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ಕರ್ನಾಟಕ ಪ್ರೌಢ ಶಿಕ್ಷಣ ಪರೀಕ್ಷಾ ಮಂಡಳಿ, ಮಲ್ಲೇಶ್ವರಂ, ಬೆಂಗಳೂರು – 560 003

KARNATAKA SECONDARY EDUCATION EXAMINATION BOARD, MALLESWARAM, BANGALORE – 560 003

ಎಸ್.ಎಸ್.ಎಲ್.ಸಿ. ಪರೀಕ್ಷೆ, ಮಾರ್ಚ್ / ಏಪ್ರಿಲ್ — 2019

S. S. L. C. EXAMINATION, MARCH/APRIL, 2019

ಮಾದರಿ ಉತ್ತರಗಳು

MODEL ANSWERS

ದಿನಾಂಕ : 25. 03. 2019] ಸಂಕೇತ ಸಂಖ್ಯೆ : **81-E**

Date : 25. 03. 2019] CODE No. : **81-E**

ವಿಷಯ : ಗಣಿತ

Subject : MATHEMATICS

(ಹೊಸ ಪಠ್ಯಕ್ರಮ / New Syllabus)

(ಖಾಸಗಿ ಅಭ್ಯರ್ಥಿ / Private Fresh)

(ಇಂಗ್ಲಿಷ್ ಭಾಷಾಂತರ / English Version)

[ಗರಿಷ್ಠ ಅಂಕಗಳು : **100**

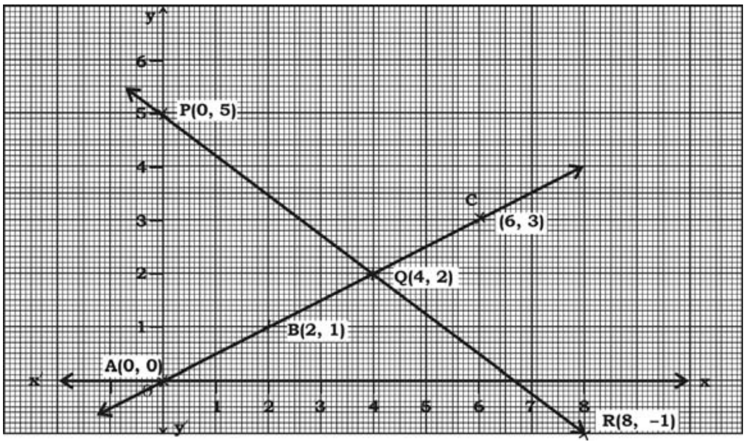
[Max. Marks : 100

Qn. Nos.	Ans. Key	Value Points	Marks allotted
I. 1.		If the *n*-th term of an arithmetic progression $a_n = 24 - 3n$, then its 2nd term is	
		(A) 18 (B) 15	
		(C) 0 (D) 2	
		Ans. :	
	(A)	18	1

Qn. Nos.	Ans. Key	Value Points	Marks allotted
2.		The lines represented by $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ are	
		(A) Intersecting lines	
		(B) Perpendicular lines to each other	
		(C) Parallel lines	
		(D) Coincident lines	
		Ans. :	
	(D)	Coincident lines	1
3.		A straight line which passes through two points on a circle is	
		(A) a chord (B) a secant	
		(C) a tangent (D) the radius	
		Ans. :	
	(B)	a secant	1
4.		If the area of a circle is 49π sq.units then its perimeter is	
		(A) 7π units (B) 9π units	
		(C) 14π units (D) 49π units	
		Ans. :	
	(C)	14π units	1
5.		"The product of two consecutive positive integers is 30." This can be expressed algebraically as	
		(A) $x(x+2) = 30$ (B) $x(x-2) = 30$	
		(C) $x(x-3) = 30$ (D) $x(x+1) = 30$	
		Ans. :	
	(D)	$x(x+1) = 30$	1

Qn. Nos.	Ans. Key	Value Points	Marks allotted
6.		If a and b are any two positive integers then HCF (a, b) × LCM (a, b) is equal to	
		(A) $a + b$	
		(B) $a - b$	
		(C) $a \times b$	
		(D) $a \div b$	
		Ans. :	
	(C)	$a \times b$	1
7.		The value of $\cos 48° - \sin 42°$ is	
		(A) 0 (B) $\frac{1}{4}$	
		(C) $\frac{1}{2}$ (D) 1	
		Ans. :	
	(A)	0	1
8.		If $P(A) = 0{\cdot}05$ then $P(\overline{A})$ is	
		(A) 0·59	
		(B) 0·95	
		(C) 1	
		(D) 1·05	
		Ans. :	
	(B)	0·95	1

Qn. Nos.	Value Points	Marks allotted
II.	Answer the following : $6 \times 1 = 6$	
	(Question Numbers 9 to 14, give full marks to direct answers)	
9.	The given graph represents a pair of linear equations in two variables. Write how many solution these pair of equations have.	



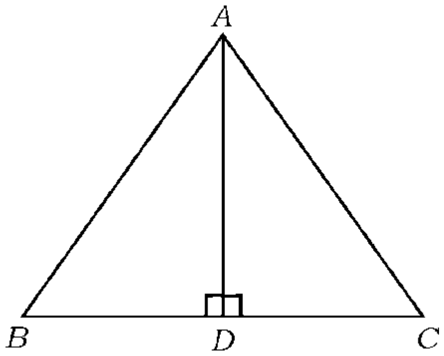
Qn. Nos.	Value Points	Marks allotted
	Ans. :	
	one or unique	1
10.	17 = 6 × 2 + 5 is compared with Euclid's Division lemma $a = bq + r$, then which number is representing the remainder ?	
	Ans. :	
	5	1
11.	Find the zeroes of the polynomial $P(x) = x^2 - 3$.	
	Ans. :	
	$x^2 - 3 = 0$	
	$(x + \sqrt{3})(x - \sqrt{3}) = 0$	
	$x = -\sqrt{3}$, $x = \sqrt{3}$	½ + ½
	Direct answer give full marks.	1

Qn. Nos.	Value Points		Marks allotted
12.	Write the degree of the polynomial $P(x) = 2x^2 - x^3 + 5$.		
	Ans. :		
	3		1
13.	Find the value of the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$.		
	Ans. :		
	$b^2 - 4ac$	½	
	$= (-4)^2 - 4 \times 2 \times 3$		
	$= 16 - 24$		
	$= -8$	½	1
14.	Write the formula to calculate the curved surface area of the frustum of a cone.		
	Ans. :		
	$\pi l (r_1 + r_2)$		1
III. 15.	Find the sum of first twenty terms of Arithmetic series 2 + 7 + 12 + ... using suitable formula.	2	
	Ans. :		
	$a = 2$ $d = 7 - 2 = 5$ $n = 20$		
	$S_n = \frac{n}{2}[2a + (n-1)d]$	½	
	$S_{20} = \frac{20}{2}[2 \times 2 + (20 - 1) \times 5]$	½	
	$= 10[4 + 19 \times 5]$		
	$= 10 \times 99$	½	
	$S_{20} = 990$	½	2

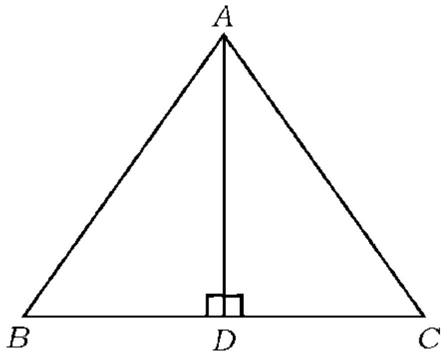
Qn. Nos.	Value Points	Marks allotted

16. In $\Delta\, ABC$, $AD \perp BC$ and $AD^2 = BD \times CD$, prove that

$$AB^2 + AC^2 = (BD + CD)^2 .$$ 2



Ans. :



In $\Delta\, ABD$

$AB^2 = AD^2 + BD^2$ ... (i) ½

In $\Delta\, ADC$

$AC^2 = AD^2 + CD^2$ ... (ii) ½

(i) + (ii)

$AB^2 + AC^2 = 2AD^2 + BD^2 + CD^2$

Put $AD^2 = BD \times CD$ } ½

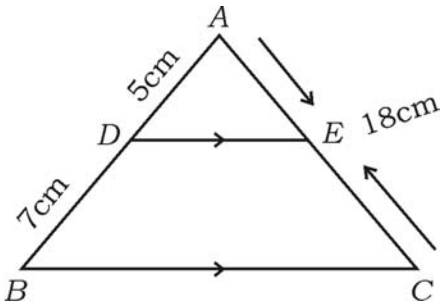
$AB^2 + AC^2 = 2BD\,.\,CD + BD^2 + CD^2$

