DATE : 12/03/2020

SET-1

Code No. 30/1/1

Aakash
Medical | IIT-JEE | Foundations
(Divisions of Aakash Educational Services Limited)
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Max. Marks : 80

Time : 3 Hrs.

Class X

Mathematics (Standard)

(CBSE 2020)

GENERAL INSTRUCTIONS :

(i) This question paper comprises four sections - A, B, C and D. This question paper carries 40 questions. All questions are compulsory.

(ii) Section A : Q. No. 1 to 20 comprises of 20 questions of one mark each.

(iii) Section B : Q. No. 21 to 26 comprises of 6 questions of two marks each.

(iv) Section C : Q. No. 27 to 34 comprises of 8 questions of three marks each.

(v) Section D : Q. No. 35 to 40 comprises of 6 questions of four marks each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of one mark each, 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to **attempt only one of the choices** in such questions.

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is not permitted.

Section-A

Q 1 - 10 are multiple choice questions. Select the most appropriate answer from the given options.

1. If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is **[1]**

(a) 10 (b) –10

(c) –7 (d) –2

Answer (b) **[1]**

Sol. Let $f(x) = x^2 + 3x + k$

$f(2) = (2)^2 + 3(2) + k = 0$

$\Rightarrow$ $4 + 6 + k = 0$

$\Rightarrow$ $k = -10$

Hence, option (b) is correct.

2. The total number of factors of a prime number is [1]

(a) 1 (b) 0

(c) 2 (d) 3

Answer (c) [1]

Sol. Total number of factors of a prime number is 2

Hence, option (c) is correct.

3. The quadratic polynomial, the sum of whose zeroes is –5 and their product is 6, is [1]

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$

(c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

Answer (a) [1]

Sol. Quadratic polynomial

$= x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$

$= x^2 - (-5)x + 6$

$= x^2 + 5x + 6$

Hence, option (a) is correct.

4. The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is [1]

(a) –2 (b) $\neq 2$

(c) 3 (d) 2

Answer (d) [1]

Sol. For no solution; $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore \quad \frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3}$

$\Rightarrow \quad \boxed{k = 2}$

Hence, option (d) is correct.

5. The HCF and the LCM of 12, 21, 15 respectively are [1]

(a) 3, 140 (b) 12, 420

(c) 3, 420 (d) 420, 3

Answer (c) [1]

Sol. $12 = 2 \times 2 \times 3$

$21 = 3 \times 7$

$15 = 5 \times 3$

$\therefore \quad \text{HCF} = 3$

$\text{L.C.M} = 2 \times 2 \times 3 \times 5 \times 7$

$= 420$

Hence, option (c) is correct.

6. The value of x for which 2x, (x + 10) and (3x + 2) are the three consecutive terms of an AP, is [1]

(a) 6 (b) –6

(c) 18 (d) –18

Answer (a) [1]

Sol. 2x, (x + 10), (3x + 2) are in A.P.

$\therefore \quad x + 10 - 2x = 3x + 2 - x - 10$

$\Rightarrow \quad x = 6$

Hence, option (a) is correct.

7. The first term of an AP is p and the common difference is q, then its 10^{th} term is **[1]**

(a) $q + 9p$ (b) $p - 9q$

(c) $p + 9q$ (d) $2p + 9q$

Answer (c) **[1]**

Sol. $\therefore$ 10^{th} term $= p + (10 - 1)q$

$$\boxed{a_{10} = p + 9q}$$

Hence, option (c) is correct.

8. The distance between the points $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$, is **[1]**

(a) $a^2 + b^2$ (b) $a^2 - b^2$

(c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$

Answer (c) **[1]**

Sol. Distance between $A(a\cos\theta + b\sin\theta, 0)$ and $B(0, a\sin\theta - b\cos\theta)$ is

$$AB = \sqrt{\left((a\cos\theta+b\sin\theta)-0\right)^2+\left(0-(a\sin\theta-b\cos\theta)\right)^2}$$

$$= \sqrt{(a\cos\theta+b\sin\theta)^2+(b\cos\theta-a\sin\theta)^2}$$

$$= \sqrt{a^2+b^2}$$

Option (c) is correct.

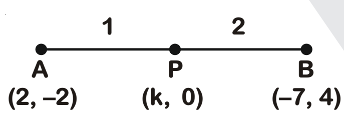
9. If the point P(k, 0) divides the line segment joining the points A(2, –2) and B(–7, 4) in the ratio 1 : 2, then the value of k is **[1]**

(a) 1 (b) 2

(c) –2 (d) –1

Answer (d) **[1]**

Sol.



$$\therefore \quad k = \frac{(1\times -7)+(2\times 2)}{1+2} \qquad \text{[Using section formula]}$$

$$\boxed{k = -1}$$

Hence, option (d) is correct.

10. The value of p, for which the points A(3, 1), B(5, p) and C(7, –5) are collinear, is **[1]**

(a) –2 (b) 2

(c) –1 (d) 1

Answer (a)

Sol. Since, points are collinear, then area of triangle formed by these points is zero.

