Quarterly Common Exam., September - 2017

Time : 3-00 Hours] Part-III MATHEMATICS [Maximum Marks : 200

Instructions:- 1. Check the question paper for fairness of printing .If there is any lack of fairness, inform the Hall Supervisor immediately.
2. Use Black or Blue ink to write and pencil to draw diagrams.

Part-A

$40 \times 1 = 40$

Note : (i) All the questions are compulsory
(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. The curve $y^2 = (x-1)(x-2)^2$ has (1) two loops between $x=0$ and $x=2$ (2) one loop between $x=0$ and $x=1$ (3) one loop between $x=1$ and $x=2$ (4) no loop
2. If $\vec{a}$, $\vec{b}$ are two unit vectors and θ is the angle between them, then $(\vec{a}+\vec{b})$ is (1) $\theta=\pi/3$ (2) $\theta=\pi/4$ (3) $\theta=\pi/2$ (4) $\theta=2\pi/3$
3. The non-parametric vector equation of a plane passing through the points whose p.v's are a, b and parallel to $\vec{v}$ is
(1) $[\vec{r}-\vec{a}, \vec{b}-\vec{a}, \vec{v}]=0$ (2) $[\vec{r}-\vec{b}, -\vec{a}, \vec{v}]=0$ (3) $[\vec{a}\,\vec{b}, \vec{v}]=0$ (4) $[\vec{r}, \vec{a}, \vec{b}]=0$
4. The vectors $2\vec{i}+3\vec{j}+4\vec{k}$ and $a\vec{i}+b\vec{j}+c\vec{k}$ are perpendicular when
(1) $a=2, b=3, c=-4$ (2) $a=4, b=4, c=5$ (3) $a=4, b=4, c=-5$ (4) $a=-2, b=3, c=4$
5. An asymptote to the curve $y^2(a+2x) = x^2(3a-x)$ is
(1) $x=3a$ (2) $x=-a/2$ (3) $x=a/2$ (4) $x=0$
6. The locus of the foot of perpendicular from the focus on any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is
(1) $x^2+y^2=a^2-b^2$ (2) $x^2+y^2=a^2$ (3) $x^2+y^2=a^2+b^2$ (4) $x=0$
7. The following two lines $\frac{x-1}{2}=\frac{y-1}{-1}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z-1}{2}$ are
(1) parallel (2) intersecting (3) skew (4) perpendicular
8. The percentage error in the 11th root of the number 28 is approximately ----- times the percentage error in 28 (1) 1/28 (2) 1/11 (3) 11 (4) 28
9. The work done by the force $\vec{F} = a\vec{i}+\vec{j}+\vec{k}$ in moving the point of application from (1, 1, 1) to (2, 2, 2) along a straight line is given to be 5 units. The value of a is
(1) –3 (2) 3 (3) 8 (4) –8
10. The asymptotes to the hyperbola $36y^2-25x^2+900=0$ are
(1) $y=\pm\frac{6}{5}x$ (2) $y=\pm\frac{5}{6}x$ (3) $y=\pm\frac{36}{25}x$ (4) $y=\pm\frac{25}{36}x$
11. If $\overrightarrow{PR} = 2\vec{i}+\vec{j}+\vec{k}, \overrightarrow{QS} = -\vec{i}+3\vec{j}+2\vec{k}$ then the area of the quadrilateral $PQRS$ is
(1) $5\sqrt{3}$ (2) $10\sqrt{3}$ (3) $\frac{5\sqrt{3}}{2}$ (4) $\frac{3}{2}$
12. The value of $[\vec{i}+\vec{j}, \vec{j}+\vec{k}, \vec{k}+\vec{i}]$ is equal to
(1) 0 (2) 1 (3) 2 (4) 4
13. The point of intersection of the tangents at $t_1 = t$ and $t_2 = 3t$ to the parabola $y^2 = 8x$ is
(1) $(6t^2, 8t)$ (2) $(8t, 6t^2)$ (3) $(t^2, 4t)$ (4) $(4t, t^2)$
14. If $u = \sin^{-1}\left(\frac{x^4+y^4}{x^2+y^2}\right)$ and $f = \sin u$ then f -is a homogenous function of degree
(1) 0 (2) 1 (3) 2 (4) 4
15. The tangents at the end of any focal chord to the parabola $y^2 = 12x$ intersect on the line
(1) $x-3=0$ (2) $x+3=0$ (3) $y+3=0$ (4) $y-3=0$
16. If $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$ for non-coplanar vectors $\vec{a}+\vec{b}+\vec{c}$ vectors $\vec{a}, \vec{b}, \vec{c}$ then
(1) $\vec{a}$ parallel to $\vec{b}$ (2) $\vec{b}$ parallel to $\vec{c}$ (3) $\vec{c}$ parallel to $\vec{a}$ (4) $\vec{a}+\vec{b}+\vec{c}=0$
17. The sum of the distances of any point on the ellipse $4x^2+9y^2=36$ from $(\sqrt{5}, 0)$ and $(-\sqrt{5}, 0)$ is (1) 4 (2) 8 (3) 6 (4) 18
18. If the normal to the rectangular hyperbola $xy = c^2$ at 't_1' meets the curve again at 't_2' then $t_1^3 t_2 =$ (1) 1 (2) 0 (3) –1 (4) –2
19. The point of intersection of the line $\vec{r}(\vec{i}-\vec{k})+t(3\vec{i}+2\vec{j}+7\vec{k})$ and the plane $\vec{r}.(\vec{i}+\vec{j}-\vec{k})=8$ is
(1) (8, 6, 22) (2) (–8, –6, –22) (3) (4, 3, 11) (4) (–4, –3, –11)