$AB^2 + AC^2 = (BD + CD)^2$ } ½

2

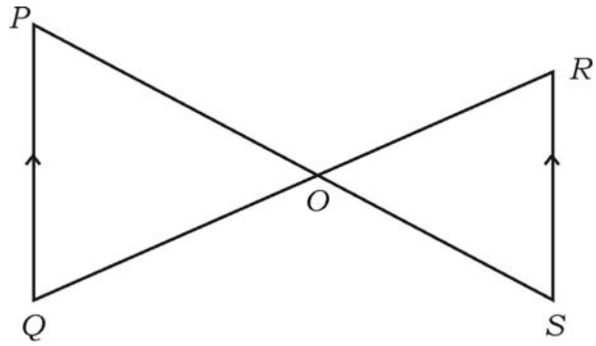
Qn. Nos.	Value Points	Marks allotted

17. In $\Delta\,ABC$, $DE \parallel BC$. If $AD = 5$ cm, $BD = 7$ cm and $AC = 18$ cm, find the length of AE. 2

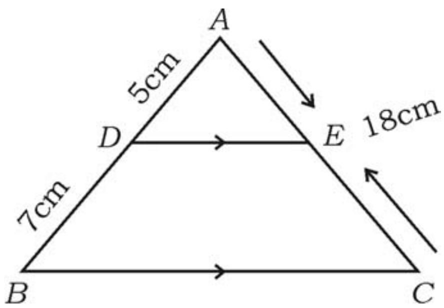


OR

In the given figure if $PQ \parallel RS$, prove that $\Delta\,POQ \sim \Delta\,SOR$.



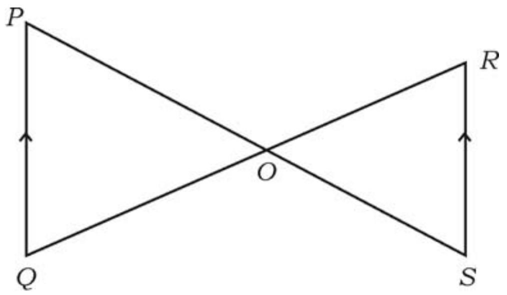
Ans. :



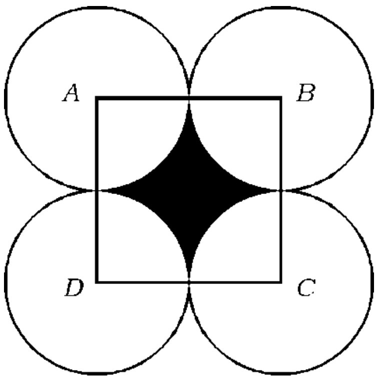
In $\Delta\,ABC$, $DE \parallel BC$

$\therefore \quad \dfrac{AD}{AB} = \dfrac{AE}{AC}$ ½

$\dfrac{5}{12} = \dfrac{AE}{18}$ ½

Qn. Nos.	Value Points		Marks allotted
	$\frac{5}{12} \times 18 = AE$	½	
	$AE = \frac{15}{2}$		
	$AE = 7 \cdot 5$ cm	½	2
	Alternate method give marks.		
	OR		
			
	In $\Delta\ POQ$ and $\Delta\ SOR$		
	$\angle P = \angle S$ (Alternate angles)		
	$\angle Q = \angle R$ (Alternate angles)	1½	
	$\angle POQ = \angle ROS$ (V.O.A.)		
	(*A.A.* criterion)		
	$\Delta\ POQ \sim \Delta\ SOR.$	½	2
18.	Solve the following pair of linear equations by any suitable method :	2	
	$x + y = 5$		
	$2x - 3y = 5.$		
	Ans. :		
	Substitution method :		
	$x + y = 5$ ... (i)		
	$2x - 3y = 5$ ... (ii)		
	$x + y = 5$		
	$y = 5 - x$	½	

Qn. Nos.	Value Points		Marks allotted
	Substitute the value of y in equation (ii) we get		
	$2x - 3(5 - x) = 5$	½	
	$2x - 15 + 3x = 5$		
	$5x - 15 = 5$		
	$5x = 5 + 15$		
	$5x = 20$		
	$x = \frac{20}{5}$		
	$x = 4$	½	
	Substituting the value of x in equation (i)		
	$x + y = 5$		
	$4 + y = 5$		
	$y = 5 - 4$		
	$y = 1$	½	2
	Elimination method :		
	$x + y = 5$		
	$x + y = 5$ ... (i) × 2		
	$2x - 3y = 5$ ... (ii)		
	$2x + 2y = 10$ ... iii		
	$2x - 3y = 5$ ... ii	½	
	(–) (+) (–) (iii) — (ii)		
	$5y = 5$		
	$y = \frac{5}{5}$ $y = 1$	½	
	Substitute the value of y in equation (i)		
	$x + y = 5$	½	
	$x + 1 = 5$		
	$x = 5 - 1$		
	$x = 4$	½	2

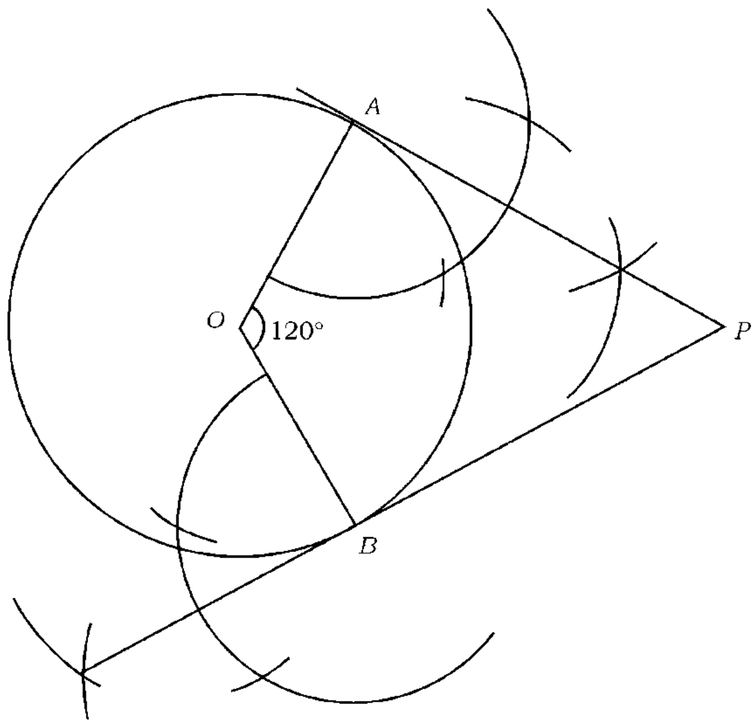
Qn. Nos.	Value Points		Marks allotted
	Cross multiplication method :		
	$\begin{array}{cccccc} & x & & y & & 1 \\ 1 & & -5 & & 1 & & 1 \\ -3 & & -5 & & 2 & & -3 \end{array}$	½	
	$\frac{x}{-5-15} = \frac{y}{-10+5} = \frac{1}{-3-2}$	½	
	$\frac{x}{-20} = \frac{y}{-5} = \frac{1}{-5}$		
	$\frac{x}{-20} = \frac{1}{-5}$		
	$-5x = -20$		
	$x = \frac{-20}{-5}$		
	$x = 4$	½	
	$\frac{y}{-5} = -\frac{1}{5}$		
	$-5y = -5$		
	$y = \frac{-5}{-5}$		
	$y = 1$	½	2
19.	In the figure, *ABCD* is a square of side 14 cm. *A, B, C* and *D* are the centres of four congruent circles such that each circle touch externally two of the remaining three circles. Find the area of the shaded region. 2		
			
	Ans. :		

