PLEASE ENSURE THAT THIS BOOKLET CONTAINS 120 QUESTIONS SERIALLY NUMBERED FROM 1 TO 120 (Printed Pages : 32)

1. The domain of the function $f(x) = \log_2(\log_3(\log_4 x))$ is

(A) $(-\infty, 4)$ (B) $(4, \infty)$ (C) $(0, 4)$ (D) $(1, \infty)$ (E) $(-\infty, 1)$

2. If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = x - 3$ and $g(x) = x^2 + 1$, then the values of x for which $g(f(x)) = 10$ are

(A) 0, −6 (B) 2, −2 (C) 1, −1 (D) 0, 6 (E) 0, 2

3. Two finite sets A and B have m and n elements respectively. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m is

(A) 7 (B) 9 (C) 10 (D) 12 (E) 13

4. If $f(x)$ satisfies the relation $2f(x) + f(1-x) = x^2$ for all real x, then $f(x)$ is

(A) $\frac{x^2+2x-1}{6}$ (B) $\frac{x^2+2x-1}{3}$ (C) $\frac{x^2+4x-1}{3}$

(D) $\frac{x^2-3x+1}{6}$ (E) $\frac{x^2+3x-1}{3}$

Space for rough work

5. The range of the function $f(x)=\frac{x^2-x+1}{x^2+x+1}$ where $x \in R$, is

(A) $(-\infty, 3]$ (B) $(-\infty, \infty)$ (C) $[3, \infty)$

(D) $[\frac{1}{3}, 3]$ (E) $(-\infty, \frac{1}{3}) \cup (3, \infty)$

6. If the area of the triangle formed by the points z, $z+iz$ and iz is 50 square units, then $|z|$ is equal to

(A) 5 (B) 8 (C) 10 (D) 12 (E) $5\sqrt{2}$

7. The locus of z such that $\arg[(1-2i)z-2+5i]=\frac{\pi}{4}$ is a

(A) line not passing through the origin
(B) circle not passing through the origin
(C) line passing through the origin
(D) circle passing through the origin
(E) circle with centre at the origin

8. If $z=\sqrt{3}+i$, then the argument of z^2e^{z-i} is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $e^{\frac{\pi}{6}}$ (D) $e^{\frac{\pi}{3}}$ (E) $\frac{\pi}{3}$

Space for rough work

9. If $\omega \neq 1$ and $\omega^3 = 1$, then $\dfrac{a\omega + b + c\omega^2}{a\omega^2 + b\omega + c} + \dfrac{a\omega^2 + b + c\omega}{a + b\omega + c\omega^2}$ is equal to

(A) 2 (B) ω (C) 2ω (D) $2\omega^2$ (E) $a + b + c$

10. The centre of a regular hexagon is at the point $z = i$. If one of its vertices is at $2+i$, then the adjacent vertices of $2+i$ are at the points

(A) $1 \pm 2i$ (B) $i + 1 \pm \sqrt{3}$ (C) $2 + i\left(1 \pm \sqrt{3}\right)$
(D) $1 + i\left(1 \pm \sqrt{3}\right)$ (E) $1 - i\left(1 \pm \sqrt{3}\right)$

11. If the roots of the equation $\dfrac{1}{x+p} + \dfrac{1}{x+q} = \dfrac{1}{r}$, $(x \neq -p,\ x \neq -q,\ r \neq 0)$ are equal in magnitude but opposite in sign, then $p + q$ is equal to

(A) r (B) $2r$ (C) r^2 (D) $\dfrac{1}{r}$ (E) $\dfrac{2}{r}$

12. The solution of the equation $(3+2\sqrt{2})^{x^2-8} + (3+2\sqrt{2})^{8-x^2} = 6$ are

(A) $3 \pm 2\sqrt{2}$ (B) ± 1 (C) $\pm 3\sqrt{3}$, $\pm 2\sqrt{2}$
(D) ± 7, $\pm\sqrt{3}$ (E) ± 3, $\pm\sqrt{7}$

13. If $2 - i$ is a root of the equation $ax^2 + 12x + b = 0$ (where a and b are real), then the value of ab is equal to

(A) 45 (B) 15 (C) −15 (D) −45 (E) 25

Space for rough work

14. If one root of the equation $lx^2 + mx + n = 0$ is $\frac{9}{2}$ (l, m and n are positive integers) and $\frac{m}{4n} = \frac{l}{m}$, then $l + n$ is equal to

(A) 80 (B) 85 (C) 90 (D) 95 (E) 100

15. If $x^2 + 4ax + 2 > 0$ for all values of x, then a lies in the interval

(A) $(-2, 4)$ (B) $(1, 2)$ (C) $\left(-\sqrt{2}, \sqrt{2}\right)$

(D) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (E) $(-4, 2)$

16. If a, b, c are in G.P. and x, y are arithmetic mean of a, b and b, c respectively, then $\frac{1}{x} + \frac{1}{y}$ is equal to

(A) $\frac{2}{b}$ (B) $\frac{3}{b}$ (C) $\frac{b}{3}$ (D) $\frac{b}{2}$ (E) $\frac{1}{b}$

17. A student read common difference of an A.P. as -3 instead of 3 and obtained the sum of first 10 terms as -30. Then the actual sum of first 10 terms is equal to