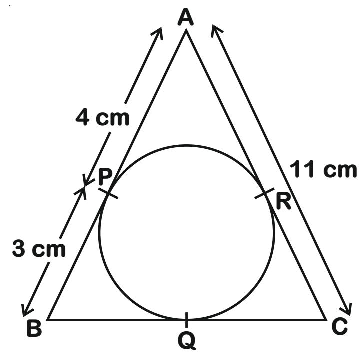
$$\frac{1}{2}\left|3(p+5)+5(-5-1)+7(1-p)\right| = 0$$

$\Rightarrow$ $p = -2$

Hence, option (a) is correct **[1]**

In Q.Nos. 11 to 15, fill in the blanks. Each question is of 1 mark.

11. In Fig. 1, ΔABC is circumscribing a circle, the length of BC is _______ cm. **[1]**



Answer : 10 cm **[1]**

Sol. BP = BQ = 3 cm

AR = AP = 4 cm

RC = AC – AR = 7 cm

RC = QC = 7 cm

$\therefore$ BC = 7 + 3 = 10 cm

12. Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} =$ ____. **[1]**

Answer : $\frac{1}{9}$ **[1]**

Sol. $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \left(\frac{1}{3}\right)^2$

$= \frac{1}{9}$

13. ABC is an equilateral triangle of side 2a, then length of one of its altitude is ________. **[1]**

Answer : $\sqrt{3}a$ **[1]**

Sol. Length of altitude of an equilateral triangle = $\frac{\sqrt{3}}{2} \times \text{side}$

$\therefore \frac{\sqrt{3}}{2} \times 2a = \sqrt{3}a$

14. $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ =$ _______. **[1]**

Answer : 2 **[1]**

Sol. $\frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec}(90^\circ - 59^\circ)$

$\Rightarrow \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59^\circ \cdot \sec 59^\circ$

$\Rightarrow$ 1 + 1

$\Rightarrow$ 2

15. The value of $\left(\sin^2\theta + \dfrac{1}{1+\tan^2\theta}\right) =$ ______. [1]

Answer : 1 [1]

Sol. $\sin^2\theta + \dfrac{1}{\sec^2\theta} = \sin^2\theta + \cos^2\theta = 1$

(using $\sec^2\theta - \tan^2\theta = 1$)

OR

The value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) =$ ______. [1]

Answer : 1 [1]

Sol. $(1+\tan^2\theta)(1-\sin^2\theta)$

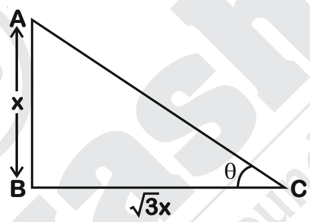
$\Rightarrow \sec^2\theta \times \cos^2\theta$

$\Rightarrow 1$

(16 – 20) Answer the following:

16. The ratio of the length of a vertical rod and the length of its shadow is $1:\sqrt{3}$. Find the angle of elevation of the sun at that moment? [1]

Sol. In $\triangle ABC$,



$\tan\theta = \dfrac{AB}{BC}$

$\Rightarrow \tan\theta = \dfrac{x}{\sqrt{3}x}$ [½]

$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$

$\therefore \boxed{\theta = 30^\circ}$ [½]

17. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes? [1]

Sol. Let r_1, r_2 and h_1, h_2 be the radius and height of two cones resectively

According to the question,

$\dfrac{r_1}{r_2} = \dfrac{3}{1}$ and $\dfrac{h_1}{h_2} = \dfrac{1}{3}$ [½]

$$\therefore \frac{\text{Volume of Cone}_1}{\text{Volume of Cone}_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$= \left(\frac{3}{1}\right)^2 \times \left(\frac{1}{3}\right)$$

$$= \frac{3}{1}$$ [½]

18. A letter of English alphabet is chosen at random. What is the probability that the chosen letter is a consonant. [1]

Sol. n(s) = Total number of alphabets in English = 26.

n(E) = Total number of consonant in English alphabet = 21 [½]

$\therefore$ Probability (Chosen letter is a consonant) $= \dfrac{n(E)}{n(s)}$

$= \dfrac{21}{26}$ [½]

19. A die is thrown once. What is the probability of getting a number less than 3? **[1]**

Sol. Total number of outcomes = 6

Number of favourable outcomes = 2 **[½]**

P(getting a number less than 3) = $\frac{2}{6}$

$= \frac{1}{3}$ **[½]**

OR

If the probability of winning a game is 0.07, what is the probability of losing it? **[1]**

Sol. Required probability = 1 – Probability of winning a game **[½]**

= 1 – 0.07

= 0.93 **[½]**

20. If the mean of the first n natural number is 15, then find n. **[1]**

Sol. $\text{Mean} = \frac{1+2+3+4...+n}{n}$

$\Rightarrow \frac{\left(\frac{n(n+1)}{2}\right)}{n} = 15$ **[½]**

$\Rightarrow \frac{n+1}{2} = 15$

$\Rightarrow$ n = 29 **[½]**

Section-B

21. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P. **[2]**

Sol. Common difference must be equal

$\therefore$ $(a^2 + b^2) - (a - b)^2 = (a + b)^2 - (a^2 + b^2)$ **[½]**

$\Rightarrow$ $(a^2 + b^2) - (a^2 + b^2 - 2ab) = (a^2 + b^2 + 2ab) - a^2 - b^2$ **[½]**

$\Rightarrow$ $a^2 + b^2 - a^2 - b^2 + 2ab = a^2 + b^2 + 2ab - a^2 - b^2$ **[½]**

$\Rightarrow$ 2ab = 2ab **[½]**

Hence, $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P.

