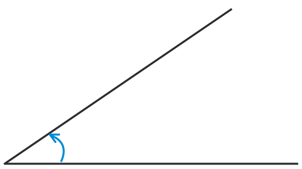
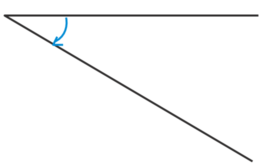
Angle in Trigonometry

The rotation of a ray from one position to another along the circumference of a circle is known as an angle. The initial position of the ray is called initial ray and the final position of the ray is called terminal ray.

Positive angles: If the rotation of a ray from one position to another is in anti-clockwise, then the angle is known as positive angle.



Negative angles: If the rotation of a ray from one position to another is in anti-clockwise, then the angle is known as positive angle.



1 Right angle $= 90 \text{ degrees } (90^0)$

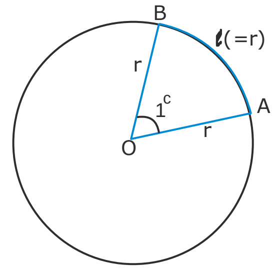
$1^0 = 60 \text{ minutes } (60')$

$1' = 60 \text{ seconds } (60'')$

Radian

One radian is the angle subtended at the centre by a positive arc equal in length to the radius of the circle.

Thus in the circular system, the circular measure of an angle = the no. of radians contained in it. It is denoted by '^{c}'.



$$\mathbf{1\,radian = 1^c = \frac{180}{\pi}\,degree} = 57^0\,16'\,22''\text{ (approx.)}$$

$$1\text{ degree} = 1^0 = \left(\frac{\pi}{180}\right)^c = 0.01746\text{ radians (approx.)}$$

Note2: Sexagesimal system. Expression in the form $x^0\,y'\,z''$ (x degree y minutes z seconds) is called sexagesimal system.

Note3:

a) The angle between two consecutive digits in a clock is 30^0 or $\left(\frac{360}{12}\right)^0$.

b) The hour hand subtends an angle of 30^0 or $\left(\frac{360}{12}\right)^0$ in 1 minute.

c) The minute hand subtends an angle of 6^0 or $\left(\frac{360}{60}\right)^0$ in 1 minute.

Note4: In a regular Polygon

a) All interior angles, exterior angles and all sides are equal.

b) Sum of all interior angles are equal.

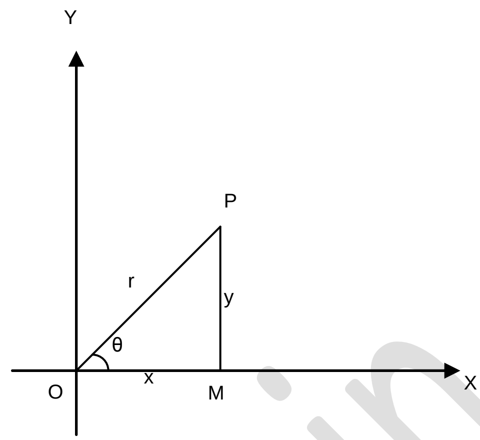
c) Sum of all exterior angles are equal.

d) Each exterior angle = $\left(\frac{360}{\text{No. of sides}}\right)^0$

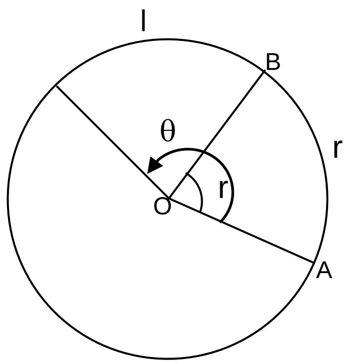
e) Each interior angle = $180 - \text{Exterior angle}$

Note5: To find the angle of a regular polygon means – find its interior angle.

Degree	30	60	90	120	180	360
Radian	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	2π



Arc, Radius and Angle relation.



Length of the arc of a circle having radius 'r' and inclination 'θ' radians is $\ell = r\theta$.

