JAIN COLLEGE, Bangalore
Mock Paper – 1, January - 2020
II PUC – Mathematics (35)

Time: 3 Hours 15 Minutes **Max. Marks: 100**

PART A

I. Answer all the following questions: **10 × 1 = 10**

1. Find whether operation * on Z^+ defined by $a * b = a^b \ \forall\, a, b \in Z^+$ is binary or not.
2. Find the value of $\cos(\sec^{-1} x + \text{coesc}^{-1} x)$, $|x| \geq 1$
3. If $\begin{bmatrix} x+2 & y-3 \\ 0 & 4 \end{bmatrix}$ is a scalar matrix, find x &y .
4. Find the value of x if $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$
5. If $\mathrm{y} = \cos(\mathrm{sinx})$ then find $\frac{dy}{dx}$.
6. Find $\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2}\right) dx$
7. Find a value of x for which $\mathrm{x}(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector.
8. If the vectors $2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}$ and $4\hat{\imath} - m\hat{\jmath} - 12\hat{k}$ are parallel, find m.
9. Define objective function in linear programming problem
10. If $\mathrm{P(E)} = 0.6, \mathrm{P(F)} = 0.3$ and $\mathrm{P(E \cap F)} = 0.2$ then find $\mathrm{P(F/E)}$.

PART B

II. Answer all the following questions : **10 × 2 = 20**

11. Verify whether the operation * defined on Q by $a * b = \frac{ab}{2}$ is associative or not.
12. Ifsin$\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$, then find the value of x.
13. Find $\cos^{-1}\left(cos\frac{7\pi}{6}\right)$
14. Find the equation of the line joining (1,2) and (3,6) using determinants.
15. Differentiate $x^{sinx}, x > 0$ with respect to x.
16. Differentiate $(x+3)^2.(x+4)^3.(x+5)^4$ with respect to x.
17. Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 3%.
18. Integrate $\frac{tan^4\sqrt{x}.sec^2\sqrt{x}}{\sqrt{x}}$ with respect to x.
19. Evaluate $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$
20. Find the order and degree of Differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y^5 = 0$
21. Find the projection of the vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$.
22. If the position of the vector of the points A and B respectively are $\hat{\imath} + \widehat{2\jmath} - 3\hat{k}$ and $\hat{\jmath} - \hat{k}$ .find the direction cosines of $\overrightarrow{\mathrm{AB}}$.
23. Find the vector and cartesian equation of the line that passes through the points $(3,-2,-5)$ and $(3,-2,6)$.
24. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that problem is solved.

PART C

III. Answer all the following questions : **10 × 3 = 30**

25. Show that the relation R in the set $\mathrm{A} = \{1,2,3,4,5\}$given by $\mathrm{R} = \{(\mathrm{a,b}): |a-b| \text{ is even}\}$ is an equivalence relation.
26. Show that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{4}{3} = \frac{\pi}{3}$
27. By using elementary transformation, find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$
28. Verify mean value theorem if $\mathrm{f(x)} = x^3 - 5x^2 - 3\mathrm{x}$ in the interval [1,3]

29. Differentiate sin^2x with respect to e^{cosx} .

30. Find the intervals in which the function f given by $\mathrm{f(x)} = 2x^3 - 3x^2 - 36\mathrm{x} + 7$ is strictly increasing.

31. Find the anti-derivative of f(x) given by $\mathrm{f(x)} = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$

32. Find $\int e^x . sinx\ dx$

33. Find the area of the region bounded by the curve $y^2 = 4\mathrm{x}$ and the line $\mathrm{x} = 3$.

34. Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

35. Find the area of the triangle where position vectors of A, B and C are $\hat{\imath} - \hat{\jmath} + 2\hat{k},\ 2\hat{\jmath} + \hat{k}, and\ \hat{\jmath} + 3\hat{k}$ respectively.

36. Three vectors $\vec{a}, \vec{b}\ and\ \vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = 0$. Evaluate $\mu = \vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}$ if $|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2.$

37. Find the shortest distance between the lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$

38. A bag contains 4 red and 4 black balls; another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

PART D

IV. Answer all the following questions : 6 × 5 = 30

39. Let $f: N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. .Show that $f: N \rightarrow S$ where S is the range of f is invertible.Find the inverse of f.

40. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}\ and\ C = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ and Calculate $AB, AC\ and\ A(B + C)$ and verify that $A(B + C) = AB + AC$

41. The cost of 4kg onion, 3kg wheat and 2kg rice is Rs.60. The cost of 2kg onion,4kg wheat and 6kg rice is Rs.90. The cost of 6kg onion, 2kg wheat and 3kg rice is Rs.70.Find the cost of each them per kg by matrix method.

42. If $y = 3\cos(logx) + 4\sin(logx)$ show that $x^2y_2 + xy_1 + y = 0.$

43. A ladder 24ft long leans against a vertical wall. The lower end is moving away of the rate of 3ft/sec. Find the rate at which the top of the ladder is moving downwards if the foot is 8ft from the wall.

44. Find the integral of $\sqrt{x^2 - a^2}$with respect x and hence evaluate $\int \sqrt{x^2 - 8x + 7}\ dx$

45. Using integration find the area of the region bounded by the triangle whose vertices are $(1,0), (2,2)\ and\ (3,1)$

46. Find the general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2logx$

47. Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.

48. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05.Find the probability that out of such 5 bulbs

    1) none

    2) not more than one

    3) more than one

    will fuse after 150 days of use.

PART E

V. Answer all the following questions : 10 × 1 = 10

49. a)Prove that $\int_0^{2a} f(x)dx = \begin{cases} \int_0^{2a} f(x)dx & , f(2a - x) = f(x) \\ 0 & , f(2a - x) = -f(x) \end{cases}$ and hence evaluate $\int_0^{2\pi} cos^5x\ dx$

    b) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)$

50. a) Minimize and maximize $z = 5x + 10y$

    subject to constraints $x + 2y \leq 120, x + y \geq 60\ and\ x \geq 0, y \geq 0.$ by graphical method.

    b) Find the value of k if $f(x) = \begin{cases} \frac{1-cos2x}{1-cosx} & , x \neq 0 \\ k & , x = 0 \end{cases}$ is continuous at x=0.

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JAIN COLLEGE, Bangalore
Mock Paper – 2, January - 2020
II PUC – Mathematics (35)

Time: 3 Hours 15 Minutes **Max. Marks: 100**

PART A

I. Answer all the following questions : **10 × 1 = 10**

1. Define Bijective function
2. Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$
3. Find the value of x, y and z if $\begin{bmatrix}4 & 3\\ x & 5\end{bmatrix} = \begin{bmatrix}y & z\\ 1 & 5\end{bmatrix}$
4. Evaluate $\begin{vmatrix}x & x+1\\ x-1 & x\end{vmatrix}$
5. Find the derivative of the function $\sec(\mathrm{Tan}(\sqrt{x}))$
6. Find $\int \sec(secx + tanx)dx$
7. Find unit vector in the direction of vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$
8. Find the direction cosines of a lines which makes equal angles with the co-ordinate axis.
9. Define constrains of linear programming problem
10. If$\mathrm{P(A)} = 0.8$ & $\mathrm{P(B/A)} = 0.4$, then find $P(A \cap B)$.

PART B

II. Answer all the following questions : **10 × 1 = 10**

11. If $\mathrm{f: R \to R}$ is defined by $\mathrm{f(x)} = x^2 - 3\mathrm{x} + 2$,find $\mathrm{f(f(x))}$
12. Prove that $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2}), \frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$
13. Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), x > 1$ in the simplest form.
14. Find the area of the triangle whose vertices are $(-2,-3), (3,2)$ and $(-1,-8)$ by using determinants
15. If $\mathrm{ax} + \mathrm{by}^2 = \cos\mathrm{y}$ then find $\frac{dy}{dx}$
16. Find the derivative of $x^x - 2^{sinx}$ with respect to x.
17. Using differentials find the approximate value of $(25)^{\frac{1}{3}}$
18. Find $\int \frac{1}{cos^2x(1-tanx)^2}dx$
19. Evaluate $\int logx\, dx$
20. Find the order and degree (if defined) of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\frac{dy}{dx} = 0$
21. If $\vec{a}$ is a unit vector such that $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 8$ find $|\vec{x}|$.
22. Show that the vector is equally inclined to the axis OX, OY and OZ respectively.
23. Find the angle between the planes whose vector equations are $\vec{r}.(2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) = 5$ and $\vec{r}.(3\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) = 3$
24. Find the probability distribution of number of heads in two tosses of a coin.