22. In Fig.2, DE || AC and DC || AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$ **[2]**

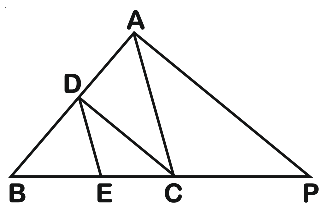


Fig.2

Sol. In ΔBAC; DE || AC

$\frac{BE}{EC} = \frac{BD}{DA}$ {By B.P.T} ...(i) **[½]**

In ΔBAP; DC || AP

$\frac{BC}{CP} = \frac{BD}{DA}$ {By B.P.T} ...(ii) [½]

From (i) and (ii), we have [½]

$\frac{BE}{EC} = \frac{BC}{CP}$ Hence Proved. [½]

OR

In Fig.3, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

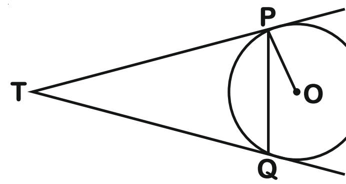


Fig.3

Sol. Join OQ.

$\angle OPQ = \angle OQP$ {OP = OQ}

$\Rightarrow \angle OPQ + \angle OQP + \angle POQ = 180°$ {Angle sum property} [½]

$\Rightarrow 2\angle OPQ = 180° - \angle POQ$ ...(i)

Also, $\angle PTQ + \angle POQ = 180°$

$\Rightarrow \angle PTQ = 180° - \angle POQ$ ...(ii) [½]

From (i) and (ii), [½]

$\angle PTQ = 2\angle OPQ$ Hence Proved. [½]

23. The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in Fig.4. If AC = 1.5 m long and CD = 3 m, find (i) $\tan\theta$ (ii) $\sec\theta + \text{cosec}\theta$ **[2]**

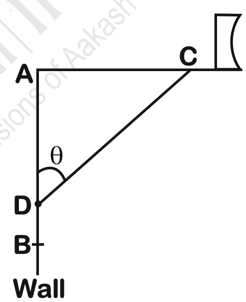
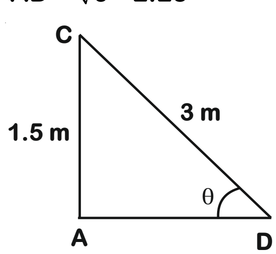


Fig.4

Sol. $AD = \sqrt{9 - 2.25}$



$= \sqrt{6.75}$

$= \frac{3\sqrt{3}}{2}$ [½]

$\therefore \quad \tan\theta = \dfrac{CA}{AD} = \dfrac{1.5}{3\sqrt{3}} \times \dfrac{2}{1} = \dfrac{1}{\sqrt{3}}$ [½]

$\sec\theta + \operatorname{cosec}\theta = \dfrac{CD}{AD} + \dfrac{CD}{CA} = 3\left[\dfrac{1\times2}{3\sqrt{3}} + \dfrac{1}{1.5}\right]$ [½]

$= 3\left[\dfrac{2}{3\sqrt{3}} + \dfrac{2}{3}\right]$

$= 6\left[\dfrac{1+\sqrt{3}}{3\sqrt{3}}\right]$

$= \dfrac{2\left(\sqrt{3}+1\right)}{\sqrt{3}}$

$= \dfrac{2}{3}\left(3+\sqrt{3}\right)$ [½]

24. A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. $\left[\text{Use}\,\pi = \dfrac{22}{7}\right]$ [2]

Sol. Length of arc = 22 cm

$\Rightarrow \quad \dfrac{2\pi r\theta}{360°} = 22$ [½]

$\Rightarrow \quad 2\times\dfrac{22}{7}\times r\times\dfrac{60°}{360°} = 22$ [½]

$\Rightarrow \quad r = \dfrac{22\times7\times6}{2\times22}$ [½]

$\Rightarrow$ r = 21 cm [½]

25. If a number x is chosen at random from the numbers –3, –2, –1, 0, 1, 2, 3. What is probability that $x^2 \le 4$? [2]

Sol. Let E be the event of getting square of a number less than or equal to 4.