(A) 240 (B) 120 (C) 300 (D) 180 (E) 480

18. If $a_1 = 1$ and $a_n = n\,a_{n-1}$, for all positive integer $n \geq 2$ then a_5 is equal to

(A) 125 (B) 120 (C) 100 (D) 24 (E) 6

Space for rough work

19. If $a_1, a_2, \dots, a_n$ are in A.P. with common difference $d \neq 0$, then $(\sin d)[\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + \cdots + \sec a_{n-1} \sec a_n]$ is equal to

(A) $\cot a_n - \cot a_1$ (B) $\cot a_1 - \cot a_n$ (C) $\tan a_n - \tan a_1$

(D) $\tan a_n - \tan a_{n-1}$. (E) $\tan a_1 - \tan a_n$

20. The value of $\frac{1}{2!} + \frac{2}{3!} + \dots + \frac{999}{1000!}$ is equal to

(A) $\frac{1000! - 1}{1000!}$ (B) $\frac{1000! + 1}{1000!}$ (C) $\frac{999! - 1}{999!}$

(D) $\frac{999! + 1}{999!}$ (E) $\frac{1000! - 999!}{1000!}$

21. $\log_e \frac{1+3x}{1-2x}$ is equal to

(A) $-5x - \frac{5x^2}{2} - \frac{35x^3}{3} - \cdots$ (B) $-5x + \frac{5x^2}{2} - \frac{35x^3}{3} + \cdots$ (C) $5x - \frac{5x^2}{2} + \frac{35x^3}{3} - \cdots$

(D) $5x + \frac{5x^2}{2} + \frac{35x^3}{3} + \cdots$ (E) $x + \frac{3x^2}{2} + \frac{5x^3}{4} + \cdots$

22. The sum of the infinite series $\frac{1}{2}\left(\frac{1}{3} + \frac{1}{4}\right) - \frac{1}{4}\left(\frac{1}{3^2} + \frac{1}{4^2}\right) + \frac{1}{6}\left(\frac{1}{3^3} + \frac{1}{4^3}\right) - \cdots$ is equal to

(A) $\frac{1}{2}\log 2$ (B) $\log\frac{3}{5}$ (C) $\log\frac{5}{3}$ (D) $\frac{1}{2}\log\frac{5}{3}$ (E) $\frac{1}{2}\log\frac{3}{5}$

23. The sum of the infinite series $\frac{2^2}{2!} + \frac{2^4}{4!} + \frac{2^6}{6!} + \cdots$ is equal to

(A) $\frac{e^2+1}{2e}$ (B) $\frac{e^4+1}{2e^2}$ (C) $\frac{(e^2-1)^2}{2e^2}$ (D) $\frac{(e^2+1)^2}{2e^2}$ (E) $\frac{(e^2-1)^2}{4e^2}$

Space for rough work

24. If $|x| < 1$, then the coefficient of x^6 in the expansion of $(1+x+x^2)^{-3}$ is

(A) 3 (B) 6 (C) 9 (D) 12 (E) 15

25. ${}^{15}C_0 \cdot {}^5C_5 + {}^{15}C_1 \cdot {}^5C_4 + {}^{15}C_2 \cdot {}^5C_3 + {}^{15}C_3 \cdot {}^5C_2 + {}^{15}C_4 \cdot {}^5C_1$ is equal to

(A) $2^{20} - 2^5$ (B) $\frac{20!}{5!\,15!}$ (C) $\frac{20!}{5!\,15!} - 1$

(D) $\frac{20!}{5!\,15!} - \frac{15!}{5!\,10!}$ (E) $\frac{15!}{5!\,10!}$

26. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3 : 5$ then the value of n is equal to

(A) 4 (B) 3 (C) 2 (D) 1 (E) 5

27. Let $[x]$ denote the greatest integer less than or equal to x. If $x = (\sqrt{3}+1)^5$, then $[x]$ is equal to

(A) 75 (B) 50 (C) 76 (D) 51 (E) 152

28. If n is a positive integer, then $5^{2n+2} - 24n - 25$ is divisible by

(A) 574 (B) 575 (C) 675 (D) 674 (E) 576

29. Let T_n denote the number of triangles which can be formed by using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 28$, then n equals

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Space for rough work

30. If α, β, γ are the cube roots of unity then the value of the determinant $\begin{vmatrix} e^{\alpha} & e^{2\alpha} & (e^{3\alpha}-1) \\ e^{\beta} & e^{2\beta} & (e^{3\beta}-1) \\ e^{\gamma} & e^{2\gamma} & (e^{3\gamma}-1) \end{vmatrix}$ is equal to

(A) −2 (B) −1 (C) 0 (D) 1 (E) 2

31. If B is a non-singular matrix and A is a square matrix such that $B^{-1}AB$ exists, then det $(B^{-1}AB)$ is equal to

(A) det (A^{-1}) (B) det (B^{-1}) (C) det (B)
(D) det (A) (E) det (AB^{-1})

32. If 1, ω, ω^2 are cube roots of unity and if $\begin{bmatrix} 1+\omega & 2\omega \\ -2\omega & -b \end{bmatrix} + \begin{bmatrix} a & -\omega \\ 3\omega & 2 \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ \omega & 1 \end{bmatrix}$, then $a^2 + b^2$ is equal to

(A) $1 + \omega^2$ (B) $\omega^2 - 1$ (C) $1 + \omega$ (D) $(1 + \omega)^2$ (E) ω^2

33. If the three linear equations

$x + 4ay + az = 0$

$x + 3by + bz = 0$

$x + 2cy + cz = 0$

have a non-trivial solution, where $a \neq 0, b \neq 0, c \neq 0$, then $ab + bc$ is equal to