20. The directrices of the hyperbola $x^2 - 4(y-3)^2 = 16$ are

(1) $y = \pm\frac{8}{\sqrt{5}}$ (2) $x = \pm\frac{8}{\sqrt{5}}$ (3) $y = \pm\frac{\sqrt{5}}{8}$ (4) $x = \pm\frac{\sqrt{5}}{8}$

21. If $A+iB = (a_1+ib_1)(a_2+ib_2)(a_3+ib_3)$ then A^2+B^2 is

(1) $a_1^2+b_1^2+a_2^2+b_2^2+a_3^2+b_3^2$ (2) $(a_1+b_2+a_3)^2+(b_1+b_2+b_3)^2$

(3) $(a_1^2+b_1^2)(a_2^2+b_2^2)(a_3^2+b_3^2)$ (4) $(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$

22. If I is the unit matrix of order n, where $k \neq 0$ is a constant then $adj(KI) =$

(1) $k^n(adj\ I)$ (2) $k(adj\ I)$ (3) $k^2(adj\ I)$ (4) $k^{n-1}(adj\ I)$

23. The value of $i+i^{22}+i^{23}+i^{24}+i^{25}$ (1) i (2) $-i$ (3) 1 (4) -1

24. The value of $\left[\frac{-1+i\sqrt{3}}{2}\right]^{100}+\left[\frac{-1+i\sqrt{3}}{2}\right]^{100}$ is (1) 2 (2) 0 (3) -1 (4) 1

25. If z represents a complex number then $\arg(z)+\arg(\bar{z})$ is (1) $\pi/4$ (2) $\pi/2$ (3) 0 (4) π

26. The function $f(x) = x^2 - 5x + 4$ is increasing

(1) $(-\infty, 1)$ (2) $(4, \infty)$ (3) $(1, 4)$ (4) everywhere

27. The system of equations $ax+y+z=0.x+by+z=0, z+y+cz=0$ has a non-trivial solution then

$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c} =$ (1) 1 (2) 2 (3) -1 (4) 0

28. If the gradient of a curve changes from positive just before P to negative just after then p in a

(1) minimum point (2) maximum point (3) inflexion point (4) discontinuous point

29. The statement If f has a local extremum (minimum or maximum) at 'c' and if $f'(c)$ exists then $f'(c) = 0$ is (1) the extreme value theorem (2) Fermat's theorem

(3) Law of mean (4) Rolle's theorem

30. If $(m-5)+i(n+4)$ is the complex conjugate at $(2m+3)+i(3n-2)$ then (n, m) are

(1) $(-½, -8)$ (2) $(-½, 8)$ (3) $(½, -8)$ (4) $(½, 8)$

31. If the rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2 then λ is

(1) 1 (2) 2 (3) 3 (4) any real number

32. If A is a square matrix of order n then $|adj\ A| =$ (1) $|A|^2$ (2) $|A|^n$ (3) $|A|^{n-1}$ (4) $|A|$

33. If ω is the nth root of unity then

(1) $1+\omega^2+\omega^4+\ldots = \omega+\omega^3+\omega^5+\ldots$ (2) $\omega^n = 0$ (3) $\omega^n = 1$ (4) $\omega = \omega^{n-1}$

34. The number of values of $(\cos\theta + i\sin\theta)^{p/q}$ where p and q are non-zero integers prime to each other is (1) p (2) q (3) $p+q$ (4) $p-q$

35. Velocity v of a particle moving along a straight line when at a distance x from the origin is given by $a+bv^2 = x^2$ where a and b are constants then the acceleration is

(1) b/x (2) a/x (3) x/b (4) x/a

36. If A and B are matrices conformable to multiplication then $(AB)^T$ is

(1) A^TB^T (2) B^TA^T (3) AB (4) BA

37. In the system of 3 linear equations with three unknowns if $\Delta = 0$ and all 2×2 minors of $\Delta = 0$ and atleast one 2×2 minor of Δ_x or Δ_y or Δ_z in non zero, then the system is

(1) consistent (2) inconsistent (3) consistent and the system reduces to two equations

(4) consistent and the system reduces to a single equation

38. If the normal to the curve $x^{2/3}+y^{2/3} = a^{2/3}$ makes are angele θ with the x axis then the slope of the normal is (1) $-\cot\theta$ (2) $\tan\theta$ (3) $-\tan\theta$ (4) $\cot\theta$

