Sample Questions

Question 1:

Find the centre and the radius of the circle

$$x^2 + y^2 + 10x + 12y - 3 = 0$$

Solution:

The given equation is

$$(x^2 + 10x) + (y^2 + 12y) = 3$$

$$(x^2 + 10x + 25) + (y^2 + 12y + 36) = 3 + 25 + 36$$

i.e. $(x + 5)^2 + (y + 6)^2 = 64$

i.e. $\{x-(-5)\}^2 + \{y-(-6)\}^2 = 8^2$

$\therefore$ Centre$(-5,-6)$, Radius $= 8$

Question 2:

Find the equation of the parabola with vertex at (0,0) and focus at (0, 2)

Solution:

$$a = 2$$

$$x^2 = 4(2)y,$$

i.e., $x^2 = 8y$.

Question 3:

Find the equation of the parabola if the curve is open rightward, vertex is (2,1) and passing through point (6, 5).

Solution:

The equation of the parabola is,

$$(y-k)^2 = 4a(x-h)$$

The vertex V(h, k) is (2, 1)

$\therefore \quad (y-1)^2 = 4a(x-2)$

But it passes through (6, 5)

$\therefore \quad 4^2 = 4a(6-2)$

$$a = 1$$

$\therefore$ The required equation is $(y-1)^2 = 4(x-2)$

Question 4 :

Find the equation of the parabola if the curve is open upward, vertex is (- 1, - 2) and the length of the latus rectum is 4.

Solution :

The equation of the parabola is,

$$(x-h)^2 = 4a(y-k)$$

Length of the latus rectum $= 4a = 4$

$$a = 1$$

The vertex V(h, k) is $(-1,-2)$

$\therefore$ The required equation is $(x+1)^2 = 4(y+2)$

Question 5 :

Find the coordinates of the foci,the vertices, the length of major axis, the minor axis, the eccentricity and the latus rectum of the ellipse

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

Solution :

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

Comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we get

$a=5, b=4$

$c=\sqrt{a^2-b^2}=\sqrt{5^2-4^2}=3$

$\therefore$

The coordinates of the foci are (-3,0),(3,0) ,

Vertices are (-5,0)and(5,0).

Length of the major axis $=$ 10 units

Length of the minor axis $=$ 6 units and the

Eccentricity $=\frac{3}{5}$

Latus rectum $= \frac{2b^2}{a}=\frac{32}{5}$

Question 7 :

Find the equation of the ellipse,whose length of the major axis is 20 and foci are $(0,\pm 6)$

Solution :

Equation of the ellipse is $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

$$a=\frac{20}{2}=10$$

$$b=\sqrt{a^2-c^2}=\sqrt{10^2-6^2}$$

$$=\sqrt{100-36}=64$$

Therefore, the equation of the ellipse is $\frac{x^2}{64}+\frac{y^2}{100}=1$

Question 8 :

Find the equation of the ellipse whose vertices are $(\pm 6,0)$ and foci are $(\pm 4,0)$

Solution :

Equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$a=6,\ c=4$$

$$a^2+b^2=c^2$$

$$b=\sqrt{a^2-c^2}=\sqrt{6^2-4^2}=\sqrt{20}$$

Therefore, the equation of the ellipse is $\frac{x^2}{36}+\frac{y^2}{20}=1$

Question 6 :

Find the coordinates of the foci, the vertices, the eccentricity and the latus rectum of the hyperbola.

$$25x^2-9y^2=225$$

Solution :

$$9y^2-4x^2=36$$

$$\frac{9y^2}{36}-\frac{4x^2}{36}=\frac{36}{36}$$

$$\frac{y^2}{4}-\frac{x^2}{9}=1$$

Comparing the given equation with $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, we get

$a=2, b=3$

$c=\sqrt{a^2+b^2}=\sqrt{4+9}=\sqrt{13}$

$\therefore$

The coordinates of the foci are $(0,-\sqrt{13})$, $(0,\sqrt{13})$,

Vertices are (0,+2)and(0,-2).

Eccentricity $=\frac{3}{5}$

Latus rectum $=\frac{2\times 3^2}{2}=9$

Question 9 :

Find the equation of the hyperbola where foci are $(0,\pm 12)$ and the length of the latus rectum is 36.

Solution :

$$c = 12$$

$$\frac{2b^2}{a} = 36$$

$$b^2 = 18a$$

$$c^2=a^2+b^2$$

$$12^2=a^2+18a$$

$$a^2+18a-144=0$$

$$a=-24,\ 6$$

we take $a=6,\ b^2=108$

Therefore, the equation of the required hyperbola is

$$\frac{y^2}{36}-\frac{x^2}{108}=1 \qquad \text{or} \qquad 3y^2-x^2=108$$