S be the sample space. Then,

S = {–3, –2, –1, 0, 1, 2, 3} [½]

$\Rightarrow$ n(S) = 7

and, E = {–2, –1, 0, 1, 2}

$\Rightarrow$ n(E) = 5. [½]

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{5}{7}$ [1]

26. Find the mean of the following distribution: [2]

Class :	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency :	5	10	10	7	8

Sol. [1]

Class	Mid-value (x_i)	Frequency (f_i)	f_ix_i
3-5	4	5	20
5-7	6	10	60
7-9	8	10	80
9-11	10	7	70
11-13	12	8	96
Total		$\Sigma f_i = 40$	$\Sigma f_ix_i = 326$

$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$ [½]

$= \frac{326}{40}$

$= 8.15$ [½]

OR

Find the mode of the following data :

Class :	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140
Frequency :	6	8	10	12	6	5	3

[2]

Sol. Here, the maximum frequency is 12 and the corresponding class is 60-80. So, 60-80 is the modal class such that l = 60, h = 20, f_0 = 12, f_1 = 10 and f_2 = 6. [1]'

$\therefore \quad \text{Mode} = 60 + \left(\frac{12-10}{2\times 12-10-6}\right)\times 20$ [½]

$= 60 + \frac{2}{8}\times 20$

= 60 + 5

= 65 [½]

Section-C

27. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$. [3]

Sol. Let α and β are the zeroes of the polynomial $f(x) = ax^2 + bx + c$.

$\therefore \quad (\alpha + \beta) = \frac{-b}{a}$ ...(i) [½]

and $\alpha\beta = \frac{c}{a}$ ...(ii) [½]

According to the question, $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the zeroes of the required quadratic polynomial

$\therefore$ Sum of zeroes of required polynomial

$S' = \frac{1}{\alpha} + \frac{1}{\beta}$

$= \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{-b}{c}$ ...(iii) [From equation (i) and (ii)] [½]

and product of zeroes of required polynomial $= \frac{1}{\alpha}\times\frac{1}{\beta}$.

$P' = \frac{1}{\alpha\beta}$

$= \frac{a}{c}$ ...(iv) [From equation (ii)] [½]

$\therefore$ Equation of the required quadratic polynomial

$= k\left(x^2 - S'x + p'\right)$, where k is any non-zero constant [½]

$= k\left(x^2 - \left(\dfrac{-b}{c}\right)x + \dfrac{a}{c}\right)$ [From equation (iii) and (iv)]

$= k\left(x^2 + \dfrac{b}{c}x + \dfrac{a}{c}\right)$ [½]

OR

Divide the polynomial f(x) = $3x^2 - x^3 - 3x + 5$ by the polynomial g(x) = $x - 1 - x^2$ and verify the division algorithm. [3]

Sol. Using long division method,

$$\begin{array}{r|l} & \quad x-2 \\ \hline -x^2+x-1 & -x^3+3x^2-3x+5 \\ & -x^3+x^2-x \\ & \;\;+\quad\;\; -\quad\;\; + \\ \hline & \qquad 2x^2-2x+5 \\ & \qquad 2x^2-2x+2 \\ & \qquad\; -\quad\;\; +\quad\;\; - \\ \hline & \qquad\qquad\qquad 3 \\ \hline \end{array}$$

[1]

Clearly, quotient q(x) = (x – 2) and remainder r(x) = 3 [1]

Now,

(Quotient × Divisor) + Remainder [½]

$= (x - 2)(-x^2 + x - 1) + 3$

$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$

$= -x^3 + 3x^2 - 3x + 5$ = Dividend [½]

Hence, the division algorithm is verified.

28. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by 2y – x = 8, 5y – x = 14 and y – 2x = 1. [3]

Sol.

$2y - x = 8$

x	0	–8
y	4	0

[½]

$5y - x = 14$

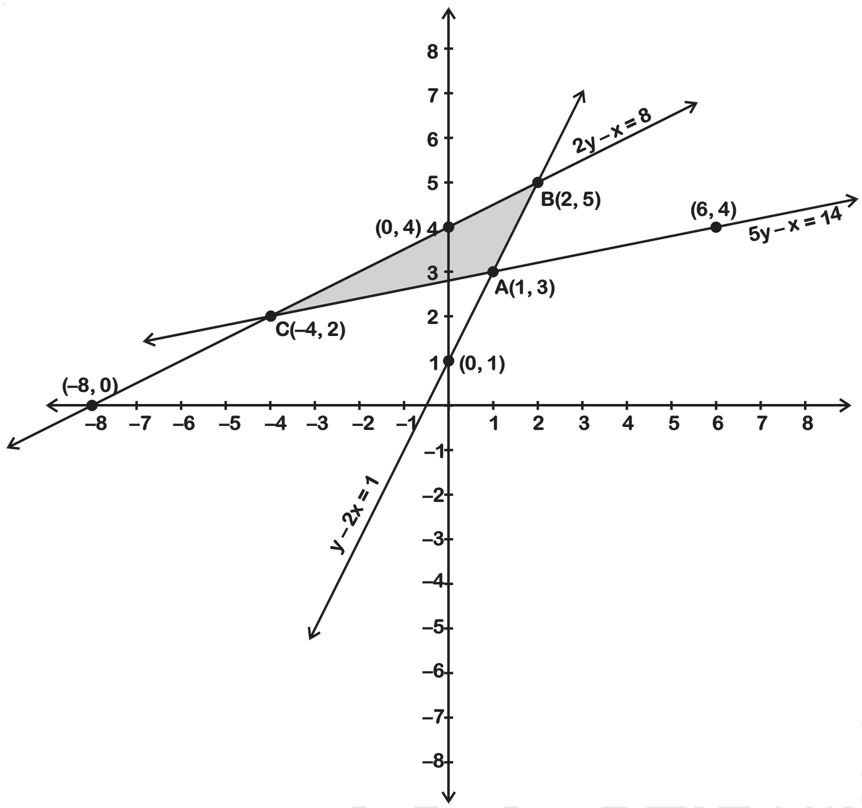
x	–4	6
y	2	4

[½]