(A) $2ac$ (B) $-ac$ (C) ac (D) $-2ac$ (E) a

34. If $A = \begin{bmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ x & x & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $A^3 - 3A^2 + 3A$ is equal to

(A) $3I$ (B) I (C) $-I$ (D) $-2I$ (E) $2I$

Space for rough work

35. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$, then the value of the determinant $|A^{2009} - 5A^{2008}|$ is

(A) −6 (B) −5 (C) −4 (D) 4 (E) 6

36. If x satisfies the inequations $2x - 7 < 11$, $3x + 4 < -5$, then x lies in the interval

(A) $(-\infty, 3)$ (B) $(-\infty, 2)$ (C) $(-\infty, -3)$ (D) $(-\infty, \infty)$ (E) $(3, \infty)$

37. The set of all real x satisfying the inequality $\frac{3-|x|}{4-|x|} \geq 0$ is

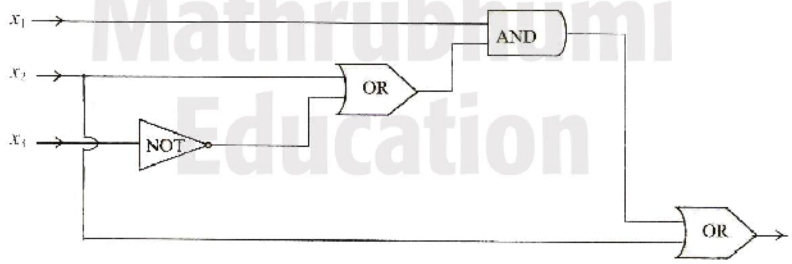
(A) $[-3, 3] \cup (-\infty, -4) \cup (4, \infty)$
(B) $(-\infty, -4) \cup (4, \infty)$
(C) $(-\infty, -3) \cup (4, \infty)$
(D) $(-\infty, -3) \cup (3, \infty)$
(E) $[-3, 3] \cup [4, \infty)$

38. Identify the false statement

(A) $\sim[p \vee (\sim q)] \equiv (\sim p) \wedge q$
(B) $[p \vee q] \vee (\sim p)$ is a tautology
(C) $[p \wedge q] \wedge (\sim p)$ is a contradiction
(D) $\sim[p \wedge (\sim p)]$ is a tautology
(E) $\sim(p \vee q) \equiv (\sim p) \vee (\sim q)$

Space for rough work

39. The Boolean expression corresponding to the combinatorial circuit is



(A) $(x_1 + x_2 \cdot x_3') \cdot x_2$

(B) $(x_1 \cdot (x_2 + x_3)) + x_2$

(C) $(x_1 \cdot (x_2 + x_3')) + x_2$

(D) $(x_1 \cdot (x_2 - x_3')) + x_3$

(E) $(x_1 + x_2' \cdot x_3) \cdot x_2$

40. In a Boolean algebra B with respect to '+' and '·', x' denotes the negation of $x \in B$. Then

(A) $x - x' = 1$ and $x \cdot x' = 1$

(B) $x + x' = 1$ and $x \cdot x' = 0$

(C) $x + x' = 0$ and $x \cdot x' = 1$

(D) $x + x' = 0$ and $x \cdot x' = 0$

(E) $x - x' = 0$ and $x \cdot x' = 0$

41. If $\cos^{-1}\left(\frac{5}{13}\right) - \sin^{-1}\left(\frac{12}{13}\right) = \cos^{-1} x$, then x is equal to

(A) 1 (B) $\frac{1}{\sqrt{2}}$ (C) 0 (D) $\frac{\sqrt{3}}{2}$ (E) -1

Space for rough work

42. The value of $\cos(\tan^{-1}(\sin(\cot^{-1} x)))$ is

(A) $\sqrt{\frac{x^2+1}{x^2-1}}$ (B) $\sqrt{\frac{1-x^2}{x^2+2}}$ (C) $\sqrt{\frac{1-x^2}{1+x^2}}$ (D) $\sqrt{\frac{x^2+1}{x^2+2}}$ (E) $\sqrt{\frac{1-x^2}{2-x^2}}$

43. If a and b are positive numbers such that $a > b$, then the minimum value of $a \sec\theta - b \tan\theta$ $(0 < \theta < \frac{\pi}{2})$ is

(A) $\frac{1}{\sqrt{a^2-b^2}}$ (B) $\frac{1}{\sqrt{a^2+b^2}}$ (C) $\sqrt{a^2+b^2}$ (D) $\sqrt{a^2-b^2}$ (E) a^2-b^2

44. If $\frac{-\pi}{2} < \sin^{-1} x < \frac{\pi}{2}$, then $\tan(\sin^{-1} x)$ is equal to

(A) $\frac{x}{1-x^2}$ (B) $\frac{x}{1+x^2}$ (C) $\frac{x}{\sqrt{1-x^2}}$ (D) $\frac{1}{\sqrt{1-x^2}}$ (E) $\frac{x}{\sqrt{x^2-1}}$

45. If $A+B=45^\circ$ then $(\cot A-1)(\cot B-1)$ is equal to

(A) 1 (B) $\frac{1}{2}$ (C) -1 (D) -2 (E) 2

46. The solution of the equation $[\sin x+\cos x]^{1+\sin 2x}=2, -\pi \le x \le \pi$ is

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$ (E) $\frac{\pi}{3}$

47. If $\sin A-\sqrt{6}\cos A=\sqrt{7}\cos A$, then $\cos A+\sqrt{6}\sin A$ is equal to

(A) $\sqrt{6}\sin A$ (B) $\sqrt{7}\sin A$ (C) $\sqrt{6}\cos A$ (D) $\sqrt{7}\cos A$ (E) $\sqrt{42}\cos A$