Qn. Nos.	Value Points		Marks allotted
	Area of the shaded region =		
	Area of square – 4 × area of quadrant	½	
	Area of a square = $(\text{side})^2$		
	$= (14)^2$		
	Area of the square = 196 cm^2	½	
	Area of a quadrant = $\frac{1}{4}\pi r^2$		
	4 × Area of quadrant = $4 \times \frac{1}{4}\pi r^2$	½	
	$= 4 \times \frac{1}{4} \times \frac{22}{7} \times 7^2$		
	4 × Area of quadrant = 22×7		
	$= 154 \text{ cm}^2$		
	Area of shaded region = 196 – 154		
	Area of shaded region = 42 cm^2	½	2
	Alternate method :		
	Area of the shaded region =		
	Area of a square – 4 × area of quadrant	½	
	Area of a square = $(\text{side})^2$		
	$= (14)^2$		
	Area of the square = 196 cm^2	½	
	Area of a quadrant = $\frac{\theta}{360^\circ} \times \pi r^2$		
	4 × area of a quadrant = $4 \times \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 7^2$		
	$= 154 \text{ cm}^2$	½	
	Area of shaded region = 196 – 154		
	Area of shaded region = 42 cm^2.	½	2
	Note : Any alternate method marks can be given.		
	[Area of shaded region = Area of a square – Area of a circle]		

Qn. Nos.	Value Points	Marks allotted
20.	Draw a circle of radius 4 cm and construct a pair of tangents such that the angle between them is 60°. 2	

Ans. :

Angle between the radius = 180° – 60° = 120° ½



Circle — ½

Radii — ½

Tangents — ½ **2**

21. Find the co-ordinates of point which divides the line segment joining the points (4, – 3) and (8, 5) in the ratio 3 : 1 internally. 2

Ans. :

Let $P(x, y)$ be the required point

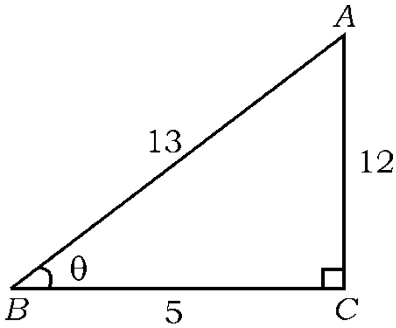
$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$ 1

OR $$P(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

Qn. Nos.	Value Points		Marks allotted
	$= \left(\frac{3 \times (8) + (4)}{3+1}, \frac{3 \times (5) + 1 \times (-3)}{3+1} \right)$	½	
	$= \left(\frac{24+4}{4}, \frac{15-3}{4} \right)$		
	$= \left(\frac{28}{4}, \frac{12}{4} \right)$		
	$(x, y) = (7, 3)$	½	2
22.	Prove that $3 + \sqrt{5}$ is an irrational number.	2	
	Ans. :		
	Let us assume $3 + \sqrt{5}$ is a rational number		
	$3 + \sqrt{5} = \frac{p}{q}$ where $p, q \in z$, $q \neq 0$	½	
	$\sqrt{5} = \frac{p}{q} - 3$		
	Rearranging this equation		
	$\sqrt{5} = \frac{p-3q}{q}$	½	
	Since p and q are integers we get $\frac{p-3q}{q}$ is rational	½	
	So $\sqrt{5}$ is rational.		
	But this contradicts the fact that $\sqrt{5}$ is rational		
	$\therefore$ $3 + \sqrt{5}$ is irrational	½	2
23.	The sum and product of the zeroes of a quadratic polynomial $P(x) = ax^2 + bx + c$ are -3 and 2 respectively. Show that $b + c = 5a$.	2	
	Ans. :		

Qn. Nos.	Value Points	Marks allotted
	Let α and β are the zeroes of the quadratic polynomial $P(x)$	
	$\alpha + \beta = -3$ ½	
	$-\frac{b}{a} = -3$	
	$-b = -3a$	
	$b = 3a$ … (i) ½	
	$\alpha\beta = 2$	
	$\frac{c}{a} = 2$	
	$c = 2a$ … (ii) ½	
	(i) + (ii) gives	
	$b + c = 3a + 2a$	
	$b + c = 5a.$ ½	2
24.	Find the quotient and the remainder when $P(x) = 3x^3 + x^2 + 2x + 5$ is divided by $g(x) = x^2 + 2x + 1$. 2	
	Ans. :	
	$x^2 + 2x + 1 \overline{)\, 3x^3 + x^2 + 2x + 5\,(}\; 3x - 5$	
	$3x^3 + 6x^2 + 3x$ 1	
	(–) (–) (–)	
	$-5x^2 - x + 5$	
	$-5x^2 - 10x - 5$	
	(+) (+) (+)	
	$9x + 10$	
	Quotient $= 3x - 5$ ½	
	Remainder $= 9x + 10$ ½	2

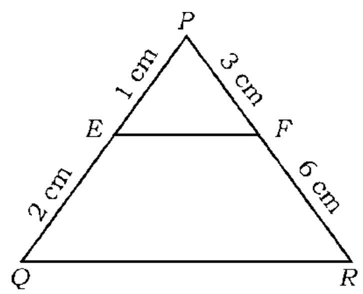
Qn. Nos.	Value Points	Marks allotted
25.	Solve $2x^2 - 5x + 3 = 0$ by using formula. 2	
	Ans. :	
	Comparing the equation with	
	$ax^2 + bx + c = 0$	
	$a = 2 \qquad b = -5 \qquad c = 3$	
	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ½	
	$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times 3}}{2 \times 2}$ ½	
	$x = \frac{5 \pm \sqrt{25 - 24}}{4}$	
	$x = \frac{5 \pm \sqrt{1}}{4}$	
	$x = \frac{5 \pm 1}{4}$	
	$x = \frac{5+1}{4}$, $\quad$ $x = \frac{5-1}{4}$ ½	
	$x = \frac{6}{4}$ $\quad$ $x = \frac{4}{4}$	
	$x = \frac{3}{2}$ $\quad$ $x = 1$ ½	2
26.	The length of a rectangular field is 3 times its breadth. If the area of the field is 147 sq.m find its length and breadth. 2	
	Ans. :	
	Let the breadth be, x	
	$\therefore$ Length $= 3x$ ½	
	$A = l \times b$	
	$147 = 3x \times x$ ½	
	$147 = 3x^2$	
	$x^2 = \frac{147}{3}$	

Qn. Nos.	Value Points		Marks allotted
	$x^2 = 49$		
	$x = \pm\sqrt{49}$		
	$x = \pm 7$	½	
	$\therefore$ Breadth $(x) = 7$ cm		
	Length $(3x) = 3 \times 7 = 21$ cm	½	2
27.	If $\sin\theta = \frac{12}{13}$ find the values of $\cos\theta$ and $\tan\theta$.	2	
	OR		
	If $\sqrt{3}\tan\theta = 1$ and θ is acute find the value of $\sin 3\theta + \cos 2\theta$.		
	Ans. :		
		½	
	$AB^2 = AC^2 + BC^2$		
	$13^2 = 12^2 + BC^2$		
	$169 = 144 + BC^2$		
	$BC^2 = 169 - 144$		
	$BC^2 = 25$ $BC = \sqrt{25}$		
	$BC = 5$	½	
	$\cos\theta = \frac{BC}{AC} = \frac{5}{13}$	½	
	$\tan\theta = \frac{AC}{BC} = \frac{12}{5}$	½	2
	OR		

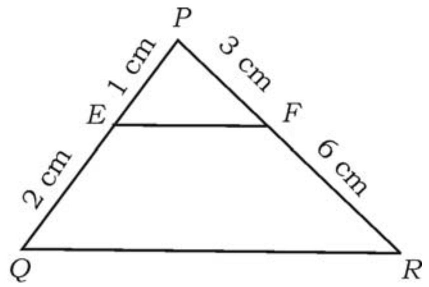
Qn. Nos.	Value Points		Marks allotted
	$\sqrt{3}\tan\theta = 1$		
	$\tan\theta = \frac{1}{\sqrt{3}}$	½	
	$\tan\theta = \tan 30°$		
	$\theta = 30°$		
	$\sin 3\theta = \sin 3\times 30° = \sin 90° = 1$		
	$\cos 2\theta = \cos 2\times 30° = \cos 60° = \frac{1}{2}$	1	
	$\sin 3\theta + \cos 2\theta = 1 + \frac{1}{2} = 1\frac{1}{2}$		
	$\sin 3\theta + \cos 2\theta = \frac{3}{2}$	½	2
28.	Prove that $\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = (\operatorname{cosec}\theta + \cot\theta)^2$.	2	
	Ans. :		
	L.H.S. $= \left(\frac{1+\cos\theta}{1-\cos\theta}\right)$		
	$= \frac{(1+\cos\theta)}{(1-\cos\theta)} \times \frac{(1+\cos\theta)}{(1+\cos\theta)}$	½	
	$= \frac{(1+\cos\theta)^2}{1^2 - \cos^2\theta}$		
	$= \frac{(1+\cos\theta)^2}{\sin^2\theta}$	½	
	$= \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$		
	$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$	½	
	$\frac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2 = \text{R.H.S.}$	½	2
	Any alternative method, marks can be awarded.		

