Jain College, Jayanagar
II PUC Mock Paper I - Jan 2020
Subject: II PUC Mathematics (35)

Duration: 3 hours 15 minutes **Max. Marks: 100**

PART-A

I. Answer all the TEN questions: **10X1=10**

1. Find whether operation * on Q defined by a*b = a+ab is communication or not?
2. Find the principal value branch of $\tan^{-1}x$.
3. Define scalar matrix.
4. If $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ find value of x.
5. If y = cos ($\sqrt{x}$). Find dy/dx.
6. Evaluate $\int e^x (\tan^{-1} x + \frac{1}{1+x^2}) dx$.
7. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$
8. If a line has directions -18, 12, -4 then find directions
9. Define corner points in LPP.
10. A fair die is rolled if E = {1, 2, 4, 6} F ={1,3} find P (E/F)

PART-B

II. Answer any TEN questions: **10X2=20**

11. Show that f: A $\rightarrow$ B & g : B$\rightarrow$C are one-one then g . f : A $\rightarrow$ C is also one-one.
12. Prove that $\tan^{-1}\left[\frac{1}{2}\right] + \tan^{-1}\left[\frac{2}{11}\right] = \tan^{-1}\left[\frac{3}{4}\right]$
13. Write is simplest from : $\tan^{-1}\left[\frac{\cos x - \sin x}{\cos x + \sin x}\right]$, 0< x < π,
14. Find the area of the triangle whose vertices are (1,0), (6,0) & (4,3) using determinant.
15. If $\sqrt{x} + \sqrt{y} = 10$ then prove that $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$
16. Find $\frac{dy}{dx}$ *if* $y = \frac{1}{\sec^{-1}(2x^2 - 1)}$.
17. Find the slope of the tangent to the curve y = x^2 – x + 1 at the point whose x co-ordinate is 2.
18. Evaluate $\int \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right) dx.$
19. Evaluate $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx.$
20. Determine the order and degree of $(y'')^2 + (y'')^3 + (y')^4 + y = 0$
21. Find the area of the triangle whose adjacent sides are determined by the vectors.
    $\vec{a} = -2\hat{i} - 5\hat{k}$ & $\vec{b} = \hat{i} - 2\hat{j} - \hat{k}$.
22. Prove that $\left[\vec{a}\ \vec{b}\ \vec{c}+\vec{d}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right] + \left[\vec{a}\ \vec{b}\ \vec{d}\right]$.
23. Find the distance of point (3,-2,1) from the plane 2x-y+2z+3=0.

24. Find the probability distribution of numbers of heads in two tosses of a coin.

PART-C

III. Answer any TEN questions: 10X3=30

25. Show that the relation R in the set Z of integers given by R = {(x,y): $|x-y|$ *is even*} is an equivalence relation.
26. Solve for x : $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$.
27. If A & B are invertible matrices of same order then, show that $(AB)^{-1} = B^{-1} A^{-1}$.
28. If x = a (θ+sinθ) y = a(1-cosθ). Prove that $\frac{dy}{dx}=\tan\left(\frac{\theta}{2}\right)$.
29. Verify mean value theorem if $f(x) = x^2 - 4x - 3$ in the interval [1,4].
30. Find the points of local maxima and minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5$.
31. If $y = (x + 3)^2 (x+4)^3 (x+5)^4$, Find dy/dx.
32. Evaluate $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$.
33. Evaluate $\int x\tan^{-1}x\ dx$.
34. Find the area under the given curves and given lines $y = x^4$, x = 1, x = 5 and x –axis.
35. Solve $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$.
36. Show that the four points A, B, C & D with position vector $4\hat{i}+5\hat{j}+\hat{k},\ -\hat{j}-\hat{k},\ 3\hat{i}+9\hat{j}+4\hat{k}$ & $4(-\hat{i}+\hat{j}+\hat{k})$ respectively are coplanar.
37. Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ & the plane 10x + 2y – 11 Z = 3
38. An urn contains 5 red & 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Morevers, 2 additional balls of the colour drawn are put in the urn & then a ball is drawn at random. What is the probability that 2^{nd} ball is red

PART-D

IV. Answer any SIX of the following: 6X5=30

39. Consider f : R +→ [4, ∞], given by $f(x) = x^2 + 4$. Show that f is invertible with inverse f^{-1} of f is given by $f^{-1}(y) = \sqrt{y-4}$, R^+ is non negative real no.
40. If $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$ prove that $A^3 - 6A^2 + 7A + 2I = 0$.
41. Solve by martix method $\begin{aligned}2x+3y+3z&=5\\x-2y+z&=-4\\3x-y-2z&=3\end{aligned}$
42. If $y = 500e^{7x} + 600e^{-7x}$ prove that $\frac{d^2y}{dx^2}=49y$.
43. A ballon which always remains spherical on inflation is being inflated by pumping in 900 cm^3 / sec. Find the rate at which the radius of the ballon increases when the radius is 15 cm?
44. Find the integral of $\int\frac{1}{a^2+x^2}dx$ with respect to x and hence evaluate $\int\frac{1}{9x^2+6x+5}dx$.
45. Using integration find the area of the circle $x^2 + y^2 = a^2$.
46. Find the general solution of the differential equation $\frac{dy}{dx}+(\sec x)y=\tan x$
47. Derive the equation in normal form in vector form & Cartesian form

PART-E

V. Answer any one of the following: 1X10=10

49. a) Solve the following LPP graphically minimize Z= 200x +500y s.t.c. $x + 2y \geq 10$, $3x+4y \leq 24$, $x \geq 0$, $y \geq 0$.

b) Prove that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)\,(b-c)(c-a)$

50. a) Prove that $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx.$ hence evaluate $\int_0^{\pi} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\, dx$.

b) Discuss the continuity of the function $f(x) = \begin{cases} 2x & if \quad x < 0 \\ 0 & if \quad 0 \leq x \leq 1 \\ 4x & if \quad x > 1 \end{cases}$

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