Class – XII
MATHEMATICS (041)
SQP Marking Scheme (2019-20)

TIME: 3 Hrs. **Maximum Marks: 80**

	SECTION A	
1	(c) 9	1
2	(a) 3 × p	1
3	(b)p=3,q=$\frac{27}{2}$	1
4	(b)0.25	1
5	(c) (2,3)	1
6	(b)$\frac{\pi}{3}$	1
7	(c) $\frac{8}{15}$	1
8	(b) $\frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + c$	1
9	(a) 0	1
10	(b) $\vec{r} = (-\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j})$	1
11	$g\left(\left[-\frac{5}{4}\right]\right) = g(-2) = 2$	1
12	2	1
13	$y = 2$	1
14	$\frac{-3}{2}$ OR decreasing at rate of 72 units/sec.	1
15	2 units OR $\frac{5}{7}(-2\hat{i} - 3\hat{j} + 6\hat{k})$	1
16	Apply$R_1 \to R_1 + R_2$ $\begin{vmatrix} l+m+n & m+n+l & n+l+m \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$ $= 2(l+m+n)\begin{vmatrix} 1 & 1 & 1 \\ n & l & m \\ 1 & 1 & 1 \end{vmatrix}$; yes (l + m + n) is a factor	1
17	$\int_{-2}^{2}(x^3+1)dx = \int_{-2}^{2}(x^3)dx + \int_{-2}^{2}1dx = I_1 + I_2$ $= 0 + [x]_{-2}^{2}$ (As I_1 is odd function) =2+2 = 4	1

18	Let $x + \sin x = t$ So $(1 + \cos x)dx = dt$ $I = 3\int \frac{dt}{t} = 3\log\|t\| + c = 3\log\|(x + \sin x)\| + c$ or directly by writing formula $$\int \frac{f'(x)}{f(x)}dx = \log\|f(x)\| + c$$ **OR** $\int \cos 4x\, dx = \frac{\sin 4x}{4} + c$	1
19	let $(1 + x^2) = t$ so $2xdx = dt$ $\Longrightarrow I = \frac{1}{2}\int e^t dt \quad = \frac{1}{2}e^t + c \;= \frac{1}{2}e^{(1+x^2)} + c$	1
20	$\frac{dy}{dx} = e^x e^y$ $\Longrightarrow \frac{dy}{e^y} = e^x dx$ integrating both sides $\Rightarrow -e^{-y} + c = e^x$ $\Rightarrow \mathbf{e^x + e^{-y} = c}$	1
	SECTION B	
21	$= \sin^{-1}\left(\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}}\right)$ if $-\frac{\pi}{4} < x < \frac{\pi}{4}$ $=\sin^{-1}\left(\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}\right)$ if $-\frac{\pi}{4} + \frac{\pi}{4} < x + \frac{\pi}{4} < \frac{\pi}{4} + \frac{\pi}{4}$ $=\sin^{-1}\left(\sin\left(x + \frac{\pi}{4}\right)\right)$ if $0 < \left(x + \frac{\pi}{4}\right) < \frac{\pi}{2}$ i.e. principal values $=\left(x + \frac{\pi}{4}\right)$	1 1
	OR Let 2 divides$(a - b)$ and 2 divides $(b - c)$: where $a, b, c \in Z$ So 2 divides$[(a - b) + (b - c)]$ 2 divides $(a - c)$: Yes relation R is transitive [0] = {0, ± 2, ± 4, ± 6,...}	1 1
22	$y = ae^{2x} + be^{-x}$..................(1) $\frac{dy}{dx} = 2ae^{2x} - be^{-x}$ ....(2) $\frac{d^2y}{dx^2} = 4ae^{2x} + be^{-x}$ ....(3) putting values on LHS $=\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$ $=(4ae^{2x} + be^{-x}) - (2ae^{2x} - be^{-x}) - 2(ae^{2x} + be^{-x})$ $=4ae^{2x} + be^{-x} - 2ae^{2x} + be^{-x} - 2ae^{2x} - 2be^{-x}$ $=0$	1 1

