\right) = \frac{1}{4}\left(\frac{2c^3}{3\sqrt{3}}\right)$$

$\Rightarrow \quad V = \frac{c^3}{6\sqrt{3}}$ cubic units

36. Given equation of plane is

$2x + y - 2z + 3 = 0$ ..... (1)

$\because$ dr's of the plane are 2, 1, –2

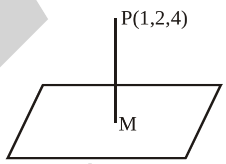
$\therefore$ dr's of line normal to plane are 2, 1, –2

$\Rightarrow$ Equation of line PM is

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2} = \lambda(\text{say})$$

$$\left[\because \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}\right]$$

$\therefore$ Coordinates of M = $(2\lambda + 1, \lambda + 2, -2\lambda + 4)$



Since point M lies on the plane (1)

$\therefore \quad 2(2\lambda + 1) + (\lambda + 2) - 2(-2\lambda + 4) + 3 = 0$

$\Rightarrow \quad 4\lambda + 2 + \lambda + 2 + 4\lambda - 8 + 3 = 0$

or $\quad 9\lambda - 1 = 0 \quad \Rightarrow \lambda = \frac{1}{9}$

$\therefore$ Foot of perpendicular $= \left(\frac{2}{9}+1, \frac{2}{9}+2, \frac{-2}{9}+4\right) = \left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$

and Length of perpendicular from (1, 2, 4)

i.e. $PM = \left|\frac{2(1)+2-2(4)+3}{\sqrt{4+1+4}}\right| = \left|\frac{2+2-8+3}{\sqrt{9}}\right| = \frac{1}{3}$ units