Qn. Nos.	Value Points	Marks allotted
29.	A cubical die numbered from 1 to 6 are rolled twice. Find the probability of getting the sum of numbers on its faces is 10. 2	
	Ans. :	
	$n(S) = 36$ ½	
	$n(A) = \{(5,5)\ (4,6)\ (6,4)\} = 3$ ½	
	$P(A) = \frac{n(A)}{n(S)}$ ½	
	$= \frac{3}{36}$ ½	2
30.	The radii of two circular ends of a frustum of a cone shaped dustbin are 15 cm and 8 cm. If its depth is 63 cm, find the volume of the dustbin. 2	
	Ans. :	
	$r_1 = 15$ cm $r_2 = 8$ cm $h = 63$ cm	
	Volume of dustbin (V) $= \frac{1}{3}\pi h\ (r_1^2 + r_2^2 + r_1 r_2)$ ½	
	$= \frac{1}{3} \times \frac{22}{7} \times 63\ (15^2 + 8^2 + 15 \times 8)$ ½	
	$= 66\ (225 + 64 + 120)$ ½	
	$= 66 \times 409$	
	Volume of dustbin (V) = 26994 cm^3. ½	2
31.	If x, 13, y and 3 are in arithmetic progression, find the values of x and y. 2	
	Ans. :	
	Let $x = a + 3d$, $13 = a + 2d$ $y = a + d$ and $a = 3$ ½	
	$a + 2d = 13$	
	$3 + 2d = 13$	
	$2d = 13 - 3$	

Qn. Nos.	Value Points		Marks allotted
	$d = \frac{10}{2}$		
	$d = 5$	½	
	$x = a + 3d$		
	$= 3 + 3 \times 5$		
	$= 3 + 15$		
	$x = 18$	½	
	$y = a + d$		
	$= 3 + 5$		
	$y = 8$	½	2
32.	In $\Delta\, PQR$, E and F are points on PQ and PR respectively. If $PE = 1$ cm, $QE = 2$ cm, $PF = 3$ cm and $RF = 6$ cm, show that $EF \parallel QR$.	2	



Ans. :



$\frac{PE}{EQ} = \frac{1}{2}$	... (i)	½	
$\frac{PF}{FR} = \frac{3}{6} = \frac{1}{2}$	... (ii)	½	
from (i) and (ii)			
$\frac{PE}{EQ} = \frac{PF}{FR}$		½	
$\therefore \quad EF \parallel QR$		½	2

Qn. Nos.	Value Points	Marks allotted

33. Find the HCF and LCM of 6 and 20. 2

Ans. :

2 | 6
3 | 3
    1

2 | 20
2 | 10
5 | 5
    1

Factors of 6 = 2×3 ½

Factors of 20 = $2^2 \times 5$ ½

H.C.F. = 2 ½

L.C.M. = $2^2 \times 3 \times 5$

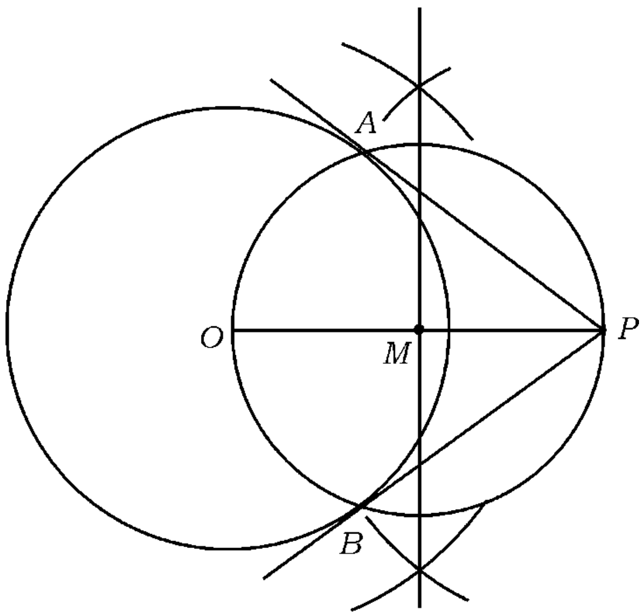
= $4 \times 3 \times 5$

L.C.M. = 60 ½ | 2

Any correct alternate method may be given marks.

34. Draw a tangent to a circle of radius 3 cm from a point 5 cm away from its centre. 2

Ans. :



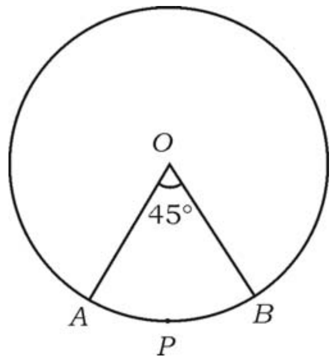
Circle — ½

Bisecting *OP* — ½

Tangents — 1 | 2

Qn. Nos.	Value Points		Marks allotted
35.	In a circle of radius 21 cm an arc subtends an angle of 60° at the centre. Find the length of the arc.	2	
	Ans. :		
	$r = 21$ cm $\theta = 60°$	½	
	Length of the arc $= \frac{\theta}{360°} \times 2\pi r$	½	
	$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21$	½	
	Length of the arc = 22 cm	½	2
36.	Express the given equation in the standard form $(x-2)^2 + 1 = 2x + 3$.	2	
	Ans. :		
	$(x-2)^2 + 1 = 2x + 3$		
	$x^2 - 2 \times x \times 2 + 2^2 + 1 = 2x + 3$	½	
	$x^2 - 4x + 4 + 1 = 2x + 3$	½	
	$x^2 - 4x + 5 = 2x + 3$		
	$x^2 - 4x + 5 - 2x - 3 = 0$	½	
	Standard form = $x^2 - 6x + 2 = 0$	½	2
37.	Write the probability of sure event and impossible event.	2	
	Ans. :		
	Probability of sure event — 1		
	Probability of Impossible event — 0		2

Qn. Nos.	Value Points	Marks allotted
38.	Find the area of the sector of a circle with radius 4 cm and of angle 45°. (use $\pi = 3{\cdot}14$). 2	



Ans. :

$\theta = 45°$ $r = 4$ ½

Area of the sector $= \frac{\theta}{360°} \times \pi r^2$ ½

$= \frac{45°}{360°} \times 3 \cdot 14 \times 4^2$ ½

$= \frac{1}{8} \times 3 \cdot 14 \times 16$

Area of the sector $= 6{\cdot}28\ \text{cm}^2$ ½ — 2

39. Find the distance between (3, 4) from the origin. 2

Ans. :

Co-ordinates of origin (0, 0) ½

$(x_1, y_1) = (0, 0)$ $(x_2, y_2) = (3, 4)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ½

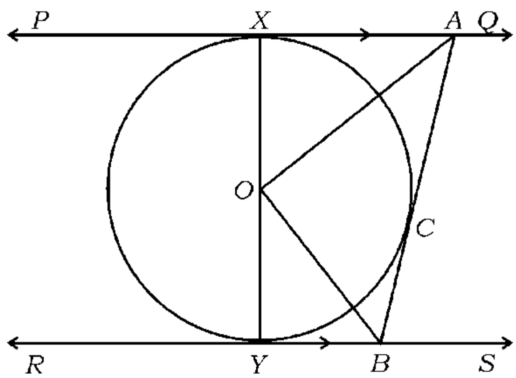
$= \sqrt{(3-0)^2 + (4-0)^2}$ ½

$= \sqrt{3^2 + 4^2}$

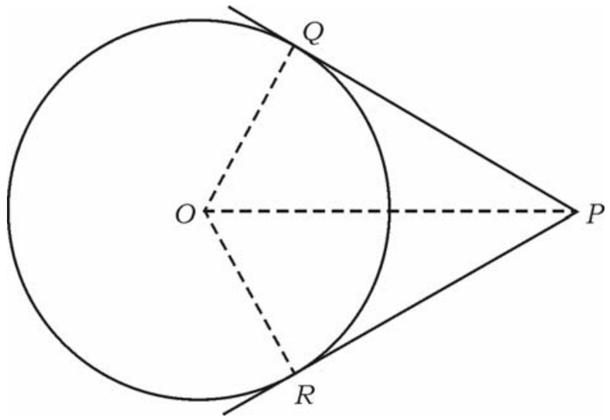
$= \sqrt{9 + 16}$

$= \sqrt{25}$

$d = 5$ units. ½ — 2

Qn. Nos.	Value Points	Marks allotted
40.	Two coins are tossed together. Find the probability of getting at least one head. 2	
	Ans. :	
	$n(S) = 4$ ½	
	$n(A) = \{(H, H), (H, T) (T, H)\} = 3$ ½	
	$P(A) = \frac{n(A)}{n(S)}$ ½	
	$P(A) = \frac{3}{4}$ ½	2
IV. 41.	Prove that "the lengths of tangents drawn from an external point to a circle are equal". 3	
	OR	
	In the given figure *PQ* and *RS* are two parallel tangents to a circle with centre *O* and another tangent *AB* with point of contact *C* intersecting *PQ* at *A* and *RS* at *B*. Prove that $\angle AOB = 90°$.	
		