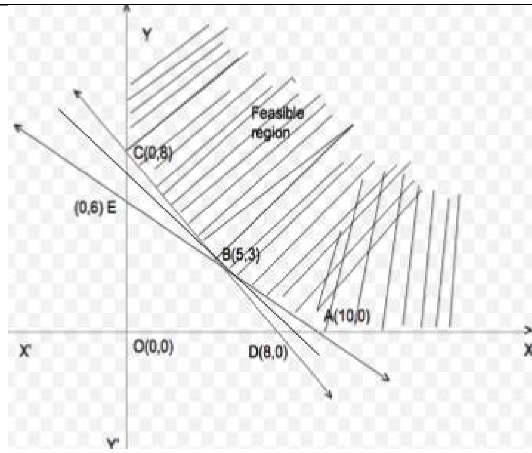
23	$x^2 = 2y$ ………(1) $\Rightarrow 2x\frac{dx}{dt} = 2\frac{dy}{dt}$ (given $\frac{dy}{dt} = \frac{dx}{dt}$) $\Rightarrow 2x\frac{dx}{dt} = 2\frac{dx}{dt}$ $\Rightarrow x = 1$ from (1) $y = \frac{1}{2}$ so point is $\left(1, \frac{1}{2}\right)$	1 1
24	$= (\vec{a} - \vec{b}).\{(\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})\}$ $=(\vec{a} - \vec{b}).\{\vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{c} \times \vec{c} + \vec{c} \times \vec{a}\}$ $=(\vec{a} - \vec{b}).\{\vec{b} \times \vec{c} - \vec{b} \times \vec{a} + \vec{c} \times \vec{a}\}$ ……$(\vec{c} \times \vec{c} = 0)$ $= (\vec{a} - \vec{b}).\{\vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a}\}$ $= \vec{a}.(\vec{b} \times \vec{c}) + \vec{a}.(\vec{a} \times \vec{b}) + \vec{a}.(\vec{c} \times \vec{a}) - \vec{b}.(\vec{b} \times \vec{c}) - \vec{b}.(\vec{a} \times \vec{b}) - \vec{b}.(\vec{c} \times \vec{a})$ $= \vec{a}.(\vec{b} \times \vec{c}) + 0 + 0 - 0 - 0 - \vec{b}.(\vec{c} \times \vec{a})$ $= \vec{a}.(\vec{b} \times \vec{c}) - \vec{b}.(\vec{c} \times \vec{a})$ $=0$ (STP remains same if vectors $\vec{a}, \vec{b}, \vec{c}$ are changed in cyclic order) **OR** $(\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = 0$ $\Rightarrow \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{a} + \vec{b}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} + \vec{c}.\vec{b} + \vec{c}.\vec{c} = 0.$ $\Rightarrow \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + \lvert\vec{c}\rvert^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Rightarrow 3^2 + 5^2 + 7^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ $\Rightarrow 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -(9 + 25 + 49)$ $\Rightarrow (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -\frac{83}{2}$	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$
25	Vector in the direction of first line $\vec{b} = (3\hat{i} + 4\hat{j} + 5\hat{k})$ Vector in the direction of second line $\vec{d} = (4\hat{i} - 3\hat{j} + 5\hat{k})$ Angle θ between two lines is given by $\cos\theta = \frac{\vec{b}.\vec{d}}{\lvert\vec{b}\rvert\lvert\vec{d}\rvert}$ $\cos\theta = \frac{(3\hat{i} + 4\hat{j} + 5\hat{k}).(4\hat{i} - 3\hat{j} + 5\hat{k})}{\lvert(3\hat{i} + 4\hat{j} + 5\hat{k})\rvert\lvert(4\hat{i} - 3\hat{j} + 5\hat{k})\rvert}$ $\Rightarrow \cos\theta = \frac{12 - 12 + 25}{\sqrt{9 + 16 + 25}\sqrt{9 + 16 + 25}}$ $\Rightarrow \cos\theta = \frac{25}{\sqrt{50}\sqrt{50}}$ $\Rightarrow \cos\theta = \frac{1}{2}$ $\Rightarrow \theta = \frac{\pi}{3}$	1 $\frac{1}{2}$ $\frac{1}{2}$
26	$P(A)=\frac{80}{100} = \frac{4}{5}$, $P(B)=\frac{90}{100} = \frac{9}{10}$ P(Agree)=P(Both speaking truth or both telling lie) $=P(AB \text{ or } \bar{A}\bar{B})$	1

