	Course:	II PU
SRI BHAGAWAN MAHAVEER JAIN COLLEGE Vishweshwarapuram, Bangalore.	Subject:	Mathematics
	Max. Marks:	100
Mock Question Paper 2 – January 2020	Duration:	3:15hrs.

Instructions:
(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.
(ii) Use the graph sheet for the question on linear programming in PART E.

PART-A

I Answer All the questions. 10 x 1 = 10

1. Let ' $*$ ' be a binary operation defined on Q by $a*b = \dfrac{ab}{4}$. Find the identity element

2. Find the value of $\cot\left[\tan^{-1}(x) + \cot^{-1}(x)\right]$

3. Construct a 2×2 matrix $A = \left[a_{ij}\right]$ whose elements are given by $a_{ij} = \dfrac{i}{j}$.

4. Give an example of a second order matrix which is both symmetric a skew-symmetric matrix.

5. Find $\dfrac{dy}{dx}$ if $y = \log(\cos e^x)$.

6. Find the anti-derivative of 3^x with respect to x.

7. For what value of λ, the vectors $\vec{a} = 2i - 3\lambda j + k$ and $\vec{b} = i - j - 2k$ are perpendicular to each other

8. If a line makes an angle 90^0, 60^0, 30^0 with positive direction of x, y and z axis respectively. Find its direction cosines.

9. Define the term corner point in LPP.

10. If $P(A) = 0.3$, $P(\text{not } B) = 0.4$ and A and B independent events find P(A and not B)

PART-B

II. Answer any TEN questions. 10x2=20

11. Define a binary operation on a set verify whether ' $*$ ' defined on Z, by $a*b = ab+1$ is a binary operation or not

12. Evaluate $\sin\left[\dfrac{\pi}{3} + \sin^{-1}\left(\dfrac{1}{2}\right)\right]$

13. Show that $3\sin^{-1} x = \sin^{-1}\left[3x - 4x^3\right]$; $x \in \left[\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

14. Without expanding evaluate $\begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix}$

15. If $y + \sin y = \cos x$. find $\dfrac{dy}{dx}$.

16. Find $\dfrac{dy}{dx}$ if $x = a\sec\theta,\ y = b\tan\theta$

17. Find the equation of the tangent to the parabola $y^2 = 4ax$ at $\left(at^2, 2at\right)$

18. Evaluate $\displaystyle\int \dfrac{2 - 3\sin x}{\cos^2 x} dx$

19. Evaluate $\displaystyle\int \left[\log(\log x) + \dfrac{1}{\left(\log x\right)^2}\right] dx$

20. Find the order and degree of the differential equation $\left(\dfrac{dy}{dx}\right)^3 - \left(\dfrac{dy}{dx}\right)^2 - y = \sin x.$

21. Find the vector equation of the line passing thro the points $(-1,0,2)$ and $(3,4,6)$

22. Find the area of parallelogram whose adjacent sides are given by $\vec{a} = 3i + j + 4k$ and $\vec{b} = i - j + k$

23. Find a vector in the direction of the vector $\vec{a} = 2i + 3j + k$ that has magnitude 7 units.

24. Two coins are tossed once. Find P(E/F) where E: no tail appears F: no head appears

PART-C

III. Answer any TEN questions.   **10x3=30**

25. Verify whether the function $f : A \to B$ where $A = R - \{3\}$ and $B = R - \{1\}$ defined by $f(x) = \dfrac{x - 2}{x - 3}$ is