	Ans. :	

Qn. Nos.	Value Points		Marks allotted



½

Data : O is the centre of the circle P is an external point
PQ and PR are the tangents ½

To prove : $PQ = PR$ ½

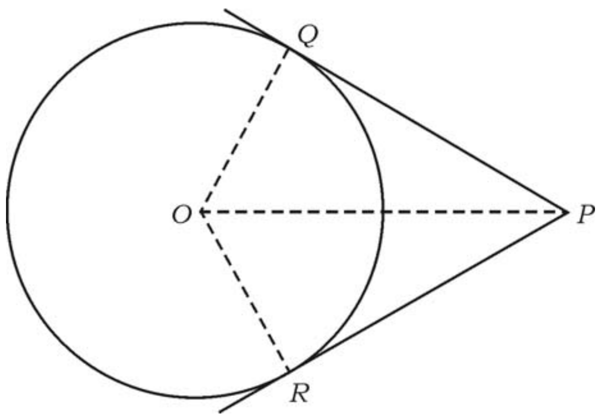
Construction : OQ, OR and OP are joined ½

Proof : In $\Delta\ POQ$ and $\Delta\ POR$
$\angle PQO = \angle PRO$ (Radius drawn at the point of contact is perpendicular to the tangent)
$hyp\ \ OP = hyp\ \ OP$ (Common side)
$OQ = OR$ (Radii of same circle)

$\therefore$ $\Delta\ POQ \equiv \Delta\ POR$ (R.H.S. theorem) ½

$\therefore$ $PQ = PR$ ½ | 3

Alternate method :



½

Qn. Nos.	Value Points		Marks allotted

Proof : We are given a circle with centre O a point P lying outside the circle and two tangents PQ and PR on the circle from P. ½

We are required to prove that $PQ = PR$ ½

For this we join OP, OQ and OR.

Then $\angle OQP$ and $\angle ORP$ are right angles because these are angles between the radii and tangents. ½

According to theorem 4·1 they are right angles

Now in right angles $\angle OQP$ and $\angle ORP$

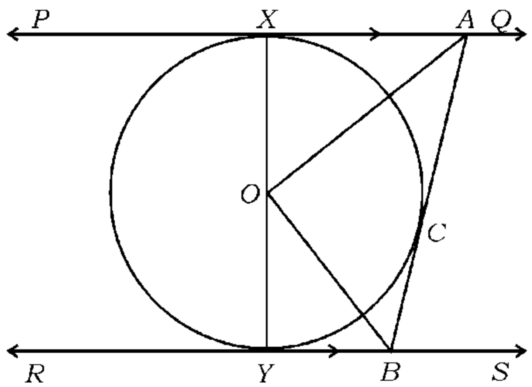
$OQ = OR$ (Radii of same circle) ½

$OP = OP$ (common)

Therefore $\Delta\, OQP = \Delta\, ORP$ (R.H.S.)

This gives $PQ = PR$. ½ — 3

OR



Let $\angle OAB = x$

$\therefore \quad \angle OAX = x$

$\angle OBA = y$ ½

$\angle OBY = y$

$PQ \parallel RS$

$\therefore \quad \angle XAB + \angle YBA = 180°$

$2x + 2y = 180°$ 1

$2(x + y) = 180°$

Qn. Nos.	Value Points		Marks allotted
	$x + y = \frac{180^\circ}{2}$		
	$x + y = 90^\circ$		
	In ΔAOB		
	$\angle OAB + \angle OBA + \angle AOB = 180^\circ$	1	
	$x + y + \angle AOB = 180^\circ$		
	$90^\circ + \angle AOB = 180^\circ \quad (\because x + y = 90^\circ)$		
	$\angle AOB = 180^\circ - 90^\circ$		
	$\angle AOB = 90^\circ$	½	3

42. Calculate the median of the following frequency distribution table : 3

Class-interval	*Frequency* (f_i)
1 — 4	6
4 — 7	30
7 — 10	40
10 — 13	16
13 — 16	4
16 — 19	4

$\Sigma f_i = 100$

OR

Calculate the mode for the following frequency distribution table.

Class-interval	*Frequency* (f_i)
10 — 25	2
25 — 40	3
40 — 55	7
55 — 70	6
70 — 85	6
85 — 100	6

$\Sigma f_i = 30$

Ans. :

Qn. Nos.	Value Points	Marks allotted

Class-interval	*Frequency*	*Cumulative frequency*
1 — 4	6	6
4 — 7	30	36
7 — 10	40	76
10 — 13	16	92
13 — 16	4	96
16 — 19	4	100

½

$$\frac{n}{2} = \frac{100}{2} = 50$$

Lower limit of median class $l = 7$

C.F. of class preceding median class $c.f. = 36$ 1

Frequency of median class $f = 40$

Class size $h = 3$

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$ ½

$$= 7 + \left[\frac{50 - 36}{40}\right] \times 3$$ ½

$$= 7 + \left[\frac{14}{40}\right] \times 3$$

$$= 7 + \frac{21}{20}$$

$$= 7 + 1{\cdot}05$$

Median $= 8{\cdot}05$ ½ 3

OR

Lower limit $l = 40$

Frequency of modal class $f_1 = 7$

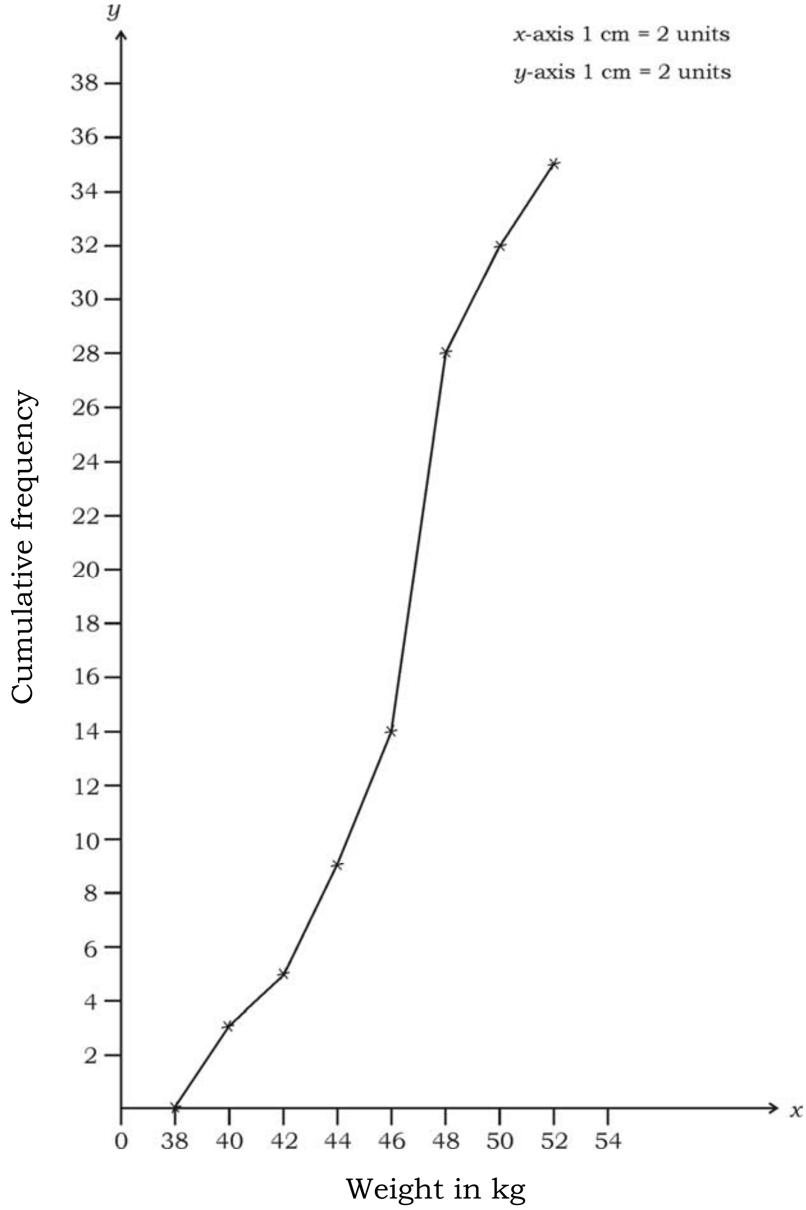
Frequency of preceding modal class $f_0 = 3$