	$= P(A)P(B) or P(\bar{A})P(\bar{B})$ $=\left(\frac{4}{5}\right)\left(\frac{9}{10}\right)+\left(\frac{1}{5}\right)\left(\frac{1}{10}\right)$ $=\frac{36+1}{50}=\frac{37}{50}$ $=\frac{74}{100}=74\%$	1
	SECTION C	
27	Let $y = f(x) = \frac{2x+3}{x-3}$ ……………………….(1) Let $x_1, x_2 \in A = R - \{3\}$ Let $f(x_1) = f(x_2)$ $\Rightarrow \frac{2x_1+3}{x_1-3} = \frac{2x_2+3}{x_2-3}$ $\Rightarrow (2x_1+3)(x_2-3) = (2x_2+3)(x_1-3)$ $\Rightarrow (2x_1x_2 - 6x_1 + 3x_2 - 9) = (2x_1x_2 - 6x_2 + 3x_1 - 9)$ $\Rightarrow -6x_1 + 3x_2 = -6x_2 + 3x_1$ $\Rightarrow 9x_1 = 9x_2$ $\Rightarrow x_1 = x_2$ Now $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ so $f(x)$ is one-one For onto $y = \frac{2x+3}{x-3}$ $\Rightarrow xy - 3y = 2x + 3$ $\Rightarrow xy - 2x = 3y + 3$ $\Rightarrow x(y-2) = 3(y+1)$ $\Rightarrow x = \frac{3(y+1)}{(y-2)}$ ……………………….(2) equation (2) is defined for all real values of y except 2 i.e $y \in R - \{2\}$ which is same as given set $B = R - \{2\}$ (co-domain=range) Also $y = f(x)$ $f(x) = f\left(\frac{3(y+1)}{(y-2)}\right)$ $= \frac{2\left[\frac{3(y+1)}{(y-2)}\right]+3}{\frac{3(y+1)}{(y-2)}-3} \left(\text{since } f(x) = \frac{2x+3}{x-3}\right)$ $\frac{2(3y+3)+3(y-2)}{3y+3-3y+6} = \frac{9y}{9} = y$ Thus for every y ∈ B,there exists x ∈ A such that $f(x) = y$ Thus function is onto. Since $f(x)$ is one-one and onto so $f(x)$is invertible. Inverse is given by $x = f^{-1}(y) = \frac{3(y+1)}{(y-2)}$	$\frac{1}{2}$ 1 $1\frac{1}{2}$ 1
28	$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ Let $x = \sin A$, $y = \sin B$ $\sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$ $\cos A + \cos B = a(\sin A - \sin B)$ $\Longrightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\Longrightarrow \cos\left(\frac{A-B}{2}\right) = a\sin\left(\frac{A-B}{2}\right)$	$\frac{1}{2}$ 1