$y - 2x = 1$

x	0	1
y	1	3

[½]



[1½]

OR

If 4 is a zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes. [3]

Sol. Let $f(x) = x^3 - 3x^2 - 10x + 24$

$f(x)$ is divisible by $(x - 4)$ [½]

$$
\begin{array}{r|l}
 & x^2 + x - 6 \\
\hline
x - 4 & x^3 - 3x^2 - 10x + 24 \\
 & \underline{\underset{-}{x^3} \underset{+}{- 4x^2}} \\
 & x^2 - 10x + 24 \\
 & \underline{\underset{-}{x^2} \underset{+}{- 4x}} \\
 & -6x + 24 \\
 & \underline{\underset{+}{-6x} \underset{-}{+ 24}} \\
 & 0
\end{array}
$$

[1½]

$\therefore$ $x^2 + x - 6 = x^2 + 3x - 2x - 6$ [½]

$= x(x + 3) - 2(x + 3)$

$= (x - 2)(x + 3)$ [½]

$\therefore$ Other two zeroes of the given polynomial are 2 and –3.

29. In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/hr and time of flight increased by 30 minutes. Find the original duration of flight. [3]

Sol. Let the duration of the flight be x hours

$$\text{Speed} = \frac{\text{Distance}}{\text{time}} = \frac{600}{x}\,\text{km/h}$$ [½]

Duration of the flight due to slow down $= x + \frac{30}{60} = x + \frac{1}{2}$

According to question [½]

$$\frac{600}{x}-\frac{600}{x+\frac{1}{2}}=200$$ [½]

$$\Rightarrow \frac{3}{x}-\frac{3}{x+\frac{1}{2}}=1$$

$$\Rightarrow \frac{3(2x+1)-6x}{x(2x+1)}=1$$ [½]

$$\Rightarrow \frac{6x+3-6x}{x(2x+1)}=1$$

$$\Rightarrow \frac{3}{x(2x+1)}=1$$

$\Rightarrow$ $2x^2 + x - 3 = 0$ [½]

$\Rightarrow$ $2x^2 + 3x - 2x - 3 = 0$

$\Rightarrow$ $x(2x + 3) - 1\ (2x + 3) = 0$

$\Rightarrow$ $(2x + 3)\ (x - 1) = 0$

$x = 1$ [½]

Original duration of the flight is 1 hour.

30. Find the area of triangle PQR formed by the points P(–5, 7), Q(–4, –5) and R(4, 5). **[3]**

Sol. Here, $x_1 = -5$, $y_1 = 7$, $x_2 = -4$, $y_2 = -5$, $x_3 = 4$, $y_3 = 5$ [½]

$$\text{Area of } \Delta PQR=\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$ [½]

$$=\frac{1}{2}\left|-5(-5-5)-4(5-7)+4(7-(-5)\right|$$ [½]

$$=\frac{1}{2}\left|50+8+48\right|$$ [½]

$$=\frac{1}{2}\left|106\right|$$ [½]

$= 53$ [½]

$\therefore$ Area of $\Delta PQR = 53$ sq. units

OR

If the point C(–1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio 3 : 4, find the coordinates of B. **[3]**

Sol. Now,

Using section formula

3 4
A(2, 5) C(–1, 2) B(x, y)

$$\Rightarrow -1=\frac{(3\times x)+(4\times 2)}{3+4}$$ [½]

$$\Rightarrow -1=\frac{3x+8}{7}$$

$\Rightarrow$ $3x + 8 = -7$ [½]

$\Rightarrow$ $3x = -15$

$\Rightarrow$ $x = -5$

Also,

$$2 = \frac{(3 \times y) + (4 \times 5)}{3 + 4}$$ [½]

$\Rightarrow$ $$2 = \frac{3y + 20}{7}$$

$\Rightarrow$ $3y + 20 = 14$

$\Rightarrow$ $3y = -6$

$\Rightarrow$ $y = -2$ [½]

$\therefore$ Coordinates of B are $(-5, -2)$ [1]

31. In Fig.5, $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that BAC is an isosceles triangle. [3]

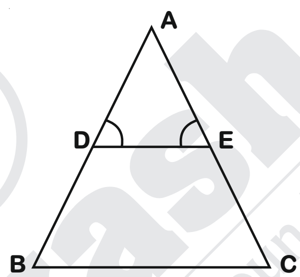
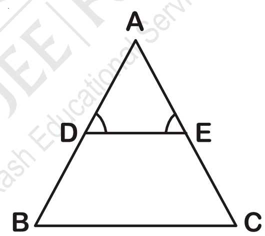


Fig.5

Sol. Given : $\angle D = \angle E$



$$\frac{AD}{DB} = \frac{AE}{EC}$$ [½]

To Prove : ΔBAC is an isosceles triangle.