Space for rough work

48. If $\tan A$ and $\tan B$ are the roots of $abx^2 - c^2x + ab = 0$ where a, b, c are the sides of the triangle ABC, then the value of $\sin^2 A + \sin^2 B + \sin^2 C$ is

(A) 1 (B) 3 (C) 4 (D) 2 (E) 5

49. In a triangle ABC, if $a = 3, b = 4, c = 5$ then the distance between its incentre and circumcentre is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{3}{2}$ (D) $\frac{5}{2}$ (E) $\frac{\sqrt{5}}{2}$

50. In triangle ABC, the value of $\dfrac{\cot\frac{A}{2}\cot\frac{B}{2} - 1}{\cot\frac{A}{2}\cot\frac{B}{2}}$ is

(A) $\frac{a}{a+b+c}$ (B) $\frac{c}{a+b+c}$ (C) $\frac{2a}{a+b+c}$ (D) $\frac{2b}{a+b+c}$ (E) $\frac{2c}{a+b+c}$

51. In a triangle ABC if $\angle A = 60$, $a = 5, b = 4$, then c is a root of the equation

(A) $c^2 - 5c - 9 = 0$ (B) $c^2 - 4c - 9 = 0$ (C) $c^2 - 10c + 25 = 0$
(D) $c^2 - 5c - 41 = 0$ (E) $c^2 - 4c - 41 = 0$

52. From the top of a tower, the angle of depression of a point on the ground is 60°. If the distance of this point from the tower is $\frac{1}{\sqrt{3}+1}$ metres, then the height of the tower is

(A) $\frac{4\sqrt{3}}{2}$ metres (B) $\frac{\sqrt{3}+3}{2}$ metres (C) $\frac{3-\sqrt{3}}{2}$ metres
(D) $\frac{\sqrt{3}}{2}$ metres (E) $\sqrt{3}+1$ metres

Space for rough work

53. The vertices of a family of triangles have integer co-ordinates. If two of the vertices of all the triangles are (0, 0) and (6, 8), then the least value of areas of the triangles is

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 3

54. A line has slope m and y-intercept 4. The distance between the origin and the line is equal to

(A) $\frac{4}{\sqrt{1-m^2}}$ (B) $\frac{4}{\sqrt{m^2-1}}$ (C) $\frac{4}{\sqrt{m^2+1}}$ (D) $\frac{4m}{\sqrt{1+m^2}}$ (E) $\frac{4m}{\sqrt{m^2-1}}$

55. One side of length $3a$ of a triangle of area a^2 square units lies on the line $x = a$. Then one of the lines on which the third vertex lies, is

(A) $x = -a^2$ (B) $x = a^2$ (C) $x = -a$ (D) $x = \frac{a}{3}$ (E) $x = -\frac{a}{3}$

56. The distance of the point (1, 2) from the line $x + y + 5 = 0$ measured along the line parallel to $3x - y = 7$ is equal to

(A) $4\sqrt{10}$ (B) 40 (C) $\sqrt{40}$ (D) $10\sqrt{2}$ (E) $2\sqrt{20}$

57. Area of the triangle formed by the lines $y = 2x$, $y = 3x$ and $y = 5$ is equal to (in square units)

(A) $\frac{25}{6}$ (B) $\frac{25}{12}$ (C) $\frac{5}{6}$ (D) $\frac{17}{12}$ (E) 6

58. Triangle ABC has vertices (0, 0), (11, 60) and (91, 0). If the line $y = kx$ cuts the triangle into two triangles of equal area, then k is equal to

(A) $\frac{30}{51}$ (B) $\frac{4}{7}$ (C) $\frac{7}{4}$ (D) $\frac{30}{91}$ (E) $\frac{25}{37}$

Space for rough work

59. If the lines $y = 3x + 1$ and $2y = x + 3$ are equally inclined to the line $y = mx + 4$, $\left(\frac{1}{2} < m < 3\right)$, then the values of m are

(A) $\frac{1}{7}\left(1 \pm 5\sqrt{3}\right)$ (B) $\frac{1}{7}\left(1 \pm 5\sqrt{5}\right)$ (C) $\frac{1}{7}\left(1 \pm 5\sqrt{2}\right)$
(D) $\frac{1}{7}\left(1 \pm 2\sqrt{5}\right)$ (E) $\frac{1}{7}\left(1 \pm 3\sqrt{2}\right)$

60. The vertices of a triangle are (3, 0), (3, 3) and (0, 3). Then the coordinates of the circumcentre are

(A) (0, 0) (B) (1, 1) (C) $\left(\frac{5}{2}, \frac{5}{2}\right)$ (D) (2, 2) (E) $\left(\frac{3}{2}, \frac{3}{2}\right)$

61. Area of the equilateral triangle inscribed in the circle $x^2 + y^2 - 7x + 9y + 5 = 0$ is

(A) $\frac{155}{8}\sqrt{3}$ square units (B) $\frac{165}{8}\sqrt{3}$ square units (C) $\frac{175}{8}\sqrt{3}$ square units
(D) $\frac{185}{8}\sqrt{3}$ square units (E) $\frac{195}{8}\sqrt{3}$ square units