	$\Rightarrow \cot\left(\frac{A-B}{2}\right) = a$ $\Rightarrow \frac{A-B}{2} = \cot^{-1} a$ $\Rightarrow A - B = 2\cot^{-1} a$ $\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$ differentiating w.r.t. x $\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ **OR**	1 1 $\frac{1}{2}$
	$x = a(\cos 2\theta + 2\theta \sin 2\theta)$ $\Rightarrow \frac{dx}{d\theta} = a(-2\sin 2\theta + 2\sin 2\theta + 4\theta\cos 2\theta)$ $\Rightarrow \frac{dx}{d\theta} = a(4\theta\cos 2\theta)$……………..(1) $y = a(\sin 2\theta - 2\theta\cos 2\theta)$ $\Rightarrow \frac{dy}{d\theta} = a(2\cos 2\theta + 4\,\theta\sin 2\theta - 2\cos 2\theta)$ $\Rightarrow \frac{dy}{d\theta} = a(4\,\theta\sin 2\theta)$………………(2) using (1)and (2) $\Rightarrow \frac{dy}{dx} = \frac{a(4\,\theta\sin 2\theta)}{a(4\theta\cos 2\theta)}$ $\Rightarrow \frac{dy}{dx} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$ Differentiating again with respect to x, we get $\Rightarrow \frac{d^2y}{dx^2} = 2\,sec^2 2\theta.\frac{d\theta}{dx}$ $\Rightarrow \frac{d^2y}{dx^2} = 2\,sec^2 2\theta.\frac{1}{a(4\theta\cos 2\theta)}$ $\left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{8}} = 2\,sec^2\frac{\pi}{4}.\frac{1}{a\left(4\frac{\pi}{8}\cos\frac{\pi}{4}\right)}$ $= \frac{8\sqrt{2}}{\pi a}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1
29	$x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$ $\Rightarrow x\frac{dy}{dx} = y + \sqrt{x^2 + y^2}$ $\Rightarrow \frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$………………………………(1) let y = vx differentiating with w.r.t. x $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ put in (1)	1

	$\Rightarrow v + x\frac{dv}{dx} = \frac{vx + \sqrt{x^2 + v^2x^2}}{x}$ $\Rightarrow v + x\frac{dv}{dx} = \frac{x(v + \sqrt{1 + v^2})}{x}$ $\Rightarrow x\frac{dv}{dx} = v + \sqrt{1 + v^2} - v$ $\Rightarrow x\frac{dv}{dx} = \sqrt{1 + v^2}$ $\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$ integrating both sides $\Rightarrow \int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$ $\Rightarrow \log\left(v + \sqrt{1 + v^2}\right) = \log x + \log c$ $\Rightarrow \log\left(v + \sqrt{1 + v^2}\right) = \log cx$ $\Rightarrow \left(v + \sqrt{1 + v^2}\right) = cx$ $\Rightarrow \left(\frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2}\right) = cx$ $\Rightarrow y + \sqrt{x^2 + y^2} = cx^2$	1 $1\frac{1}{2}$ $\frac{1}{2}$
30	Consider $I=\int_1^3 \lvert x^2 - 2x\rvert dx$ $\lvert x^2 - 2x\rvert = \begin{cases} -(x^2 - 2x) & when\ 1 \le x < 2 \\ (x^2 - 2x) & when\ 2 \le x \le 3 \end{cases}$ $I=\int_1^2 \lvert x^2 - 2x\rvert dx + \int_2^3 \lvert x^2 - 2x\rvert dx$ $I=\int_1^2 -(x^2 - 2x)\,dx + \int_2^3 (x^2 - 2x)dx$ $I=-\left[\frac{x^3}{3} - x^2\right]_1^2 + \left[\frac{x^3}{3} - x^2\right]_2^3$ $I=-\left(-\frac{4}{3} + \frac{2}{3}\right) + \left(\frac{4}{3}\right)$ $I=\frac{6}{3} = 2$	1 1 1 1
31	Let X denotes the smaller of the two numbers obtained So X can take values 1,2,3,4,5,6 P(X=1 is smaller number) P(X=1)=$\frac{6}{7_{C_2}} = \frac{6}{21} = \frac{2}{7}$ (Total cases when two numbers can be selected from first 7 numbers are 7_{C_2}) P(X=2)=$\frac{5}{7_{C_2}} = \frac{5}{21}$ P(X=3)=$\frac{4}{7_{C_2}} = \frac{4}{21}$ P(X=4)=$\frac{3}{7_{C_2}} = \frac{3}{21} = \frac{1}{7}$ P(X=5)=$\frac{2}{7_{C_2}} = \frac{2}{21}$ P(X=6)=$\frac{1}{7_{C_2}} = \frac{1}{21}$ <table><tr><td>x_i</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr></table>	$\frac{1}{2}$ 2