Proof : $\frac{AD}{DB} = \frac{AE}{EC}$ (Given) [½]

$\therefore$ DE || BC [By converse of B.P.T]

$\Rightarrow$ $\angle D = \angle B$ ...(i) [Corresponding angles] [½]

$\angle E = \angle C$ ...(ii) [Corresponding angles]

But $\angle D = \angle E$ (Given) [½]

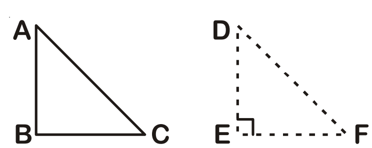
From (i) and (ii)

$\therefore$ $\angle B = \angle C$ $\Rightarrow$ AB = AC [½]

Hence, ΔBAC is an isosceles triangle. [½]

32. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle. [3]

Sol. Given : A triangle ABC such that $AC^2 = AB^2 + BC^2$



To prove : $\angle ABC = 90°$ [½]

Construction : Construct a ΔDEF such that

DE = AB, EF = BC and $\angle E = 90°$ [½]

Proof : In right ΔDEF

$DE^2 + EF^2 = DF^2$ [By pythagoras theorem] [½]

$\Rightarrow$ $AB^2 + BC^2 = DF^2$ [$\because$ DE = AB, EF = BC]

But $AB^2 + BC^2 = AC^2$ [Given] [½]

$\therefore$ $AC^2 = DF^2$

$\Rightarrow$ AC = DF

Thus in ΔABC and ΔDEF, we have

AB = DE, BC = EF and AC = DF [½]

$\therefore$ $\Delta ABC \cong \Delta DEF$ [By SSS congruency]

$\Rightarrow$ $\angle B = \angle E = 90°$ [½]

Therefore, ΔABC is right triangle, right angled at B.

Hence proved.

33. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$. [3]

Sol. $\sin\theta + \cos\theta = \sqrt{3}$

On squaring both sides, we get

$\Rightarrow$ $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$ [½]

$\Rightarrow$ $1 + 2\sin\theta\cos\theta = 3$ [½]

$\Rightarrow$ $\sin\theta\cos\theta = 1$ [½]

Now, $\tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ [½]

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ [½]

$= \frac{1}{1}$

= 1 = RHS [½]

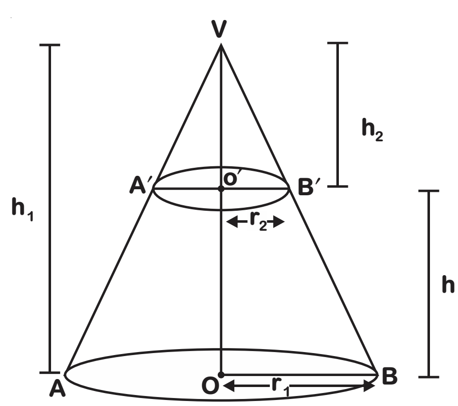
Hence Proved.

34. A cone of base radius 4 cm is divided into two parts by drawing a plane through the mid-point of its height and parallel to its base. Compare the volume of the two parts. [3]

Sol. Here, $r_1 = 4$ cm

$\Delta VO'A' \sim \Delta VOA$ (AA similarity)

Now, $\frac{r_1}{r_2} = \frac{h_1}{h_2}$ [½]



Also, $h_1 = 2h_2$

$\Rightarrow$ $\frac{r_1}{r_2} = 2$ [½]

$\Rightarrow$ $r_2 = 2$ cm

Now, $\frac{\text{Volume of smaller cone } VA'B'}{\text{Volume of frustum } ABB'A'}$ [½]

$= \frac{\frac{1}{3}\pi r_2^2 h_2}{\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)}$ [½]

$= \dfrac{r_2^2}{r_1^2 + r_2^2 + r_1 r_2}$

$= \dfrac{4}{16+4+8}$ [$\because$ h = h_2] [½]

$= \dfrac{4}{28}$

= 1 : 7 [½]

Section-D

35. Show that the square of any positive integer cannot be of the form (5q + 2) or (5q + 3) for any integer q. [4]

Sol. Let a be any positive integer and b = 5

Then, by Euclid's division Lemma

$a = 5m + r$ for some integer $m \geq 0$ and r = 0 , 1, 2, 3, 4 [½]

So, a = 5m or 5m + 1 or 5m + 2 or 5m + 3 or 5m + 4 [½]

$(5m)^2 = 25m^2 = 5(5m^2)$ [½]

= 5q, where q is any integer

$(5m + 1)^2 = 25m^2 + 10m + 1$

$= 5(5m^2 + 2m) + 1$ [½]

= 5q + 1, where, q is any integer

$(5m + 2)^2 = 25m^2 + 20m + 4$

$= 5(5m^2 + 4m) + 4$ [½]

= 5q + 4, where, q is any integer

$(5m + 3)^2 = 25m^2 + 30m + 9$

$= 5(5m^2 + 6m + 1) + 4$

= 5q + 4, where, q is any integer [½]

$(5m + 4)^2 = 25m^2 + 40m + 16$

$= 5(5m^2 + 8m + 3) + 1$ [½]

= 5q + 1, where q is any integer

Hence, square of any positive integer cannot be of the form [½]

(5q + 2) or (5q + 3) for any integer q.