Trigonometric Functions: Trigonometric functions are periodic and thus many natural phenomena are most readily studies with the help of trigonometric functions. Unlike algebraic functions, these functions are not represented by single letters, instead the abbreviation sin is used for sine function, cos is for cosine function, tan is for tangent function, cosec for cosecant function, sec is for secant function and cot is for cotangent function. For a given angle θ, it is usual to write sinθ for sin(θ), etc.

$$\sin\theta = \frac{opp.side}{hypotenuse} = \frac{y}{r}$$

$$\cos\theta = \frac{adjacent\ side}{hypotenuse} = \frac{x}{r}$$

$$\tan\theta = \frac{opp.side}{adjacent\ side} = \frac{y}{x}$$

$$\cos ec\theta = \frac{1}{\sin\theta} = \frac{r}{y}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$$

$$\cot\theta = \cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$$

Quotient Relations

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \qquad \frac{\cos\theta}{\sin\theta} = \cot\theta$$

Phythagoras' relations

i. $\cos^2\theta + \sin^2\theta = 1$

a) $\cos^2\theta = 1 - \sin^2\theta$

b) $\sin^2\theta = 1 - \cos^2\theta$

ii. $\sec^2\theta - \tan^2\theta = 1$

a) $\sec^2\theta = 1 + \tan^2\theta$

b) $\tan^2\theta = \sec^2\theta - 1$

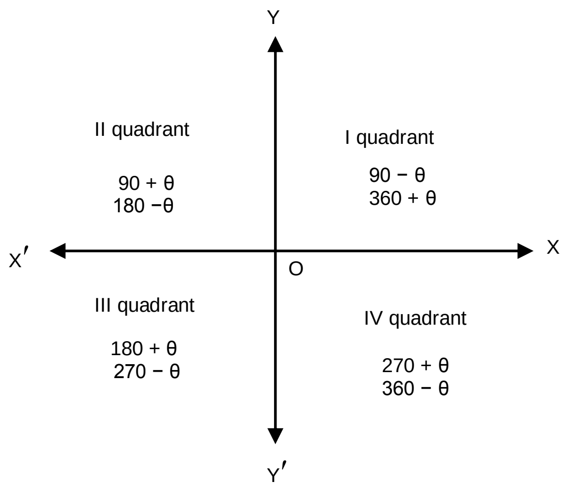
iii. $\cos ec^2\theta - \cot^2\theta = 1$

a) $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

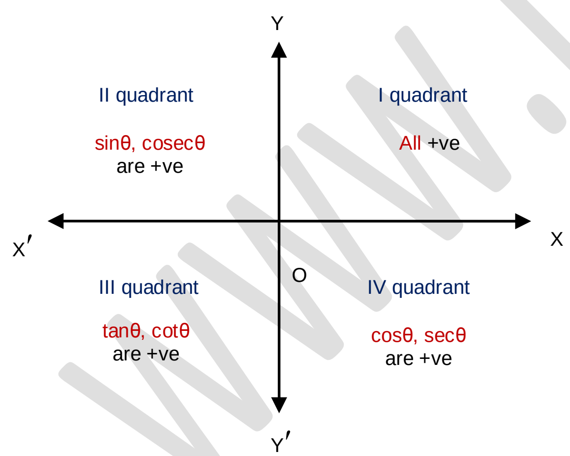
b) $\cot^2\theta = \operatorname{cosec}^2\theta - 1$

Quadrants

Two mutually perpendicular lines in a plane divide the plane into 4 regions and each region is called quadrant.



Signs of Trigonometric functions in different Quadrants



Hint for memory:-

1. **A**ll **S**tudents **T**ake **C**offee
   I II III IV
2. **A**fter **S**chool **T**uition **C**lass

Note:

a) Acute angle - angle less than 90^0 [$< 90^0$]
b) Obtuse angle - angle greater than 90^0 [$> 90^0$]
c) Right angle - angle is equal to 90^0 [$= 90^0$]

Trigonometric Ratios of particular angles.