		p_i	$\frac{6}{21}$	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{3}{21}$	$\frac{2}{21}$	$\frac{1}{21}$ | $p_i x_i$: $\frac{6}{21}$, $\frac{10}{21}$, $\frac{12}{21}$, $\frac{12}{21}$, $\frac{10}{21}$, $\frac{6}{21}$	$\frac{1}{2}$
	Mean $=\sum p_i x_i = \frac{6}{21}+\frac{10}{21}+\frac{12}{21}+\frac{12}{21}+\frac{10}{21}+\frac{6}{21}=\frac{56}{21}=\frac{8}{3}$ **OR**	1
	Let E_1 = event of selecting a two headed coin E_1 = event of selecting a biased coin, which shows 75% times Head E_3 =event of selecting a unbiased coin. A = event that tossed coin shows head. $\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$. $P\left(A/_{E_1}\right) = P$(coin showing head given that it is two headed coin) = 1 $P\left(A/_{E_2}\right) = P$(coin showing head given that it is a biased coin) $= \frac{75}{100} = \frac{3}{4}$ $P\left(A/_{E_3}\right) = P$(coin showing head given that it is unbiased coin) $= \frac{1}{2}$ By Bayes theorem P(gettingtwo headedcoin when it is known that it shows Head) $P\left(E_1/_{A}\right) = \frac{P(E_1)P\left(A/_{E_1}\right)}{P(E_1)P\left(A/_{E_1}\right)+P(E_2)P\left(A/_{E_2}\right)+P(E_1)P\left(A/_{E_3}\right)}$ $= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{1}{2}} = \frac{\frac{1}{3}\times 1}{\frac{1}{3}\left(1+\frac{3}{4}+\frac{1}{2}\right)} = \frac{\frac{1}{3}}{\frac{1}{3}\times\frac{9}{4}} = \frac{4}{9}$ Required probability$=\frac{4}{9}$	1 $\frac{1}{2}$ 1 $1\frac{1}{2}$
32	Let tailor A works for x days and tailor B works for y days Objective function : To minimize labour cost $Z = 150x + 200y$ (in ₹) Subject to constraints $6x + 10y \geq 60$ i.e. $3x + 5y \geq 30$ $4x + 4y \geq 32$ i.e. $x + y \geq 8$ $x \geq 0, y \geq 0$ consider equations to draw the graph and then we will shade feasible region $3x + 5y = 30$ $x + y = 8$	$\frac{1}{2}$ $1\frac{1}{2}$



1

corner points of feasible region are A(10,0),B(5,3) and C(0,8)
Value of Z at these corner points

Point	$Z = 150x + 200y$ (in ₹)
A(10,0)	=1500+0=1500
B(5,3)	=750+600=1350 (minimum)
C(0,8)	=0+1600=1600

So minimum value of Z is ₹1350 when tailor A works for 5 days and tailor B works for 3 days.
To check draw $150x + 200y < 1350$ i.e $3x + 4y < 27$
As there is no region common with feasible region so minimum value is ₹1350 — 1

SECTION D

33

$$\text{LHS}=\begin{vmatrix}(y+z)^2 & x^2 & x^2\\ y^2 & (z+x)^2 & y^2\\ z^2 & z^2 & (x+y)^2\end{vmatrix}$$