    one-one and onto or not. Give reason

26. Prove that $\cos^{-1}\left(4/5\right) + \cos^{-1}\left(12/13\right) = \cos^{-1}\left(33/65\right)$

27. If A and B are symmetric matrices, prove that $(AB - BA)$ is skew symmetric

28. If $x = \sqrt{a^{\sin^{-1}(t)}}, y = \sqrt{a^{\cos^{-1}(t)}}$ prove that $\dfrac{dy}{dx} = -\dfrac{y}{x}$

29. If $y = \sin^{-1}\left[\dfrac{2^{x+1}}{1 + 4^x}\right]$ Find $dy\big/dx$

30. Verify mean value theorem for the function $f(x) = x^3 - 5x^2 - 3x$ in the interval $[1,3]$

31. Evaluate $\displaystyle\int \dfrac{\left(1 + \log x\right)^2}{x} dx$

32. Evaluate $\displaystyle\int_0^2 \left(x^2 + 1\right) dx$ as a limit of a sum.

33. Find the area of the region bounded by the curve $y^2 = x$ and the line $x = 1, x = 4$ and the x-axis in the

    I quadrant

34. Form the differential equation representing the family of curve $y = mx$ where 'm' is arbitrary constant

35. Find the area of the triangle having points A(1,1,1) B(1,2,3) and C(2,3,1) as its vertices using vector method.

36. Show that the points A(-1,4,-3), B(-3,2,+5), C(-3,8,-5) and D(-3,2,1) are coplanar

37. Find the cartesian and vector equation of the line that passes through the points (3,-2,-5) and (3,-2, 6)

38. Probability that A speaks truth is 4/5. A coin is tossed 'A' reports that a head appears. Find the probability that it is actually head.

PART-D

IV. Answer any SIX of the following.   **6x5=30**

39. Let $f : R \to R$ defined by $f(x) = 4x + 3$. Show that 'f' is invertible. Find the inverse of f.

40. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ verity $A^3 - 23A - 40I = 0$

41. Solve by matrix method

$$x + y + z = 6$$
$$x - 2y + 3z = 6$$
$$x - y + z = 2$$

42. If $y = \sin^{-1} x$ prove that $(1 - x^2)\dfrac{d^2 y}{dx^2} - x\dfrac{dy}{dx} = 0$

43. Sand is pouring from a pipe at the rate of 12cm³/sec. The falling sand from a cone on the ground in such a way that the height of the cone is always 1/6th of the radius of the base. How fast is the height of the sand cone is increasing when the height is 4cms.

44. Find the integral of $\dfrac{1}{\sqrt{a^2 - x^2}}$ with respect to x and hence evaluate $\int \dfrac{dx}{\sqrt{7 - 6x - x^2}}$

45. Find the area of the region bounded by the parabola $y = x^2$ and $y^2 = x$

46. Find the particular solution of the differential equation $(1 + x^2)\dfrac{dy}{dx} + 2x.y = \dfrac{1}{(1 + x^2)}$ Given $x = 1$, $y = 0$

47. Derive the equation of the line in space passing thro a point and parallel to a given vector $\vec{b}$ both in vector and Cartesian form

48. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is 1/100 what is the probability that he will win a prize.

(i) atleast once   (ii) exactly one   (iii) atleast twice

PART-E

V. Answer any ONE of the following   **1x10=10**

49. (a) Prove that $\int\limits_a^b f(x)\,dx = \int\limits_a^c f(x)\,dx + \int\limits_c^b f(x)\,dx$ hence evaluate $\int\limits_{-1}^2 |x^3 - x|\,dx$

(b) Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

50. (a) A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hour for fabrication and 1 labour hour for finishing. Each piece of model B requires 12 labour hour for fabricating and 3 labour hour for finishing. For fabricating & finishing the maximum labour hour available are 180 and 30 respectively. The company makes a profit of Rs.8000 on each piece of model A and Rs.12,000 on each piece of model B. How many pieces of model A and model B should be manufactured per week to realize a maximum profits what is the maximum profit per week?

(b) Find the values of 'a' and 'b' such that the function

defined by $\quad f(x) = \begin{cases} 5 & if & x \le 2 \\ ax+b & if & 2 < x < 10 \\ 21 & if & x \ge 10 \end{cases}$ is a continuous function.

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