$\sin 0 = \cos 90 = 0$ $\quad$ $\cos 0 = \sin 90 = 1$

$\tan 0 = \cot 90 = 0$ $\quad$ $\cot 0 = \tan 90 = \infty$

$\sin 30 = \cos 60 = \frac{1}{2}$ $\quad$ $\cos 30 = \sin 60 = \frac{\sqrt{3}}{2}$

$\tan 30 = \cot 60 = \frac{1}{\sqrt{3}}$ $\quad$ $\cot 30 = \tan 60 = \sqrt{3}$

$\sin 45 = \cos 45 = \frac{1}{\sqrt{2}}$ $\quad$ $\tan 45 = \cot 45 = 1$

Compound Angles and Reduction Formulae

$$\sin(x+y) = \sin x\cos y + \cos x\sin y$$

$$\sin(x-y) = \sin x\cos y - \cos x\sin y$$

$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$

$$\cos(x-y) = \cos x\cos y + \sin x\sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$$

$$\cot(x+y) = \frac{\cot x\cot y - 1}{\cot x + \cot y}$$

$$\cot(x-y) = \frac{\cot x\cot y + 1}{\cot y - \cot x}$$

$$\tan(45+x) = \frac{1+\tan x}{1-\tan y}$$

$$\tan(45-x) = \frac{1-\tan x}{1+\tan x}$$

$$\sin(x+y+z) = \sum \sin x\cos y\cos z - \sin x\sin y\sin z$$

$$\cos(x+y+z)=\cos x\cos y\cos z-\sum\sin x\sin y\cos z$$

$$\tan(x+y+z)=\frac{\sum\tan x-\tan x\tan y\tan z}{1-\sum\tan x\tan y}$$

Important Formulae

$$\sin(x+y)\sin(x-y)=\sin^2 x-\sin^2 y$$

$$\cos(x+y)\cos(x-y)=\cos^2 x-\sin^2 y$$

$$\tan(x+y)\tan(x-y)=\frac{\tan^2 x-\tan^2 y}{1-\tan^2 x\tan^2 y}$$

Product Formulae.

i. $\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

ii. $\sin x-\sin y=2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

iii. $\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

iv. $\cos x-\cos y=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

Converse of Product Formulae.

i. $2\sin x\cos y=\sin(x+y)+\sin(x-y)$

ii. $2\cos x\sin y=\sin(x+y)-\sin(x-y)$

iii. $2\cos x\cos y=\cos(x+y)+\cos(x-y)$

iv. $2\sin x\sin y=\cos(x-y)-\cos(x+y)$

Related angles & General Reduction Formulae

For any angle θ, the angles $\pm\theta, 90\pm\theta, 180\pm\theta$, etc. are known as related angles and the functions expressed in terms of the related angles are known as **General Reduction Formulae.**

Reduction Formulae for $(-\theta)$

$\sin(-\theta)=-\sin\theta$: $\operatorname{cosec}(-\theta)=-\operatorname{cosec}\theta$

$\cos(-\theta)=\cos\theta$: $\sec(-\theta)=\sec\theta$

$\tan(-\theta)=-\tan\theta$:

$\cot(-\theta)=-\cot\theta$

Reduction Formulae for $(90-\theta)$

$\sin(90-\theta)=\cos\theta$: $\cos(90-\theta)=\sin\theta$

$\tan(90-\theta)=\cot\theta$: $\cot(90-\theta)=\tan\theta$

Reduction Formulae for $(90+\theta)$

$\sin(90+\theta)=\cos\theta$: $\cos(90+\theta)=-\sin\theta$

$\tan(90+\theta)=-\cot\theta$: $\cot(90+\theta)=-\tan\theta$

Reduction Formulae for $(180\pm\theta)$

$\sin(180-\theta)=\sin\theta$: $\cos(180-\theta)=-\cos\theta$

$\tan(180-\theta)=-\tan\theta$: $\cot(180-\theta)=-\cot\theta$

$\sin(180+\theta)=-\sin\theta$: $\cos(180+\theta)=-\cos\theta$

$\tan(180+\theta)=\tan\theta$: $\cot(180+\theta)=\cot\theta$

Hint: Any trigonometric function is in the form $(n.90\pm\theta)$ is numerically equal to:

1) the same function if 'n' is an even integer (2,4,6,…)
2) the corresponding co-function if 'n' is an odd integer (1,3,5, ……)

The algebraic sign in each case is same as the quadrant in which the given angle $(n.90\pm\theta)$ lies.

Function	Co-function
$\sin\theta$	$\cos\theta$
$\cos\theta$	$\sin\theta$
$\tan\theta$	$\cot\theta$
$\cot\theta$	$\tan\theta$
$\operatorname{cosec}\theta$	$\sec\theta$
$\sec\theta$	$\operatorname{cosec}\theta$

Examples:

$$\sin 150 = \sin(1.90+60) = +\cos 60 = \frac{1}{2} \quad \text{(or)}$$

$$\sin 150 = \sin(2.90-30) = +\sin 30 = \frac{1}{2}$$

$$\sin 225 = \sin(2.90+45) = -\sin 45 = -\frac{1}{\sqrt{2}}$$

$$\cos 330 = \cos(4.90-30) = +\cos 30 = \frac{\sqrt{3}}{2}$$

Note: Since T-functions are circular functions, we can add or subtract any number of 360s to the given function, its value is not changed. For example,

$$\sin 420 = \sin(420-360) = \sin 60 = \frac{\sqrt{3}}{2}$$

$$\sin(-765) = \sin(-765+2.360) = \sin(-765+720)$$
$$= \sin(-45) = -\sin 45 = -\frac{1}{\sqrt{2}}$$

Multiple Angles

$$\sin 2x = 2\sin x\cos x = \frac{2\tan x}{1+\tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1-2\sin^2 x = 2\cos^2 x - 1$$

$$= \frac{1-\tan^2 x}{1+\tan^2 x}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$1+\cos 2x = 2\cos^2 x \qquad :$$

$$1-\cos 2x = 2\sin^2 x$$

$$1+\sin 2x = (\cos x+\sin x)^2 \qquad :$$

$$1-\sin 2x = (\cos x-\sin x)^2$$

$$\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x \qquad :$$

$$\tan x = \frac{\pm\sqrt{1+\tan^2 x}-1}{\tan x}$$

$$\sin 18 = \frac{\sqrt{5}-1}{4} \quad : \quad \cos 36 = \frac{\sqrt{5}+1}{4}$$

$$\cos 18 = \frac{\sqrt{10+2\sqrt{5}}}{4} \quad : \quad \sin 36 = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\frac{1+\cos 2x}{1-\cos 2x} = \cot^2 x \qquad :$$

$$\frac{1-\sin 2x}{1+\sin 2x} = \tan^2\left(\frac{\pi}{4}-x\right)$$

$$\frac{1+\sin 2x}{1-\sin 2x} = \tan^2\left(\frac{\pi}{4}+x\right)$$

$$\sin 3x = 3\sin x - 4\sin^3 x$$

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$$

$$\cos A\cos 2A\cos 2^2A \ldots \cos 2^n A = \frac{\sin 2^n A}{2^n \sin A}$$

Sub-Multiple Angles

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = 1-2\sin^2\frac{x}{2} = 2\cos^2\frac{x}{2} - 1$$

$$= \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\tan x=\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

$$1+\cos x=2\cos^2\frac{x}{2}$$

$$1-\cos x=2\sin^2\frac{x}{2}$$

$$1+\sin x=\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2$$

$$1-\sin x=\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2$$

$$\frac{1-\cos x}{1+\cos x}=\tan^2\frac{x}{2}$$

$$\frac{1+\cos x}{1-\cos x}=\cot^2\frac{x}{2}$$

$$\frac{1-\sin x}{1+\sin x}=\tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\frac{1+\sin x}{1-\sin x}=\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

$$\frac{1-\cos x}{\sin x}=\pm\tan\frac{x}{2}=\pm\frac{\sqrt{1+\tan^2 x}-1}{\tan x}$$

$$\frac{1+\cos x}{\sin x}=\pm\cot\frac{x}{2}=\pm\frac{\sqrt{1+\tan^2 x}+1}{\tan x}$$