62. The equation of one of the diameters of the circle $x^2 + y^2 - 6x + 2y = 0$ is

(A) $x + y = 0$ (B) $x - y = 0$ (C) $3x + y = 0$ (D) $x + 3y = 0$ (E) $x + 2y = 0$

63. If two chords having lengths $a^2 - 1$ and $3(a+1)$, where a is a constant, of a circle bisect each other, then the radius of the circle is

(A) 6 (B) $\frac{15}{2}$ (C) 8 (D) $\frac{19}{2}$ (E) 10

Space for rough work

64. The equation of the parabola whose focus (3, 2) and vertex (1, 2) , is

(A) $x^2+4x-8y+12=0$ (B) $x^2-4x-8y+12=0$
(C) $y^2-8x-4y+12=0$ (D) $y^2+4y-8x+12=0$
(E) $y^2-8x-2y-17=0$

65. The sum of the distances of a point (2, -3) from the foci of an ellipse $16(x-2)^2+25(y+3)^2=400$ is

(A) 8 (B) 6 (C) 50 (D) 32 (E) 10

66. The equation of one of the tangents to $\frac{x^2}{3}-\frac{y^2}{2}=1$ which is parallel to $y=x$, is

(A) $x-y+2=0$ (B) $x+y-1=0$ (C) $x+y-2=0$
(D) $x-y+1=0$ (E) $x+y+1=0$

67. If e_1 is the eccentricity of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and e_2 is the eccentricity of the hyperbola $\frac{x^2}{9}-\frac{y^2}{7}=1$, then e_1+e_2 is equal to

(A) $\frac{16}{7}$ (B) $\frac{25}{4}$ (C) $\frac{25}{12}$ (D) $\frac{16}{9}$ (E) $\frac{23}{16}$

68. If $\vec{p}\times\vec{q}=\vec{r}$ and $\vec{q}\times\vec{r}=\vec{p}$, then

(A) $r=1,\ p=q$ (B) $p=1,\ q=1$ (C) $r=2p,\ q=2$
(D) $q=1,\ p=r$ (E) $q=1,\ r=1$

Space for rough work

69. Vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle $\theta = 120°$. If $|\vec{a}| = 1, |\vec{b}| = 2$ then $\left[(\vec{a} + 3\vec{b}) \times (3\vec{a} + \vec{b})\right]^2$ is equal to

(A) 190 (B) 275 (C) 300 (D) 320 (E) 192

70. If the projection of the vector $\vec{a}$ on $\vec{b}$ is $|\vec{a} \times \vec{b}|$ and if $3\vec{b} = \vec{i} - \vec{j} + \vec{k}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\pi/3$ (B) $\pi/2$ (C) $\pi/4$ (D) $\pi/6$ (E) 0

71. If $\vec{x} = \vec{a} + \vec{b}$, $\vec{y} = \vec{a} - \vec{b}$, $|\vec{a}| = 2$, $|\vec{b}| = 3$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, then $|\vec{x} \times \vec{y}|$ is equal to

(A) $5\sqrt{3}$ (B) 6 (C) $4\sqrt{3}$ (D) 9 (E) $6\sqrt{3}$

72. If the position vectors of three consecutive vertices of a parallelogram are $\vec{i} + \vec{j} + \vec{k}$, $\vec{i} + 3\vec{j} + 5\vec{k}$ and $7\vec{i} + 9\vec{j} + 11\vec{k}$, then the coordinates of the fourth vertex are

(A) (2, 1, 3) (B) (6, 7, 8) (C) (4, 1, 3) (D) (7, 7, 7) (E) (8, 8, 8)

73. The two variable vectors $3x\,\vec{i} + y\,\vec{j} - 3\,\vec{k}$ and $x\,\vec{i} - 4y\,\vec{j} + 4\,\vec{k}$ are orthogonal to each other, then the locus of (x, y) is

(A) hyperbola (B) circle (C) straight line
(D) ellipse (E) parabola

Space for rough work

74. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar and $\left(\vec{a}+\lambda\vec{b}\right)\cdot\left[\left(\vec{b}+3\vec{c}\right)\times\left(\vec{c}-4\vec{a}\right)\right]=0$, then the value of λ is equal to

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{5}{12}$ (D) 3 (E) $\frac{7}{12}$

75. The angle between the line $\frac{3x-1}{3}=\frac{y+3}{-1}=\frac{5-2z}{4}$ and the plane $3x-3y-6z=10$ is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) $\frac{2\pi}{3}$

76. The angle between the straight lines $\vec{r}=(2-3t)\vec{i}+(1+2t)\vec{j}+(2+6t)\vec{k}$ and $\vec{r}=(1+4s)\vec{i}+(2-s)\vec{j}+(8s-1)\vec{k}$ is

(A) $\cos^{-1}\left(\frac{\sqrt{41}}{34}\right)$ (B) $\cos^{-1}\left(\frac{21}{34}\right)$ (C) $\cos^{-1}\left(\frac{43}{63}\right)$

(D) $\cos^{-1}\left(\frac{5\sqrt{23}}{41}\right)$ (E) $\cos^{-1}\left(\frac{34}{63}\right)$

77. If Q is the image of the point $P(2, 3, 4)$ under the reflection in the plane $x-2y+5z=6$, then the equation of the line PQ is

(A) $\frac{x-2}{-1}=\frac{y-3}{2}=\frac{z-4}{5}$ (B) $\frac{x-2}{1}=\frac{y-3}{-2}=\frac{z-4}{5}$

(C) $\frac{x-2}{-1}=\frac{y-3}{-2}=\frac{z-4}{5}$ (D) $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{5}$