$Apply\ C_2 \to C_2 - C_1,\quad C_3 \to C_3 - C_1$

$$=\begin{vmatrix}(y+z)^2 & x^2-(y+z)^2 & x^2-(y+z)^2\\ y^2 & (z+x)^2-y^2 & 0\\ z^2 & 0 & (x+y)^2-z^2\end{vmatrix}$$ 1

$$=\begin{vmatrix}(y+z)^2 & (x+y+z)(x-y-z) & (x+y+z)(x-y-z)\\ y^2 & (z+x+y)(z+x-y) & 0\\ z^2 & 0 & (x+y+z)(x+y-z)\end{vmatrix}$$ 1

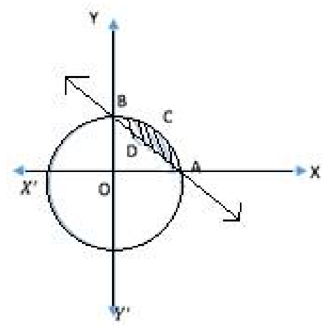
Taking $(x+y+z)$common from C_2as well as C_3

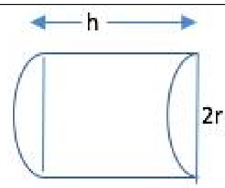
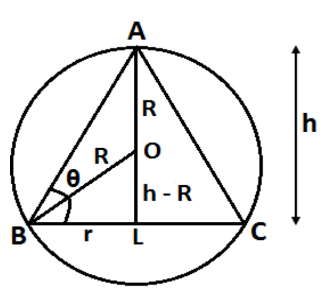
$$=(x+y+z)^2\begin{vmatrix}(y+z)^2 & (x-y-z) & (x-y-z)\\ y^2 & (z+x-y) & 0\\ z^2 & 0 & (x+y-z)\end{vmatrix}$$ 1

Apply $R_1 \to R_1 - R_2 - R_3$

$$=(x+y+z)^2\begin{vmatrix}2yz & -2z & -2y\\ y^2 & (z+x-y) & 0\\ z^2 & 0 & (x+y-z)\end{vmatrix}$$

	Apply $C_2 \to y\,C_2$ and $C_3 \to z\,C_3$ $=\frac{(x+y+z)^2}{yz}\begin{vmatrix} 2yz & -2yz & -2yz \\ y^2 & (yz+yx-y^2) & 0 \\ z^2 & 0 & (zx+zy-z^2) \end{vmatrix}$ Apply $C_2 \to C_2 + C_1$ and $C_3 \to C_3 + C_1$ $=\frac{(x+y+z)^2}{yz}\begin{vmatrix} 2yz & 0 & 0 \\ y^2 & (yz+yx) & y^2 \\ z^2 & z^2 & (zx+zy) \end{vmatrix}$ expanding along R_1 $=\left(\frac{(x+y+z)^2}{yz}\right)2yz[(yz+yx)(zx+zy)-y^2z^2]$ $=2(x+y+z)^2[xyz^2+x^2yz+xy^2z+y^2z^2-y^2z^2]$ $=2xyz(x+y+z)^2(x+y+z)$ $=2xyz(x+y+z)^3$ **OR**	1 1 1
	** $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ $\lvert A\rvert = 2(-2)-3(2-0)+4(1-0) = -6 \neq 0$ $\therefore A^{-1}$ exists Cofactors $A_{11}=-2 \quad A_{12}=-2 \quad A_{13}=1$ $A_{21}-2 \quad A_{22}=4 \quad A_{23}=-2$ $A_{31}=4 \quad A_{32}=4 \quad A_{33}=-5$ $Adj\,A = \begin{bmatrix} -2 & -2 & 1 \\ -2 & 4 & -2 \\ 4 & 4 & -5 \end{bmatrix}'$ $Adj\,A = \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$ $A^{-1} = \frac{Adj\,A}{\lvert A\rvert} = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$ System of equations can be written as $AX = B$ Where $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$ Now $AX = B$ $\Rightarrow X = A^{-1}B$ $\Rightarrow X = \frac{1}{-6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}\begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$	1 2 1 $\frac{1}{2}$