Qn. Nos.	Value Points	Marks allotted
	Succeeding modal class $f_2 = 6$	
	Class size $h = 15$ 1	
	Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$ ½	
	$= 40 + \left[\frac{7-3}{14-6-3}\right] \times 15$ ½	
	$= 40 + \left[\frac{4}{5}\right] \times 15$	
	$= 40 + \frac{4}{5} \times 15$ ½	
	$= 40 + 12$	
	Mode $= 52$ ½	3
43.	During the medical check-up of 35 students of a class, their weights were recorded as follows. Draw a less than type of ogive for the given data : 3	

Weight (in kg)	*Number of students*
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Ans. :

Qn. Nos.	Value Points	Marks allotted



	x and y axis scale —	½	
	Plotting points —	1½	
	Drawing graph —	1	3

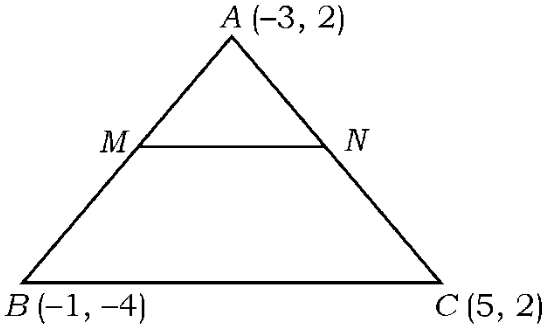
Note : Scale, x-axis, y-axis can be changed.

44. The seventh term of an Arithmetic progression is four times its second term and twelveth term is 2 more than three times of its fourth term. Find the progression. 3

OR

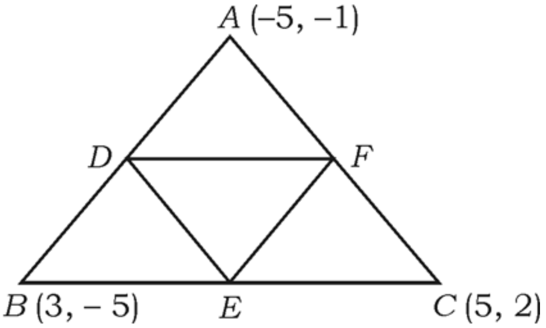
Qn. Nos.	Value Points	Marks allotted
	A line segment is divided into four parts forming an Arithmetic progression. The sum of the lengths of 3rd and 4th parts is three times the sum of the lengths of first two. If the length of fourth part is 14 cm, find the total length of the line segment.	
	Ans. :	
	$a_7 = T_7 = 4(T_2)\ a_2$ $a + 6d = 4(a + d)$	½
	$a + 6d = 4a + 4d$	
	$6d - 4d = 4a - a$	
	$2d = 3a$ ... (i)	½
	$a_{12} = T_{12} = 3T_4\ (a_4) + 2$	
	$a + 11d = 3(a + 3d) + 2$	
	$a + 11d = 3a + 9d + 2$	
	$11d - 9d = 3a - a + 2$	
	$2d = 2a + 2$ ... (ii)	½
	substituting (i) in (ii)	
	$3a = 2a + 2$	
	$3a - 2a = 2$	
	$a = 2$	½
	$2d = 3a$	
	$2d = 3 \times 2$	
	$2d = 6$	
	$d = \frac{6}{2}$	
	$d = 3$	½

Qn. Nos.	Value Points		Marks allotted
	$\therefore$ The required sequence		
	$a, \quad a+d, \quad a+2d$		
	$2, \quad 2+3, \quad 2+2\times 3$		
	The required sequence 2, 5, 8	½	3
	OR		
	Let the four parts of the line segment be		
	$a-3d, \; a-d, \; a+d, \; a+3d$	½	
	According to the data		
	$(a+d+a+3d) = 3(a-3d+a-d)$		
	$2a+4d = 3(2a-4d)$	½	
	$2(a+2d) = 3\times 2(a-2d)$		
	$a+2d = 3a-6d$		
	$2d+6d = 3a-a$		
	$2a = 8d$		
	$a = \frac{8d}{2}$		
	$a = 4d$	½	
	$a+3d = 14$	½	
	$4d+3d = 14$		
	$7d = 14$		
	$d = \frac{14}{7}$		
	$d = 2$		
	$a = 4d$		
	$a = 4\times 2$		
	$a = 8$	½	

Qn. Nos.	Value Points	Marks allotted
	∴ Length of the line segment =	
	$= a - 3d + a - d + a + d + a + 3d$	
	$= 4a$	
	$= 4 \times 8 = 32$ cm. ½	3
45.	The vertices of a ΔABC are $A(-3, 2)$, $B(-1, -4)$ and $C(5, 2)$. If M and N are the mid-points of AB and AC respectively, show that $2MN = BC$. 3	
	OR	
	The vertices of a ΔABC are $A(-5, -1)$, $B(3, -5)$, $C(5, 2)$. Show that the area of the ΔABC is four times the area of the triangle formed by joining the mid-points of the sides of the triangle ABC.	
	Ans. :	
		
	Co-ordinates of $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	
	$= \left(\frac{-1-3}{2}, \frac{-4+2}{2} \right)$	
	Co-ordinates of $M = (-2, -1)$ 1	
	Co-ordinates of $N = \left(\frac{5-3}{2}, \frac{2+2}{2} \right)$	
	$= \left(\frac{2}{2}, \frac{4}{2} \right)$	
	Co-ordinates of $N = (1, 2)$	
	Length of $MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ½	

Qn. Nos.	Value Points		Marks allotted
	$= \sqrt{(1+2)^2 + (2+1)^2}$		
	$= \sqrt{3^2 + 3^2}$	½	
	$= \sqrt{9+9} = \sqrt{18}$		
	$= \sqrt{9 \times 2} = 3\sqrt{2}$		
	$MN = 3\sqrt{2}$		
	Length of $BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$		
	$= \sqrt{(5+1)^2 + (2+4)^2}$		
	$= \sqrt{6^2 + 6^2}$		
	$= \sqrt{36 + 36}$		
	$= \sqrt{72}$		
	$= \sqrt{36 \times 2}$		
	$BC = 6\sqrt{2}$	½	
	$2MN = 2 \times 3\sqrt{2}$		
	$= 6\sqrt{2}$		
	$\therefore \quad 2MN = BC$	½	3
	OR		

Qn. Nos.	Value Points	Marks allotted



$(x_1, y_1) = (-5, -1)$, $(x_2, y_2) = (3, -5)$, $(x_3, y_3) = (5, 2)$

Area of triangle ABC =

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-5(-5-2) + 3(2+1) + 5(-1+5)]$ ½

$= \frac{1}{2}[5 \times (-7) + 3 \times 3 + 5 \times 4]$

$= \frac{1}{2}[35 + 9 + 20]$

$= \frac{1}{2} \times 64$ ½

Area of ΔABC = 32 sq.units

Co-ordinates of $D = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-5+3}{2}, \frac{-1-5}{2}\right)$