Values of $\sin 22\frac{1}{2}^0, \cos 22\frac{1}{2}^0, \tan 22\frac{1}{2}^0, \cot 22\frac{1}{2}^0$

$$\sin 22\frac{1}{2}^0=\frac{\sqrt{2-\sqrt{2}}}{2} \qquad \cos 22\frac{1}{2}^0=\frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\tan 22\frac{1}{2}^0=\sqrt{2}-1 \qquad \cot 22\frac{1}{2}^0=\sqrt{2}+1$$

Values of $\sin 9^0$, $\cos 9^0$, $\tan 9^0$

$$\sin 9^0=\frac{\sqrt{3+\sqrt{5}}-\sqrt{5-\sqrt{5}}}{4}$$

$$\cos 9^0=\frac{\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}}{4}$$

$$\tan 9^0=\frac{\sqrt{3+\sqrt{5}}-\sqrt{5-\sqrt{5}}}{\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}}$$

Values of $\sin 15^0$, $\cos 15^0$, $\tan 15^0$

$$\sin 15^0=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos 15^0=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\tan 15^0=2-\sqrt{3}$$

Values of $\sin 75^0$, $\cos 75^0$, $\tan 75^0$, $\cot 75^0$

$$\sin 75=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\cos 75=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\tan 75=\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\cot 75=\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\sin\theta\sin\left(\frac{\pi}{3}-\theta\right)\sin\left(\frac{\pi}{3}+\theta\right)=\frac{1}{4}\sin 3\theta$$

$$\cos\theta\cos\left(\frac{\pi}{3}-\theta\right)\cos\left(\frac{\pi}{3}+\theta\right)=\frac{1}{4}\cos 3\theta$$

$$\tan\theta\tan\left(\frac{\pi}{3}-\theta\right)\tan\left(\frac{\pi}{3}+\theta\right)=\tan 3\theta$$

$$\cos 36-\cos 72=\frac{1}{2} \qquad : \quad \cos 36\cos 72=\frac{1}{4}$$

Trigonometric Equations.

The general value of θ if

i. $\sin\theta=0$ is $\theta = n\pi$, $n\in Z$

ii. $\cos\theta=0$ is $\theta = (2n+1).\frac{\pi}{2}$, $n\in Z$

iii. $\tan\theta=0$ is $\theta = n\pi$, $n\in Z$

The general value of θ if

i. $\sin\theta = \sin\alpha$ is $\theta = n\pi + (-1)^n.\alpha$, $n\in Z$

ii. $\cos\theta = \cos\alpha$ is $\theta = 2n\pi \pm \alpha$, $n\in Z$

iii. $\tan\theta = \tan\alpha$ is $\theta = n\pi + \alpha$, $n\in Z$

The general value of θ if

i. $\sin^2\theta = \sin^2\alpha$ is θ = nπ ± α

ii. $\cos^2\theta = \cos^2\alpha$ is θ = nπ ± α

iii. $\tan^2\theta = \tan^2\alpha$ is θ = nπ ± α

The general value of θ if $a\cos\theta + b\sin\theta = c$

dividing throughout by $\sqrt{a^2+b^2}$ then

$$\frac{a}{\sqrt{a^2+b^2}}\cos\theta + \frac{b}{\sqrt{a^2+b^2}}\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$$

……. (1)

knowing the values of 'a' and 'b' we can find the value of the angle α such that $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$ and $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$. Also

find the angle β such that $\cos\beta = \frac{c}{\sqrt{a^2+b^2}}$.