39. The 'c' of Lagranges mean value theorem for the function $f(x) = x^2+2x-1$ $a=0, b=1$ is

(1) -1 (2) 1 (3) 0 (2) ½

40. The value of $\lim_{x\to0}\frac{a^x-b^x}{c^x-d^x}$ (1) ∞ (2) 0 (3) $\log\frac{ab}{cd}$ (4) $\frac{\log\frac{a}{b}}{\log\frac{c}{d}}$

Part - B $10\times 6=60$

Note : (i) Answer any ten questions
(ii) Question No. 55 is compulsory and choose nine questions from remaining

41. Find the adjoint of matrix $A=\begin{bmatrix}1 & 2\\ 3 & -5\end{bmatrix}$ and verify the result $A(adj\ A)=(adj\ A)=|A|I$

42. Find the rank of the matrix $\begin{bmatrix}1 & -2 & 3 & 4\\ -2 & 4 & -1 & -3\\ -1 & 2 & 7 & 6\end{bmatrix}$

43. With usual notation prove that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.

44. Show that the diameter of a sphere subtends a right angle at a point on the surface.

45. (i) For any vector $\vec{a}$ Prove that $\vec{i}\times(\vec{a}\times\vec{i})+\vec{j}\times(\vec{a}\times\vec{j})+\vec{k}\times(\vec{a}\times\vec{k})=2\vec{a}$.

(ii) Find the angle between the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{-2}$ and the plane $3x+4y+z+5=0$

46. P represents the variable complex number z. Find the locus of P if $\mathrm{Re}\left(\frac{\bar{z}+1}{z-i}\right)=0$

47. State and prove the triangle inequality of complex numbers.

48. Solve $6x^4-25x^3+32x^2+3x-10=0$ given that one of the roots is $2-i$

49. Find the equation of the rectangular hyperbola which has its centre at $(2,1)$ of its asymptotes $3x-y-5=0$ and which passes through the point $(1,-1)$

50. (i) Find the equation of the parabola whose vertex is $(1,2)$ and the equation of the latus rectum is $x=3$
(ii) Find the equation of chord of contact of tangents from the point $(2,4)$ to the ellipse $2x^2+5y^2=20$

51. Resistance to motion, F of a moving vehicle is given by $F=\frac{5}{x}+100x$. Determine the the minimum value of resistance.

52. Apply Rolle's theorem to find points on curve $y=-1+\cos x$ where the tangent is parallel to x axis in $[0,2\pi]$

53. Prove that the inequality $(1+x)^n>1+nx$ is true whenever $x>0$ and $n>1$

54. The time of swing T of a pendulum of length 'l' is given by $T=k\sqrt{l}$ where k is a constant. Determine the percentage error in the time of swing if the length of pendulum changes from 32.1 cm. to 32.0 cm.

55. a) Solve the system of equations : $2x+3y=8;4x+6y=16$, using determinant method (or)

b) If $U=xy^2\sin\left(\frac{x}{y}\right)$ using Euler's theorem prove that $x\frac{\partial x}{\partial y}+y\frac{\partial u}{\partial y}=3u$

Part - C $10 \times 10 = 100$

Note : (i) Answer any ten questions
(ii) Question No. 70 is compulsory and choose nine questions from remaining

56. Solve by matrix inversion method : $2x - y + z = 7, 3x + y - 5z = 13, x + y + z = 5$

57. For what value of μ the equations $x + y + 3z = 0, 4x + 3y + \mu z = 0, 2x + y + 2z = 0$ have a (i) trivial solution (ii) non-trivial solution

58. Attitudes of a triangle are concurrent - prove by vector method.

59. Show that the lines $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{z-2}{1} = \frac{y-1}{2} = \frac{-z-1}{1}$ intersect and find their point of intersection.

60. If α and β are the roots of the equation $x^2 - 2px + (p^2 + q^2) = 0$ and $\tan\theta = \frac{q}{y+p}$

show that $\frac{(y+\alpha)^n + (y+\beta)^n}{\alpha + \beta} = q^{n-1} \frac{\sin n\theta}{\sin^n \theta}$

61. Solve the equation $x^9 + x^5 - x^4 - 1 = 0$.

62. Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3 m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

63. Find the eccentricity, centre, foci and vertices of the hyperbola
$x^2 - 4y^2 + 6x + 16y - 11 = 0$ and draw the diagram.

64. Prove that the line $3x - y - 5 = 0$ touches the hyperbola $2x^2 - 3y^2 = 6$ and find its point of contact.

65. Gravel is being dumped from a conveyor belt at a rate of 30 ft3/min and its coarsened such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

66. Find the absolute maximum and absolute minimum values of
$f(x) = x - 2\sin x, 0 \le x \le 2\pi$

67. Find the intervals of concavity and the points of inflection of the function
$y = 12x^2 - 2x^3 - x^4$

68. If $w = u^2 e^v$ where $u = \frac{x}{y}$ and $v = y \log x$, find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$

69. Trace the curve : $y = x^3$

70. a) Derive the equation of the plane in the intercept form (or)
b) The arch of a bridge is in the shape of a semi-ellipse having a horizontal span of 40 ft and 16 ft high at the centre. How high is the arch, 9 ft from the right or left of the centre ?