	$\Rightarrow X = \frac{1}{-6}\begin{bmatrix} -34-6+28 \\ -34+12+28 \\ 17-6-35 \end{bmatrix}$ $\Rightarrow X = \frac{1}{-6}\begin{bmatrix} -12 \\ 6 \\ -24 \end{bmatrix}$ $\Rightarrow X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $\Rightarrow x = 2, \quad y = -1, \quad z = 4$	$1\frac{1}{2}$
34	$x^2 + y^2 = 1$………………..(1) $x + y = 1$…………………..(2) solving (1) and(2) $x^2 + (1-x)^2 = 1$ $x^2 + x^2 - 2x + 1 = 1$ $2x^2 - 2x = 0$ $2x(x-1) = 0$ $x = 0 \; or \; x = 1$  Required area = shaded area ACBDA =area(OACBO)– area(OADBO) $=\int_0^1 (y_{circle} - y_{line})dx$ $\int_0^1 \sqrt{1-x^2}dx - \int_0^1 (1-x)dx$ $=\left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x - \frac{x^2}{2}\right]_0^1$ $\left[\left(0 + \frac{1}{2}.\frac{\pi}{2}\right) - 0\right] - \left[\left(1 - \frac{1}{2}\right)\right]$ $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ square units	1 1 1 $1\frac{1}{2}$ $1\frac{1}{2}$
35	Let r be the radius and $h$be the height of half cylinder Volume $=\frac{1}{2}\pi r^2 h = V$(constant)……………..(1)	$\frac{1}{2}(fig)$

	 	$1\frac{1}{2}$
	Total surface area of half cylinder is	
	S=$2\left(\frac{1}{2}\pi r^2\right) + \pi rh + 2rh$…………………(2)	
	From (1) put the value of h in (2)	1
	S=$(\pi r^2) + \pi r\left(\frac{2V}{\pi r^2}\right) + 2r\left(\frac{2V}{\pi r^2}\right)$	
	S=$(\pi r^2) + \left(\frac{1}{r}\right)\left[\frac{4V}{\pi} + 2V\right]$	
	$\frac{ds}{dr} = (2\pi r) + \left(\frac{-1}{r^2}\right)\left[\frac{4V}{\pi} + 2V\right]$ ……………..(3)	
	For maxima/minima $\frac{ds}{dr} = 0$	
	$\Rightarrow (2\pi r) + \left(\frac{-1}{r^2}\right)\left[\frac{4V}{\pi} + 2V\right] = 0$	1
	$\Rightarrow (2\pi r) = \left(\frac{1}{r^2}\right)\left[\frac{4V + 2V\pi}{\pi}\right]$	
	$\Rightarrow \pi r^3 = V\left[\frac{2 + \pi}{\pi}\right]$	1
	$\Rightarrow$ V=$\frac{\pi^2 r^3}{\pi+2}$………………..(4)	
	From (1) and (4)	
	$\Rightarrow \frac{1}{2}\pi r^2 h = \frac{\pi^2 r^3}{\pi + 2}$	1
	$\Rightarrow \frac{h}{2r} = \frac{\pi}{\pi + 2}$	
	$\Rightarrow height{:}diameter = \pi{:}\pi + 2$	
	Differentiating (3) with respect to r	
	$\frac{d^2s}{dr^2} = (2\pi) + \left(\frac{2}{r^3}\right)\left[\frac{4V}{\pi} + 2V\right]$=positive (as all quantities are +ve)	
	so S is minimum when	
	$height{:}diameter = \pi{:}\pi + 2$	
	OR	
	Let 2r be the base and h be the height of triangle ,which is inscribed in a circle of radius R	
	Area of triangle=$\frac{1}{2}(base)(height)$	
	A $=\frac{1}{2}(2r)(h) = rh$…………(1)	1
	 	$\frac{1}{2}(fig)$
	Area being positive quantity, A will be maximum or minimum if A^2 is	