Then (1) becomes,

$$\cos\alpha\cos\theta + \sin\alpha\sin\theta = \cos\beta$$

i.e., $\cos\theta\cos\alpha + \sin\theta\sin\alpha = \cos\beta$

i.e., $\cos(\theta-\alpha) = \cos\beta$

$\therefore$ $\theta - \alpha = 2n\pi \pm \beta$

$\therefore$ $\theta = 2n\pi + \alpha \pm \beta$

Trigonometric Identities

i. In any ΔABC, A+B+C = π

Then A + B = π − C

C = π − (A+B)

ii. In any ΔABC, A+B+C = π

Then $\frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\frac{C}{2} = \frac{\pi}{2} - \left(\frac{A+B}{2}\right)$

Tips1:

1. $\sin\theta = 0 \Rightarrow \theta = 0, \pi, 2\pi, 3\pi, \ldots$
2. $\tan\theta = 0 \Rightarrow \theta = 0, \pi, 2\pi, 3\pi, \ldots$
3. $\cos\theta = 1 \Rightarrow \theta = 0, 2\pi, 4\pi, \ldots$
4. $\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$
5. $\cos\theta = -1 \Rightarrow \theta = \pi, 3\pi, 5\pi, \ldots$

Tips2:

- *sinx is positive*
  Then $x = x$ or $(\pi - x)$
- sinx is *negative*
  Then $x = -x$ or $(\pi + x)$
- *cosx is positive*
  Then $x = x$ or $(2\pi - x)$
- *cosx is negative*
  Then $x = (\pi - x)$ or $(\pi + x)$
- *tanx is positive*
  Then $x = x$ or $(\pi + x)$
- *tanx is negative*
  Then $x = (\pi - x)$ or $(-x)$

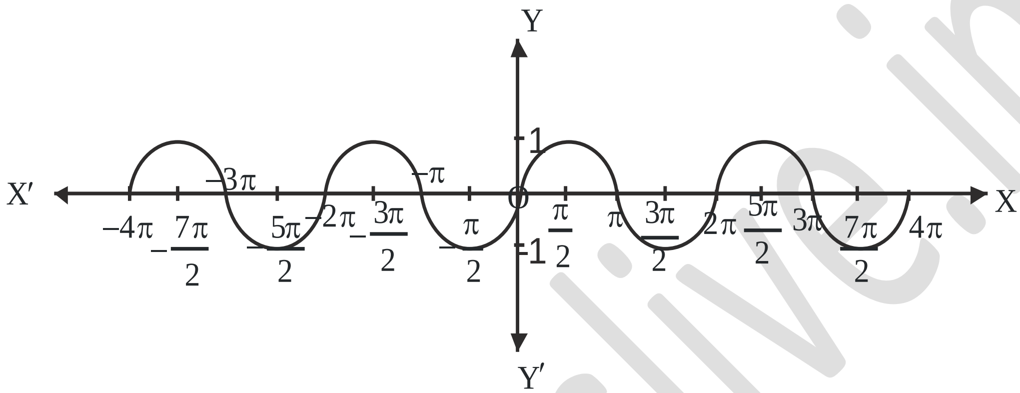
Domain and Range of t-functions:

f(x)	**Domain**	**Range**
$\sin x$	R	$[-1,+1]$
$\cos x$	R	$[-1,+1]$
$\tan x$	$R-(2n+1)\frac{\pi}{2}, n\in Z$	R
$\operatorname{cosec} x$	$R-n\pi, n\in Z$	$(-\infty,-1]\cup[1,\infty)$
$\sec x$	$R-(2n+1)\frac{\pi}{2}, n\in Z$	$(-\infty,-1]\cup[1,\infty)$
$\cot x$	$R-n\pi, n\in Z$	R