(E) $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z+4}{5}$

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78. The distance of the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=5$ from the point $(-1,-5,-10)$ is

(A) 13 (B) 12 (C) 11 (D) 8 (E) 7

79. If the direction cosines of a line are $\left(\frac{1}{c},\frac{1}{c},\frac{1}{c}\right)$, then

(A) $0<c<1$ (B) $c>2$ (C) $c=\pm\sqrt{2}$ (D) $c=\pm\sqrt{3}$ (E) $c=\pm 3$

80. The vector form of the sphere $2(x^2+y^2+z^2)-4x+6y+8z-5=0$ is

(A) $\vec{r}\cdot[\vec{r}-(2\vec{i}+\vec{j}+\vec{k})]=\frac{2}{5}$

(B) $\vec{r}\cdot[\vec{r}-(2\vec{i}-3\vec{j}-4\vec{k})]=\frac{1}{2}$

(C) $\vec{r}\cdot[\vec{r}-(2\vec{i}+3\vec{j}+4\vec{k})]=\frac{5}{2}$

(D) $\vec{r}\cdot[\vec{r}+(2\vec{i}-3\vec{j}-4\vec{k})]=\frac{5}{2}$

(E) $\vec{r}\cdot[\vec{r}-(2\vec{i}-3\vec{j}-4\vec{k})]=\frac{5}{2}$

81. If the lines $\frac{1-x}{3}=\frac{y-2}{2\alpha}=\frac{z-3}{2}$ and $\frac{x-1}{3\alpha}=y-1=\frac{6-z}{5}$ are perpendicular, then the value of α is

(A) $\frac{-10}{7}$ (B) $\frac{10}{7}$ (C) $\frac{-10}{11}$ (D) $\frac{10}{11}$ (E) $\frac{10}{9}$

82. The distance between the lines $\vec{r}=(4\vec{i}-7\vec{j}-9\vec{k})+t(3\vec{i}-7\vec{j}+4\vec{k})$ and $\vec{r}=(7\vec{i}-14\vec{j}-5\vec{k})+s(-3\vec{i}+7\vec{j}-4\vec{k})$ is equal to

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 17 (E) 0

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83. If the variance of 1, 2, 3, 4, 5 $\cdots$, 10 is $\frac{99}{12}$, then the standard deviation of 3, 6, 9, 12, $\cdots$, 30 is

(A) $\frac{297}{4}$ (B) $\frac{3}{2}\sqrt{33}$ (C) $\frac{3}{2}\sqrt{99}$ (D) $\sqrt{\frac{99}{12}}$ (E) $\frac{3\sqrt{3}}{2}$

84. The mean of the values 0, 1, 2, 3, ...n with the corresponding weights ${}^nC_0, {}^nC_1, \cdots, {}^nC_n$ respectively, is

(A) $\frac{n+1}{2}$ (B) $\frac{n-1}{2}$ (C) $\frac{2^n-1}{2}$ (D) $\frac{2^n+1}{2}$ (E) $\frac{n}{2}$

85. A complete cycle of a traffic light takes 60 seconds. During each cycle the light is green for 25 seconds, yellow for 5 seconds and red for 30 seconds. At a randomly chosen time, the probability that the light will not be green is

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{4}{12}$ (D) $\frac{7}{12}$ (E) $\frac{3}{4}$

86. If the random variable X takes the values $x_1, x_2, x_3 ... x_{10}$ with probabilities $P(X = x_i) = ki$, then the value of k is equal to

(A) $\frac{1}{10}$ (B) $\frac{1}{15}$ (C) $\frac{1}{55}$ (D) 10 (E) 55

87. Let α and β be the roots of $ax^2+bx+c=0$. Then $\lim_{x \to \alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(A) 0 (B) $\frac{1}{2}(\alpha-\beta)^2$ (C) $\frac{a^2}{2}(\alpha-\beta)^2$ (D) $(\alpha-\beta)$ (E) 1

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88. The number of discontinuities of the greatest integer function $f(x)=[x],\quad x\in\left(-\frac{7}{2},100\right)$ is equal to

(A) 104 (B) 100 (C) 102 (D) 101 (E) 103

89. If $f(x)=\begin{cases}\frac{3\sin\pi x}{5x}, & x\neq 0\\ 2k, & x=0\end{cases}$ is continuous at $x=0$, then the value of k is equal to

(A) $\frac{3\pi}{10}$ (B) $\frac{3\pi}{5}$ (C) $\frac{\pi}{10}$ (D) $\frac{3\pi}{2}$ (E) $\frac{2\pi}{3}$

90. If a function f satisfies $f(f(x))=x+1$ for all real values of x and if $f(0)=\frac{1}{2}$, then $f(1)$ is equal to

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2 (E) 0

91. If $y=\log_2\log_2(x)$ then $\frac{dy}{dx}=$

(A) $\frac{\log_2 e}{\log_e x}$ (B) $\frac{\log_2 e}{x\log_x 2}$ (C) $\frac{\log_2 x}{\log_e 2}$ (D) $\frac{\log_2 e}{\log_2 x}$ (E) $\frac{\log_2 e}{x\log_e x}$

92. If $\frac{d}{dx}(f(x))=\frac{1}{1+x^2}$ then $\frac{d}{dx}\left(f(x^3)\right)$ is

(A) $\frac{3x}{1+x^3}$ (B) $\frac{3x^2}{1+x^6}$ (C) $\frac{-6x^5}{(1+x^6)^2}$ (D) $\frac{-6x^5}{1+x^6}$ (E) $\tan^{-1}x$