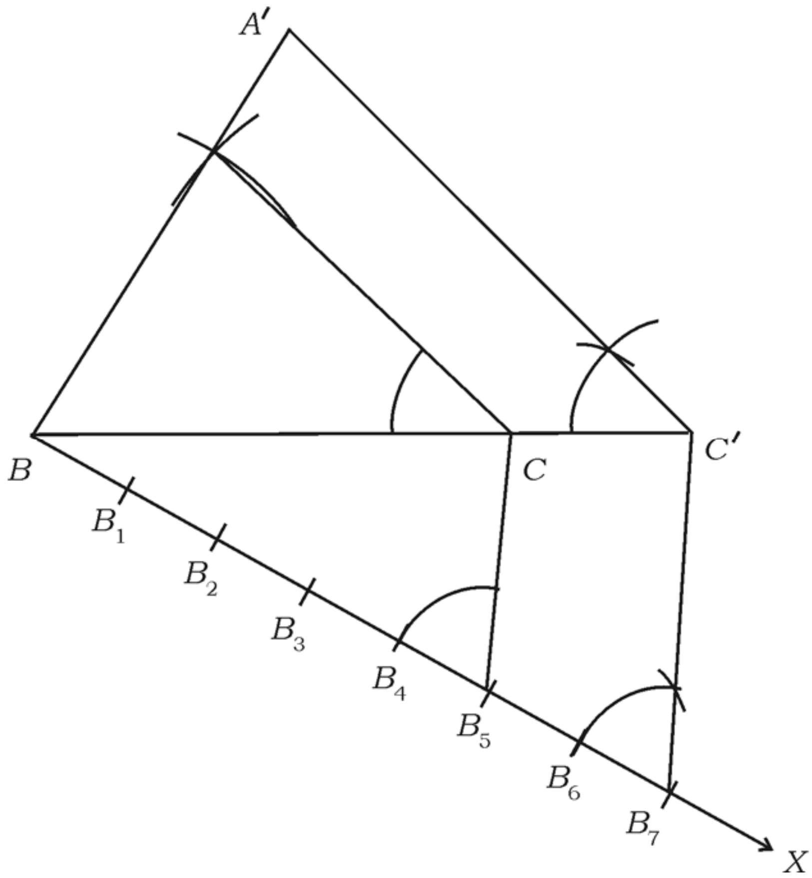
$= \left(\frac{-2}{2}, \frac{-6}{2}\right)$

Co-ordinates of $D = (-1, -3)$

Qn. Nos.	Value Points	Marks allotted
	Co-ordinates of $E = \left(\frac{3+5}{2}, \frac{-5+2}{2} \right)$	
	$= \left(\frac{8}{2}, \frac{-3}{2} \right)$	
	Co-ordinates of $E = \left(4, \frac{-3}{2} \right)$	
	Co-ordinates of $F = \left(\frac{-5+5}{2}, \frac{-1+2}{2} \right)$ 1	
	$= \left(\frac{0}{2}, \frac{1}{2} \right)$	
	Co-ordinates of $F = \left(0, \frac{1}{2} \right)$	
	$(x_1, y_1) = (-1, -3)$ $(x_2, y_2) = \left(4, -\frac{3}{2} \right)$ $(x_3, y_3) = \left(0, \frac{1}{2} \right)$	
	Area of ΔDEF =	
	$= \frac{1}{2}\left[-1\left(\frac{-3}{2} - \frac{1}{2} \right) + 4\left(\frac{1}{2} + 3 \right) + 0\left(-3 + \frac{3}{2} \right) \right]$	
	$= \frac{1}{2}\left[-1 \times (-2) + 4 \times \frac{7}{2} + 0 \right]$ ½	
	$= \frac{1}{2}[2 + 14]$	
	$= \frac{1}{2} \times 16$	
	ΔDEF = 8 sq. units	
	$\therefore$ Area of ΔABC = 4 × area of ΔDEF	
	32 = 4 × 8 ½	
	32 = 32	3
	Note : Any alternate method can be given marks.	

Qn. Nos.	Value Points	Marks allotted
46.	Construct a triangle with sides 5 cm, 6 cm and 7 cm and then construct another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle. 3	

Ans. :



Constructing given triangle	1	
Drawing acute angle line and dividing into 7 parts	½	
Drawing parallel lines (one pair)	½	
Drawing parallel line (another pair)	½	
Triangle $A^{l}BC^{l}$	½	3

Qn. Nos.	Value Points	Marks allotted

V. 47. Find the solution of the following pairs of linear equation by the graphical method : 4

$2x + y = 6$

$2x - y = 2$

Ans. :

$2x + y = 6$

$y = 6 - 2x$

x	0	1	2
y	6	4	2

$2x - y = 2$

$y = 2x - 2$

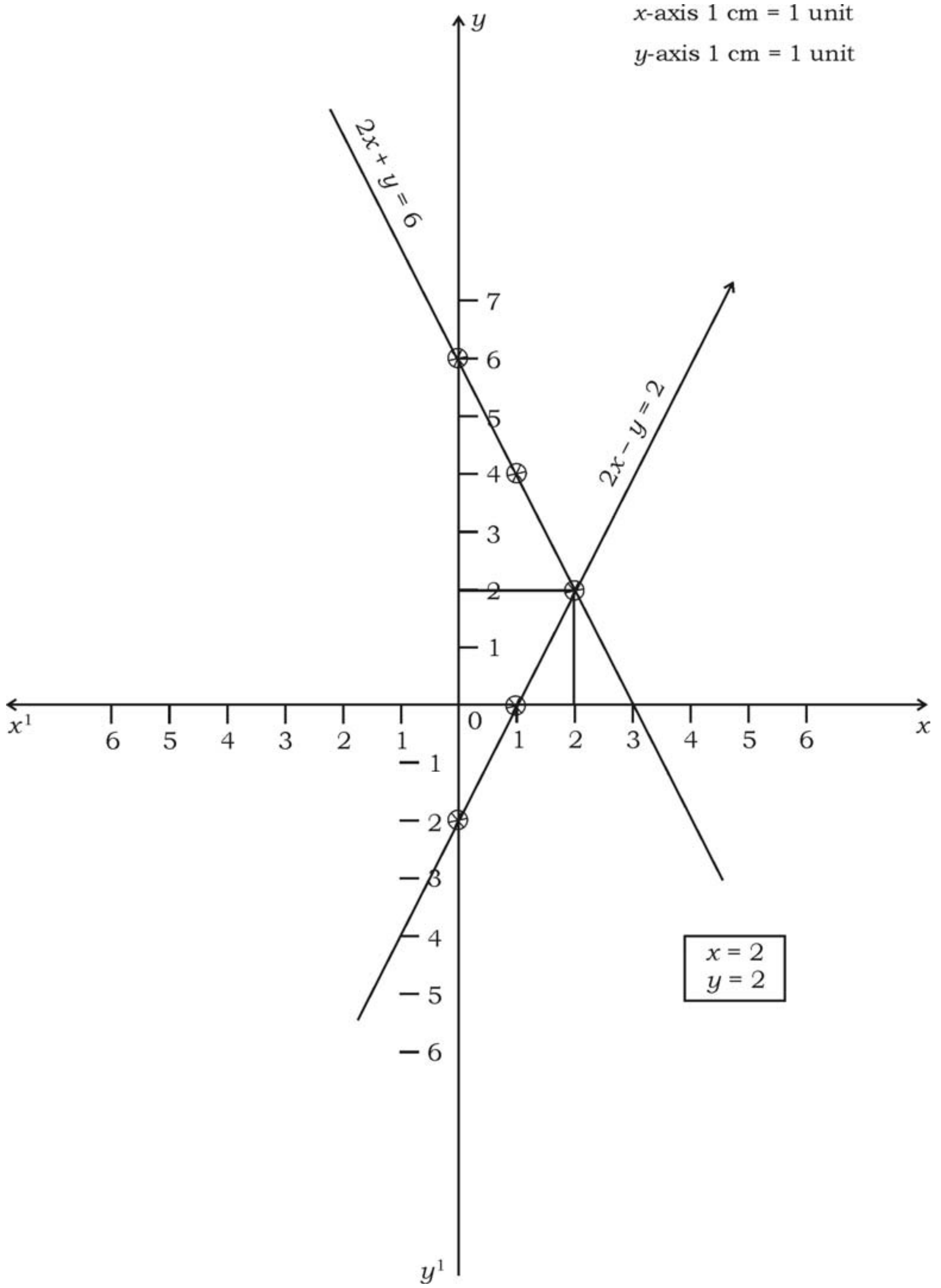
x	0	1	2
y	– 2	0	2

Tables — 2

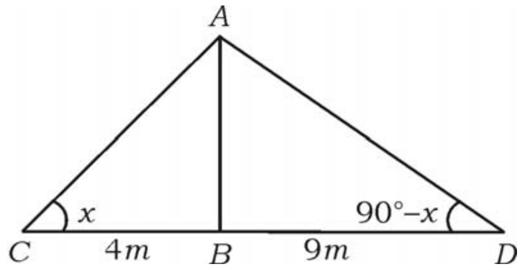
Drawing or Plotting 2 straight lines — 1

Identifying Intersecting straight line points and answer — 1 | 4

Note : For each line any *two* points may be taken.