Graph of t-functions

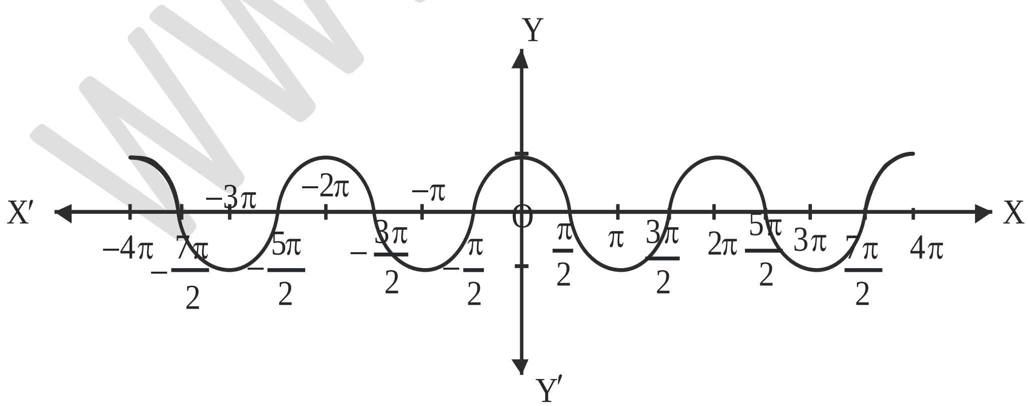
1. $f(x) = \sin x$

-4π	$-\frac{7\pi}{2}$	-3π	$-\frac{5\pi}{2}$	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
0	1	0	-1	0	1	0	-1	0	1	0	-1	0	1	0	-1	0



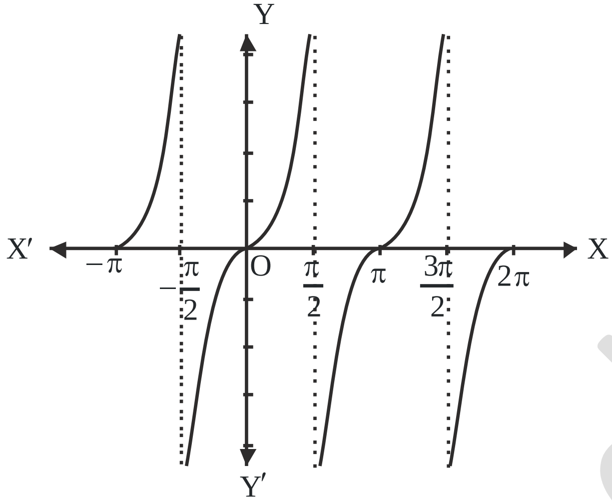
2. $f(x) = \cos x$

-4π	$-\frac{7\pi}{2}$	-3π	$-\frac{5\pi}{2}$	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
1	0	-1	0	1	0	-1	0	1	0	-1	0	1	0	-1	0	1