Space for rough work

93. If $y = \sin\{\cos^{-1}[\sin(\cos^{-1} x)]\}$, then $\frac{dy}{dx}$ at $x = \frac{1}{2}$ is equal to

(A) 0 (B) -1 (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{3}}$ (E) 1

94. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{x^2 y^3}$ (B) $\frac{1}{xy^3}$ (C) $\frac{1}{x^2 y^2}$ (D) $\frac{1}{x^3 y}$ (E) $\frac{-1}{x^3 y}$

95. If $y = \sec^{-1}[\operatorname{cosec} x] + \operatorname{cosec}^{-1}[\sec x] + \sin^{-1}[\cos x] + \cos^{-1}[\sin x]$, then $\frac{dy}{dx}$ is equal to

(A) 0 (B) 2 (C) -2 (D) -4 (E) 1

96. If $y = e^x \cdot e^{x^2} \cdot e^{x^3} \cdots e^{x^n} \cdots$, for $0 < x < 1$, then $\frac{dy}{dx}$ at $x = \frac{1}{2}$ is

(A) e (B) $4e$ (C) $2e$ (D) $3e$ (E) $5e$

97. The derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\cos^{-1}\sqrt{1-x^2}$ is

(A) $\frac{\sqrt{1-x^2}}{1+x^2}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{2}{\sqrt{1-x^2}(1+x^2)}$

(D) $\frac{2}{1+x^2}$ (E) $\frac{2\sqrt{1-x^2}}{1+x^2}$

Space for rough work

98. If the curves $\frac{x^2}{a^2}+\frac{y^2}{12}=1$ and $y^2=8x$ intersect at right angles, then the value of a^2 is equal to

(A) 16 (B) 12 (C) 8 (D) 4 (E) 2

99. If the function $f(x)=x^3-12ax^2+36a^2x-4$ $(a>0)$ attains its maximum and minimum at $x=p$ and $x=q$ respectively and if $3p=q^2$, then a is equal to

(A) $\frac{1}{6}$ (B) $\frac{1}{36}$ (C) $\frac{1}{3}$ (D) 18 (E) 6

100. The equation of the tangent to the curve $y=4e^{-\frac{x}{4}}$ at the point where the curve crosses y-axis is equal to

(A) $3x+4y=16$ (B) $4x+y=4$ (C) $x+y=4$
(D) $4x-3y=-12$ (E) $x-y=-4$

101. The diagonal of a square is changing at the rate of 0.5 cm/sec. Then the rate of change of area, when the area is 400 cm^2, is equal to

(A) $20\sqrt{2}$ cm^2/sec (B) $10\sqrt{2}$ cm^2/sec (C) $\frac{1}{10\sqrt{2}}$ cm^2/sec
(D) $\frac{10}{\sqrt{2}}$ cm^2/sec (E) $5\sqrt{2}$ cm^2/sec

Space for rough work

102. The equation of the tangent to the curve $x^2 - 2xy + y^2 + 2x + y - 6 = 0$ at $(2, 2)$ is

(A) $2x + y - 6 = 0$ (B) $2y + x - 6 = 0$ (C) $x + 3y - 8 = 0$

(D) $3x + y - 8 = 0$ (E) $x + y - 4 = 0$

103. The angle between the curves $y = a^x$ and $y = b^x$ is equal to

(A) $\tan^{-1}\left(\left|\frac{a-b}{1+ab}\right|\right)$ (B) $\tan^{-1}\left(\left|\frac{a+b}{1-ab}\right|\right)$

(C) $\tan^{-1}\left(\left|\frac{\log b + \log a}{1 + \log a \log b}\right|\right)$ (D) $\tan^{-1}\left(\left|\frac{\log a + \log b}{1 - \log a \log b}\right|\right)$

(E) $\tan^{-1}\left(\left|\frac{\log a - \log b}{1 + \log a \log b}\right|\right)$

104. Let $f(x) = (x-7)^2(x-2)^7$, $x \in [2, 7]$. The value of $\theta \in (2, 7)$ such that $f'(\theta) = 0$ is equal to

(A) $\frac{49}{4}$ (B) $\frac{53}{9}$ (C) $\frac{53}{7}$ (D) $\frac{49}{9}$ (E) $\frac{45}{7}$

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105. $\int \left(\sqrt[3]{x}\right)\left(\sqrt[5]{1+\sqrt[3]{x^4}}\right)dx$ is equal to

(A) $\left(1+x^{\frac{3}{4}}\right)^{\frac{6}{5}}+C$ (B) $\left(1+x^{\frac{4}{3}}\right)^{\frac{6}{5}}+C$ (C) $\frac{5}{8}\left(1+x^{\frac{4}{3}}\right)^{\frac{6}{5}}+C$

(D) $\frac{1}{6}\left(1+x^{\frac{4}{3}}\right)^{6}+C$ (E) $\frac{15}{8}\left(1+x^{\frac{4}{3}}\right)^{\frac{6}{5}}+C$

106. If $u=-f''(\theta)\sin\theta+f'(\theta)\cos\theta$ and $v=f''(\theta)\cos\theta+f'(\theta)\sin\theta$, then

$$\int\left[\left(\frac{du}{d\theta}\right)^2+\left(\frac{dv}{d\theta}\right)^2\right]^{\frac{1}{2}}d\theta=$$