PART C

III. Answer all the following questions : **10 × 3 = 30**

25. If function $\mathrm{f: R \to R}$ and $\mathrm{g: R \to R}$ are given by$\mathrm{f(x)} = |x|$ and $\mathrm{g(x)} = [x]$ (where [x] is greatest integer function) find $\mathrm{fog}\left(\frac{-1}{2}\right)$ and $\mathrm{gof}\left(\frac{-1}{2}\right)$
26. If $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ then find the values of x.
27. By using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix}1 & 3\\ 2 & 7\end{bmatrix}$
28. Differentiate $(logx)^{cosx}$ with respect to 'x'.
29. If$\mathrm{x} = \sqrt{a^{\sin^{-1}t}}$ and $\mathrm{y} = \sqrt{a^{\cos^{-1}t}}$ then prove that $\frac{dy}{dx} = \frac{-y}{x}$.
30. Find two numbers whose product is 100 and whose sum is minimum
31. Evaluate $\int \frac{2x}{x^2+3x+2}dx$

32. Evaluate $\int x.\tan^{-1} x\, dx$
33. Fine area of the region bounded by the curve $y^2 = 4\mathrm{x}, \mathrm{y} -$ axis and the line $\mathrm{y} = 3$.
34. Form the differential equation representing the family of curves $\mathrm{y} = \mathrm{a}\sin(\mathrm{x} + \mathrm{b})$ where a,b are arbitrary constants.
35. Show that the position vector of the point p, which divides the line joining the points A&B having position vector a and b internally in the ratio m:n is $\frac{m\vec{b}+n\vec{a}}{m+n}$
36. Find x, such that the four points $\mathrm{A}(3,2,1), \mathrm{B}(4,\mathrm{x},5), \mathrm{c}(4,2,-2)$ and $\mathrm{D}(6,5,-1)$ are coplanar.
37. Find the vector equation of the plane passing through the points $R(2,5,-3), S(-2,-3,5)\ and\ T(5,3,-3)$.
38. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event "the sum of numbers on the dice is 4."

PART D

IV. Answer all the following questions : **6 × 5 = 30**

39. Consider $f: R^+ \to [5,\infty)$ given by $f(x) = 9x^2 + 6x - 5$ .Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$
40. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ prove that $A^3 - 6A^2 + 7A + 2I = 0$
41. Solve the following system of equations by matrix method $x + y + z = 6, y + 3z = 11\ and\ x - 2y + z = 0$
42. If $y = 3e^{2x} + 2e^{3x}$ show that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.
43. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8times as fast as the x-coordinate.
44. Find the integral of $\frac{1}{x^2-a^2}$ with respect to x and hence evaluate $\int \frac{1}{x^2-16} dx$
45. Using integration find the area of the triangular region whose sides have equations $y = 2x + 1, y = 3x + 1\ and\ x = 4$.
46. Find the general solution of the differential equation $e^x tan\, y\, dx + (1 - e^x)sec^2 y dy = 0$
47. Derive the equation of a line in space passing through a given point and parallel to a given vector in both vector and Cartesian form.
48. A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize exactly one.

PART –E

V. Answer all the following questions : **1 × 10 =10**

49. a) Prove that $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$and hence evaluate $\int_0^4 |x-1|dx$

    b) Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$
50. a)Minimize and maximize $z = 600x + 400y$
    subject to constraints $x + 2y \le 12, 2x + y \le 12\ 4x + 5y \ge 20\ and\ x \ge 0, y \ge 0$.
    by graphical method

    b)Find the value of k if $f(x) = \begin{cases} \frac{kcosx}{\pi-2x}, & x \ne \frac{\pi}{2} \\ 3, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$

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