Qn. Nos.	Value Points	Marks allotted
		
48.	The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Find the height of the tower. 4	
	Ans. :	

Qn. Nos.	Value Points	Marks allotted



½

Let AB be tower

$\angle ACB = x°$

$\therefore \angle ADB = 90° - x$ ½

In ΔABC

$\tan x = \frac{AB}{BC}$

$\tan x = \frac{AB}{4}$ ... (i) ½

In ΔADB

$\tan(90° - x) = \frac{AB}{9}$

$\cot x = \frac{AB}{9}$ ... (ii) ½

(i) × (ii)

$\tan x \times \cot x = \frac{AB}{4} \times \frac{AB}{9}$ 1½

$\tan x \times \frac{1}{\tan x} = \frac{AB^2}{36}$

$1 = \frac{AB^2}{36}$

$AB^2 = 36$

$AB = \pm\sqrt{36}$ $AB = \pm 6$

$\therefore$ Height of the tower $AB = 6$ m. ½ 4

Note : C and D can be taken on the same side of AB also.

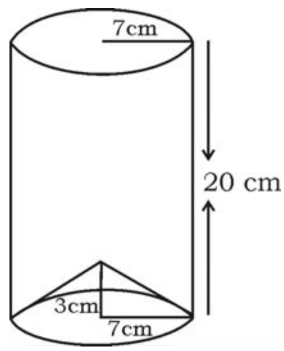
Alternate method :

$\cot x = \frac{AB}{9}$ $\frac{1}{\tan x} = \frac{AB}{9}$ $\frac{1}{\frac{AB}{4}} = \frac{AB}{9}$

$\frac{4}{AB} = \frac{AB}{9}$ $AB^2 = 36$ $AB = \pm 6$

$AB = 6$ m.

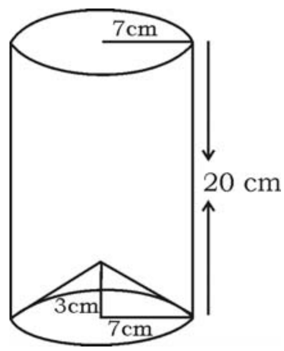
Qn. Nos.	Value Points	Marks allotted
49.	The bottom of a right cylindrical shaped vessel made from metallic sheet is closed by a cone shaped vessel as shown in the figure. The radius of the circular base of the cylinder and radius of the circular base of the cone each is equal to 7 cm. If the height of the cylinder is 20 cm and height of cone is 3 cm, calculate the cost of milk to fill completely this vessel at the rate of Rs. 20 per litre. 4	



OR

A hemispherical vessel of radius 14 cm is fully filled with sand. This sand is poured on a level ground. The heap of sand forms a cone shape of height 7 cm. Calculate the area of ground occupied by the circular base of the heap of the sand.

Ans. :



Volume of the vessel is equal to

Volume of the cylinder – Volume of cone ½

Volume of the cylinder = $\pi r^2 h$ ½

$$= \frac{22}{7} \times 7^2 \times 20$$

Volume of the cylinder = 3080 cm^3 ½

Qn. Nos.	Value Points		Marks allotted
	Volume of the cone $= \frac{1}{3}\pi r^2 h$	½	
	$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 3$		
	Volume of the cone $= 154\ \text{cm}^3$	½	
	Volume of vessel = Volume of cylinder – volume of cone		
	$= 3080 - 154$		
	$= 2926\ \text{cm}^3$	½	
	$= \frac{2926}{1000} = 2{\cdot}926$ litres.	½	
	∴ Cost of milk to fill this vessel at the rate of Rs. 20 per litre		
	$= 2{\cdot}926 \times 20$		
	$= 58{\cdot}520$		
	= Rs. 58·520	½	4
	OR		
	Volume of the hemisphere $= \frac{2}{3}\pi r^3$	½	
	Volume of the cone $= \frac{1}{3}\pi r^2 h$	½	
	Hemisphere *Cone*		
	$r = 14$ cm $h = 7$ cm.		
	Volume of hemisphere = Volume of cone		
	$\frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$	½	
	$2 \times (14)^3 = r^2 \times 7$		
	$r^2 = \frac{2 \times (14)^3}{7}$		
	$= \frac{2 \times 14 \times 14 \times 14}{7}$	1	
	$r^2 = 196 \times 4$		
	$r^2 = 784$		

Qn. Nos.	Value Points	Marks allotted

$r = \sqrt{784}$

$r = 28$ cm

$\therefore$ The area occupied by the circular base of the heap of the sand on the ground $= \pi r^2$ ½

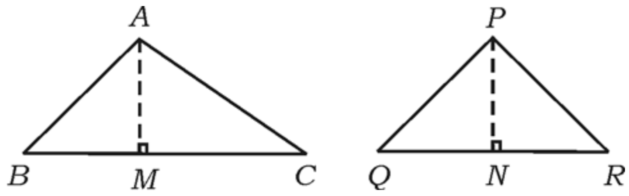
$= \dfrac{22}{7} \times (28)^2$

$= \dfrac{22}{7} \times 28 \times 28$ ½

$= 2464 \text{ cm}^2$ ½ **4**

50. Prove that "the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides". 4

Ans. :



½

Data : $\Delta ABC \sim \Delta PQR$ ½

To prove : $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \dfrac{BC^2}{QR^2}$ ½

Construction : Draw $AM \perp BC$ and $PN \perp QR$ ½

Proof : In ΔAMB and ΔPQN

$\underline{|ABM} = \underline{|PQN}$ (Data)

$\underline{|AMB} = \underline{|PNQ} = 90°$ (Construction)

$\Delta AMB \sim \Delta PQN$ ½

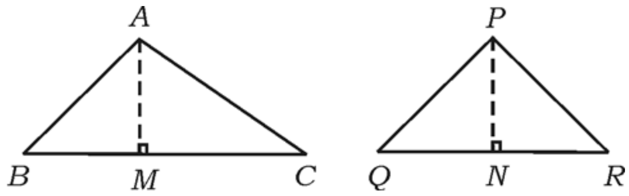
$\therefore \dfrac{AM}{PN} = \dfrac{AB}{PQ}$ *A.A* criteria

But $\dfrac{BC}{QR} = \dfrac{AB}{PQ}$ Data

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR}$ ½

Qn. Nos.	Value Points	Marks allotted
	$\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \dfrac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$ ½	
	$= \dfrac{BC}{QR} \times \dfrac{AM}{PN}$	
	$= \dfrac{BC}{QR} \times \dfrac{BC}{QR}$, $\left[\dfrac{AM}{PN} = \dfrac{BC}{QR} \right]$	
	$= \dfrac{BC^2}{QR^2}$	
	$\therefore \quad \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \dfrac{BC^2}{QR^2}$ ½	4

Alternate method :



½

Data : We are given two triangles *ABC* and *PQR* such that

$\Delta ABC \sim \Delta PQR$ ½

We need to prove that

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$ ½

For finding areas of two triangles

Draw altitudes *AM* and *PN* of the triangles ½

Now $\dfrac{ar(ABC)}{ar(PQR)} = \dfrac{\frac{1}{2} BC \times AM}{\frac{1}{2} QR \times PN}$

$= \dfrac{BC}{QR} \times \dfrac{AM}{PN}$ ... (i) ½

Now in ΔABM and ΔPQN

Qn. Nos.	Value Points		Marks allotted
	$\angle B = \angle Q$ (As $\Delta ABC \sim \Delta PQR$)		
	$\angle M = \angle N$ (each is of $90°$)		
	$\Delta ABM \sim \Delta PQN$ (*A. A.* criterion)	½	
	Therefore $\frac{AM}{PN} = \frac{AB}{PQ}$ ... (ii)		
	Also $\Delta ABC \sim \Delta PQR$ (given)		
	$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ... (iii)	½	
	Therefore $\frac{ar(ABC)}{ar(PQR)} = \frac{AB}{PQ} \times \frac{AM}{PN}$		
	From (i) and (iii)		
	$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ (from (i) and (iii))		
	$= \left(\frac{AB}{PQ}\right)^2$		
	Now using (iii) we get		
	$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$	½	4