(A) $f(\theta)-f''(\theta)+C$
(B) $f(\theta)+f''(\theta)+C$
(C) $f'(\theta)+f''(\theta)+C$
(D) $f'(\theta)-f''(\theta)+C$
(E) $f(\theta)+f'(\theta)+C$

Space for rough work

107. $\int \frac{e^{6\log_e x} - e^{5\log_e x}}{e^{4\log_e x} - e^{3\log_e x}}\, dx$ is equal to

(A) $\frac{x^3}{3}+C$ (B) $\frac{x^2}{2}+C$ (C) $\frac{x^2}{3}+C$ (D) $\frac{-x^3}{3}+C$ (E) $x+C$

108. $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to

(A) $e^x\left(\frac{1-x}{1+x^2}\right)+C$ (B) $e^x\left(\frac{1}{1+x^2}\right)+C$ (C) $e^x\left(\frac{1+x}{1+x^2}\right)+C$

(D) $e^x\left(\frac{1-x}{\left(1+x^2\right)^2}\right)+C$ (E) $e^x\left(\frac{1}{\left(1+x^2\right)^2}\right)+C$

109. $\int \frac{x^4-1}{x^2\left(x^4+x^2+1\right)^{\frac{1}{2}}}\, dx$ is equal to

(A) $\sqrt{\frac{x^4+x^2+1}{x}}+C$ (B) $\frac{x^2}{\sqrt{x^4+x^2+1}}+C$ (C) $x\left(x^4+x^2+1\right)^{\frac{3}{2}}+C$

(D) $\frac{\sqrt{x^4+x^2+1}}{x}+C$ (E) $\sqrt{x^4+x^2+1}+C$

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110. $\int \frac{\cos x - \sin x}{1 + 2\sin x \cos x} dx$ is equal to

(A) $-\frac{1}{\cos x - \sin x} + C$ (B) $\frac{\cos x + \sin x}{\cos x - \sin x} + C$ (C) $-\frac{1}{\sin x + \cos x} + C$

(D) $\frac{x}{\sin x + \cos x} + C$ (E) $\tan x \sec x + C$

111. $\int \frac{1}{x}(\log_{ex} e)\, dx$ is equal to

(A) $\log_e(1 - \log_e x) + C$ (B) $\log_e(\log_e ex - 1) + C$ (C) $\log_e(\log_e x - 1) + C$

(D) $\log_e(\log_e x + x) + C$ (E) $\log_e(1 + \log_e x) + C$

112. The value of $\int_1^e 10^{\log_e x} dx$ is equal to

(A) $10 \log_e(10e)$ (B) $\frac{10e - 1}{\log_e 10e}$ (C) $\frac{10e}{\log_e 10e}$

(D) $(10e)\log_e(10e)$ (E) $\frac{10}{\log_e(10e)}$

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113. The area between the curve $y = 1 - |x|$ and the x-axis is equal to

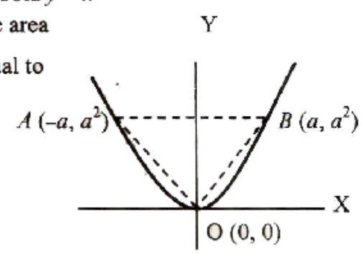
(A) 1 sq.unit (B) $\frac{1}{2}$ sq.unit (C) $\frac{1}{3}$ sq.unit (D) 2 sq.units (E) 3 sq.units

114. The value of $\int_{e^{-1}}^{e} \frac{dt}{t(1+t)}$ is equal to

(A) 0 (B) $\log\left(\frac{e}{1+e}\right)$ (C) $\log\left(\frac{1}{1+e}\right)$

(D) $\log(1+e)$ (E) 1

115. The figure shows a triangle AOB and the parabola $y = x^2$
The ratio of the area of the triangle AOB to the area of the region AOB of the parabola $y = x^2$ is equal to



(A) $\frac{3}{5}$ (B) $\frac{3}{4}$ (C) $\frac{7}{8}$ (D) $\frac{5}{6}$ (E) $\frac{2}{3}$

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116. The value of $\int_{-2}^{4} |x+1|\, dx$ is equal to

(A) 12 (B) 14 (C) 13 (D) 16 (E) 15

117. The solution of $\cos y \frac{dy}{dx} = e^{x+\sin y} + x^2 e^{\sin y}$ is

(A) $e^x - e^{-\sin y} + \frac{x^3}{3} = C$ (B) $e^{-x} - e^{-\sin y} + \frac{x^3}{3} = C$ (C) $e^x + e^{-\sin y} + \frac{x^3}{3} = C$

(D) $e^x - e^{\sin y} - \frac{x^3}{3} = C$ (E) $e^x - e^{\sin y} + \frac{x^3}{3} = C$

118. The order and degree of the differential equation $\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{4}} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$ is

(A) (2, 4) (B) (2, 3) (C) (6, 4) (D) (6, 9) (E) (2, 12)

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119. The integrating factor of the differential equation $(y \log y)dx = (\log y - x)dy$ is

(A) $\frac{1}{\log y}$ (B) $\log(\log y)$ (C) $1 + \log y$ (D) $\frac{1}{\log(\log y)}$ (E) $\log y$

120. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{x + y^2}$ is

(A) $y = -x^2 - 2x - 2 + ce^x$ (B) $y = x^2 + 2x + 2 - ce^x$
(C) $x = -y^2 - 2y + 2 - ce^y$ (D) $x = -y^2 - 2y - 2 + ce^y$
(E) $x = y^2 + 2y + 2 - ce^y$

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