OR

Prove that one of every three consecutive positive integers is divisible by 3. [4]

Sol. Let n, (n + 1), (n + 2) be three consecutive positive integers. [1]

Then by Euclid's division Lemma

$n = 3q + r$ for some integer $q \geq 0$ and r = 0, 1, 2 [1]

Case (i) when n = 3q :

In this case,

n is divisible by 3 but (n + 1) and (n + 2) are not divisible by 3 [½]

Case (ii) when n = 3q + 1,

In this case,

n + 2 = 3q + 1 + 2 = 3(q + 1) is divisible by 3 but n and (n + 1) are not divisible by 3. [½]

Case (iii) when n = 3q + 2,

In this case,

n + 1 = 3q + 2 + 1 = 3(q + 1) is divisible by 3 but n and (n + 2) are not divisible by 3. [½]

Hence, one of n, (n + 1) and (n + 2) is divisible by 3. [½]

36. The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7 : 15. Find the numbers. [4]

Sol. Let the four consecutive numbers in A.P. are (a – 3d), (a – d), (a + d) and (a + 3d). [½]

$\therefore$ According to the condition given,

(a – 3d) + (a – d) + (a + d) + (a + 3d) = 32

$\Rightarrow$ 4a = 32

$\Rightarrow$ a = 8 ...(i) [1]

and, according to the 2nd condition given,

$$\frac{(a-3d)\times(a+3d)}{(a-d)\times(a+d)}=\frac{7}{15}$$ [½]

$$\Rightarrow \frac{(8-3d)\times(8+3d)}{(8-d)\times(8+d)}=\frac{7}{15}$$

$$\Rightarrow \frac{64-9d^2}{64-d^2}=\frac{7}{15}$$ [½]

$\Rightarrow$ $15(64 - 9d^2) = 7(64 - d^2)$

$\Rightarrow$ $128d^2 = 512$

$\Rightarrow$ $d^2 = 4$

$\Rightarrow$ d = ±2 [½]

$\therefore$ Numbers are 2, 6, 10 and 14 or 14, 10, 6 and 2. [1]

OR

Solve : 1 + 4 + 7 + 10 + ... + x = 287 [4]

Sol. Here 1, 4, 7, 10, ... x is an A.P.

With first term a = 1 and common difference d = 3. [½]

Let there be n terms in the A.P. Then,

x = n^{th} term

$\Rightarrow$ x = 1 + (n – 1) × 3 [½]

= 3n – 2 ...(i)

Now, 1 + 4 + 7 + 10 + ... + x = 287

$$\Rightarrow \frac{n}{2}[1+x]=287 \qquad \left[S_n=\frac{n}{2}(a+l)\right]$$ [½]

$$\Rightarrow \frac{n}{2}[1+3n-2]=287$$

$\Rightarrow$ $3n^2 - n - 574$

$\Rightarrow$ $3n^2 - n - 574 = 0$ [1]

$\Rightarrow$ $3n^2 - 42n + 41n - 574 = 0$

$\Rightarrow$ 3n (n – 14) + 41 (n – 14) = 0

$\Rightarrow$ (n – 14) (3n + 41) = 0

$\Rightarrow$ n – 14 = 0 $[\because 3n + 41 \neq 0]$

$\Rightarrow$ n = 14 **[½]**

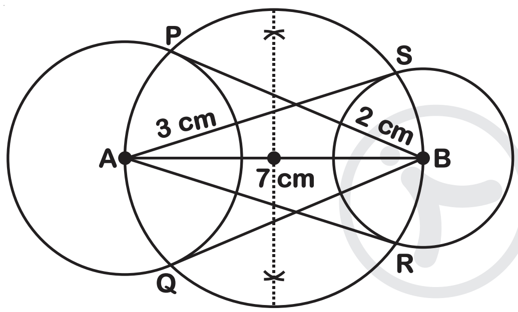
Putting n = 14 in eqn (i), we get

x = 3 × 14 – 2

x = 40 **[1]**

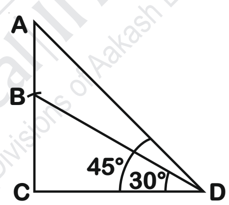
37. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle. **[4]**

Sol. **[4]**



38. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. $\left(\text{Take } \sqrt{3} = 1.73\right)$ **[4]**

Sol. AB = height of flag-staff = 6 m



Let BC = height of tower = h m **[½]**

In ΔBCD

$\frac{BC}{CD} = \tan 30^\circ$ **[½]**

$\Rightarrow \frac{h}{CD} = \frac{1}{\sqrt{3}} \Rightarrow CD = h\sqrt{3}$ ...(i) **[½]**