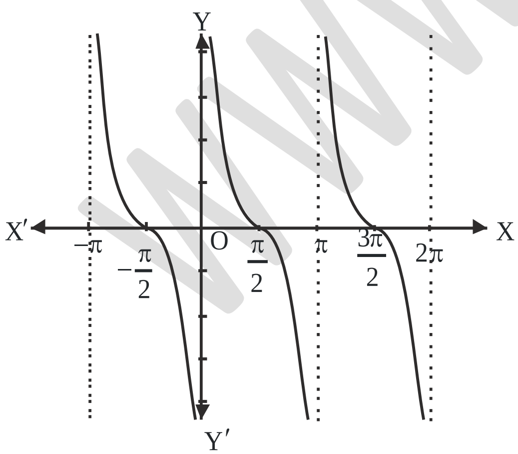
3. $f(x) = \tan x$

$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
-1	$-\infty$	1	∞	0	∞	0



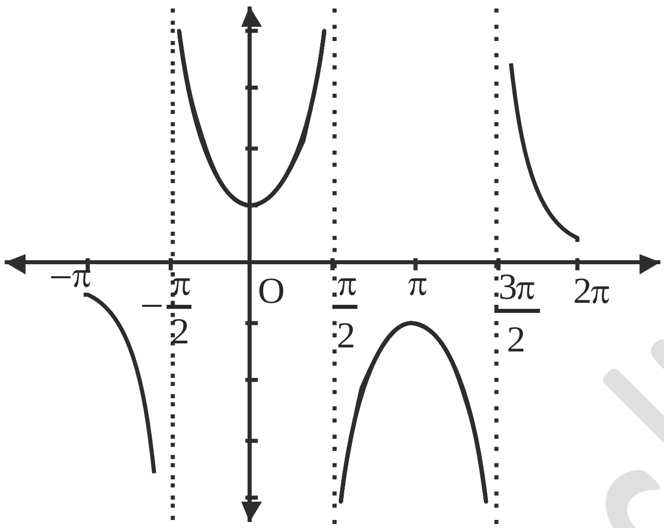
4. $f(x) = \cot x$

$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
∞	0	∞	0	$-\infty$	0	$-\infty$



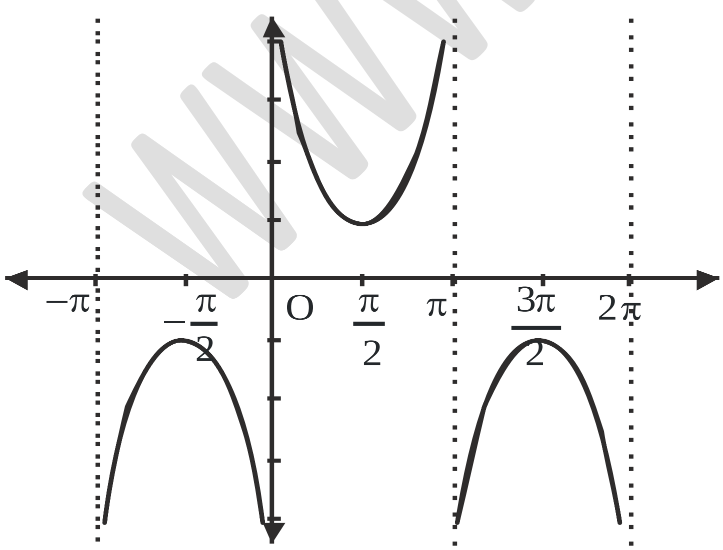
5. $f(x) = \sec x$

$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
-1	∞	1	∞	-1	∞	1



6. $f(x) = \operatorname{cosec} x$

$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$-\infty$	-1	∞	1	∞	-1	∞



Properties of Δles

i. **Sine rule**: In any ΔABC, $\frac{a}{sin\,A}=\frac{b}{sin\,B}=\frac{c}{sin\,C}$, where a, b, c are the sides and A, B, C are the angles.

ii. **Cosine rule**: In any Δ ABC,

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = c^2 + a^2 - 2ca\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

iii. **Tangent rule** (Napier's Formula)

In any Δ ABC,

$$tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}cot\frac{A}{2}$$

$$tan\left(\frac{C-A}{2}\right)=\frac{c-a}{c+a}cot\frac{B}{2}$$

$$tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}cot\frac{C}{2}$$

Note:

- In any ΔABC, $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ where 'R' is the circumradius of the circum circle.

  $a=2R\sin A$ $\quad b=2R\sin B$ $\quad c=2R\sin C$

- $R=\frac{a}{2\sin A}=\frac{b}{2\sin B}=\frac{c}{2\sin C}$

- In any ΔABC,

  1. $a=2R\sin A$: $b=2R\sin B$: $c=2R\sin C$
  2. $a^2=\frac{b^2+c^2-a^2}{2bc}$: $b^2=\frac{c^2+a^2-b^2}{2ca}$: $c^2=\frac{a^2+b^2-c^2}{2ab}$