	maximum or minimum. $Z = A^2 = r^2h^2$……………………..(2) Now In triangle OLB $BL^2 = OB^2 - OL^2$ In ΔOBD $Z = A^2 = r^2 h^2$ $r^2 = R^2 - (h - R)^2 \Rightarrow r^2 = 2hR - h^2$ $put\ in$ (2) $Z = h^2(2hR - h^2)$ $\Rightarrow Z = (2h^3R - h^4)$ $\Rightarrow \frac{dZ}{dh} = 6h^2R - 4h^3$……………(3) For maxima/minima $\frac{dZ}{dh} = 0$ $\Rightarrow 6h^2R - 4h^3 = 0$ $\Rightarrow 6R = 4h(h \neq 0)$ $\Rightarrow h = \frac{3R}{2}$ differentiating (3) w.r.t. h $\Rightarrow \frac{d^2Z}{dh^2} = 12hR - 12h^2$ $\Rightarrow \left.\frac{d^2Z}{dh^2}\right]_{h=\frac{3R}{2}} = 12\left(\frac{3R}{2}\right)R - 12\left(\frac{3R}{2}\right)^2$ $= 18R^2 - 27R^2 = -ve$ so Z=A^2 is maximum when $h = \frac{3R}{2}$ $\Rightarrow A$ is maximum when $h = \frac{3R}{2}$ when $h = \frac{3R}{2}$, $r^2 = 2hR - h^2 = 2R.\frac{3R}{2} - \left(\frac{3R}{2}\right)^2$ $r^2 = \frac{3R^2}{4}$ $r = \frac{\sqrt{3}R}{2}$ $\tan\theta = \frac{h}{r} = \frac{\frac{3R}{2}}{\frac{\sqrt{3}R}{2}} = \sqrt{3}\theta = \frac{\pi}{3}$ Triangle ABC is equilateral triangle	$\frac{1}{2}$ 1 1 1 1
36	Let $P(x, y, z)$ be any point on the plane in which $A(2,1,2)$ and $B(4, -2,1)$lie. $\therefore \overrightarrow{AP}\ and\ \overrightarrow{AB}$lie on required plane. Also required plane is perpendicular to given plane $\vec{r}.(\hat{\imath} - 2\hat{k}) = 5$ ∴normal to given plane $\overrightarrow{n_1} = (\hat{\imath} - 2\hat{k})$ lie on required plane. $\Rightarrow \overrightarrow{AP}, \overrightarrow{AB}and\overrightarrow{n_1}$ are coplanar. Where$\overrightarrow{AP} = (x - 2)\hat{\imath} + (y - 1)\hat{\jmath} + (z - 2)\hat{k}$ $\overrightarrow{AB}$ = =$2\hat{\imath} - 3\hat{\jmath} - \hat{k}$ ⇒Scaler triple product $[\overrightarrow{AP}\quad \overrightarrow{AB}\quad \overrightarrow{n_1}] = 0$	1 1 1

	$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z-2 \\ 2 & -3 & -1 \\ 1 & 0 & -2 \end{vmatrix} = 0$$ $\Rightarrow (x-2)(6-0)-(y-1)(-4+1)+(z-2)(0+3)=0$ $\Rightarrow 6x-12+3y-3+3z-6=0$ $\Rightarrow 2x+y+z=7$………………….(1) Line passing through points $L(3,4,1)$and $M(5,1,6)$ is $\Rightarrow \frac{x-3}{2}=\frac{y-4}{-3}=\frac{z-1}{5}=\lambda$………….(2) ⇒General point on the line is $Q(2\lambda+3,-3\lambda+4,5\lambda+1)$ As line (2) crosses plane (1) so point Q should satisfy equation(1) $$\therefore 2(2\lambda+3)+(-3\lambda+4)+(5\lambda+1)=7$$ $$4\lambda+6-3\lambda+4+5\lambda+1=7$$ $$6\lambda=-4$$ $$\lambda=-\frac{2}{3}$$ $Q(-\frac{4}{3}+3,2+4,-\frac{10}{3}+1)=Q\left(\frac{5}{3},6,-\frac{7}{3}\right)$	1 1 1