In ΔACD, $\frac{AC}{CD} = \tan 45^\circ$ **[½]**

$\Rightarrow \frac{h+6}{CD} = 1 \Rightarrow h = CD - 6$

$\Rightarrow h = h\sqrt{3} - 6$ [From (i)] **[½]**

$\Rightarrow h(\sqrt{3} - 1) = 6$

$\Rightarrow h = \frac{6}{\sqrt{3} - 1}$ **[½]**

$\Rightarrow h = 3(\sqrt{3} + 1)$ **[½]**

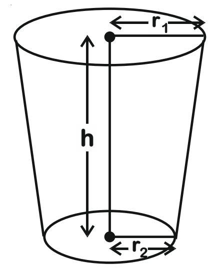
$h = 3 \times 2.73$

$h = 8.19$ m [½]

$\therefore$ Height of the tower is 8.19 m

39. A bucket in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm, respectively. Find the capacity of the bucket. Also find the cost of milk which can completely fill the bucket at the rate of ₹40 per litre. $\left(\text{Use } \pi = \frac{22}{7}\right)$ [4]

Sol. Here



$r_1 = 20$ cm

$r_2 = 10$ cm [1]

$h = 30$ cm

Volume of the bucket $= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$

$= \frac{1}{3} \times \frac{22}{7} \times 30[400 + 100 + 200]$ [1]

$= \frac{1}{3} \times \frac{22}{7} \times 30 \times 700$

$= 22000 \text{ cm}^3$ [1]

$= 22$ litres (1000 cm^3 = 1 litre)

Cost of milk = ₹40 × 22 [1]

= ₹880

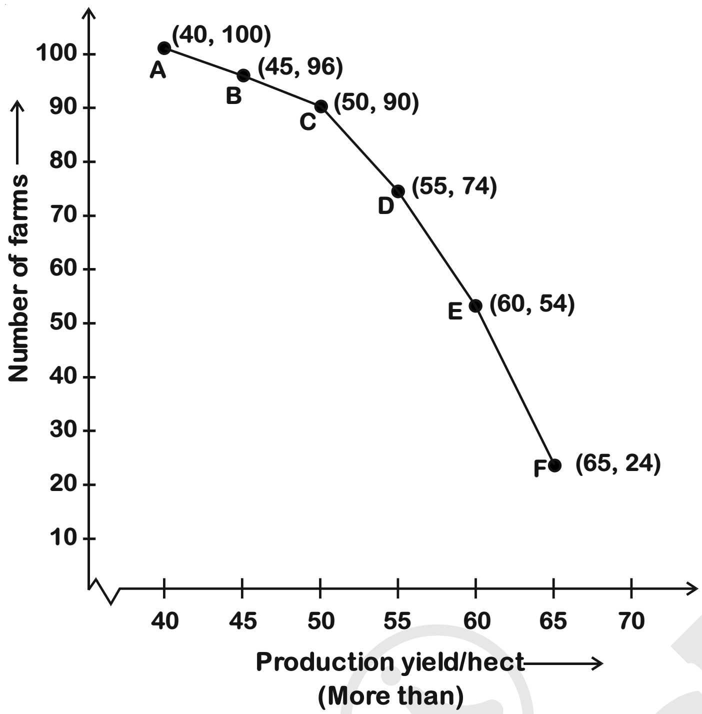
40. The following table gives production yield per hectare (in quintals) of wheat of 100 farms of a village :

Production yield/hect.	40-45	45-50	50-55	55-60	60-65	65-70
No. of farms	4	6	16	20	30	24

Change the distribution to 'a more than' type distribution and draw its ogive. [4]

Sol. [2]

Production yield/hect	Number of farms	Production yield more than/hect	Cumulative frequency
40–45	4	40	100
45–50	6	45	96
50–55	16	50	90
55–60	20	55	74
60–65	30	60	54
65–70	24	65	24



[2]

OR

The median of the following data is 525. Find the values of x and y, if total frequency is 100 : [4]

Class :	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Frequency :	2	5	x	12	17	20	y	9	7	4

Sol. [1]

Class	Frequency F_i	c.f.
0–100	2	2
100–200	5	7
200–300	x	7 + x
300–400	12	19 + x
400–500	17	36 + x
500–600	20	56 + x
600–700	y	56 + x + y
700–800	9	65 + x + y
800–900	7	72 + x + y
900–1000	4	76 + x + y = N

Here N = 100

$\Rightarrow$ 76 + x + y = 100

x + y = 24 ...(i) [½]

Median = 525

Median class = 500 – 600

l = 500, h = 100

f = 20

c.f. = 36 + x [½]

$$\text{Median} = l + \left[\frac{\frac{N}{2} - \text{c.f.}}{f}\right] \times h$$ [½]

$$\Rightarrow \quad 525 = 500 + \left[\frac{50 - 36 - x}{20}\right] \times 100$$ [½]

⇒ 25 = (14 – x)5

⇒ 14 – x = 5

⇒ x = 9 [½]

Now from (i)

9 + y = 24

y = 15 [½]

❑ ❑ ❑