Chapter – 9

Mechanical Properties of Solids

Properties of Solids

1. Solids have a definite shape and size.
2. Solids are crystalline or amorphous.
3. The density of solids is slightly higher than their liquid states.

Elastic Behaviour of Solids

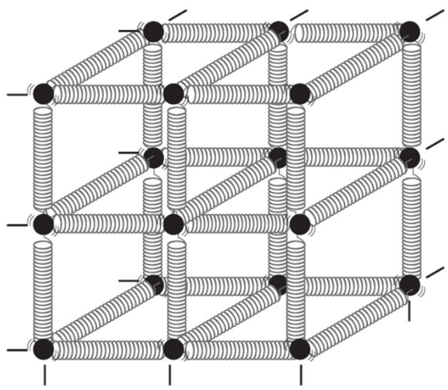


Fig. 9.1 Spring-ball model for the illustration of elastic behaviour of solids.

Solids are not perfectly rigid.

A rigid body generally means a hard solid object having a definite shape and size. But in reality solid bodies can be stretched, compressed and bent. Solid bodies are not perfectly rigid.

Elastic and Plastic Bodies

Elasticity: -

The property of a body by virtue of which, it tends to regain its original size and shape when the applied force is removed is known as elasticity and the deformation caused is known as elastic deformation.

Eg: - Steel is an elastic body.

Plasticity: - The body which has no tendency to regain its original shape and get permanently deformed is called plastic body. This property is known as plasticity.

Eg: clay, wax, etc. are plastic bodies.

Stress

A force which changes the length, shape or volume of a body is called a deforming force.

When an elastic body is subjected to a deforming force, a **restoring force** is developed in the body. This restoring force is equal in magnitude but opposite in direction to the applied force.

"**The restoring force per unit area is known as stress**"

If F is the applied force and A is the area of cross-section of the body, then

$$\textbf{stress} = \frac{F}{A}$$

The S.I unit of stress is **Nm^{-2} or Pascal [Pa]**.

Its dimensional formula is **$ML^{-1}T^{-2}$**

Types of stress

Stress is of three types.

1. Linear Stress (longitudinal or tensile stress)

It is the stress developed, when the applied force produces a change in the length of the body.

2. Volume stress (or Bulk stress)

It is the stress developed in the body, when the applied force produces a change in the volume of the body.

3. Shearing Stress (or tangential stress)

It is the stress developed in the body, when the applied force produces, a change in shape of the body.

Strain

The deforming force applied on a body produces generally a change in its dimensions and the body is said to be strained.

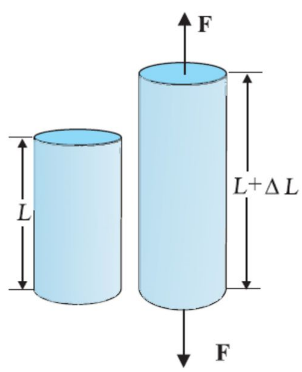
Strain is defined as the ratio of change in dimension to the original dimension.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Strain has **no unit and dimension**.

Types of Strain

1. Longitudinal Strain

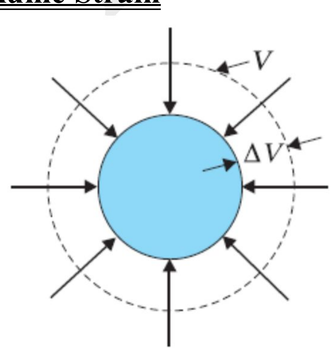


If the deforming force produces a change in length, the strain produced in the body is called longitudinal strain or tensile strain or linear strain.

$$\text{Longitudinal Strain} = \frac{\text{Change in length}}{\text{Original length}}$$

$$= \frac{\Delta L}{L}$$

Volume Strain

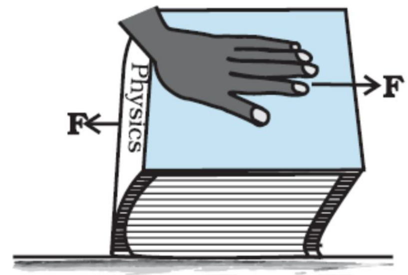


If the deforming force produces a change in volume, the strain produced in the body is called volume strain.

$$\text{Volume Strain} = \frac{\text{Change in Volume}}{\text{Original Volume}}$$

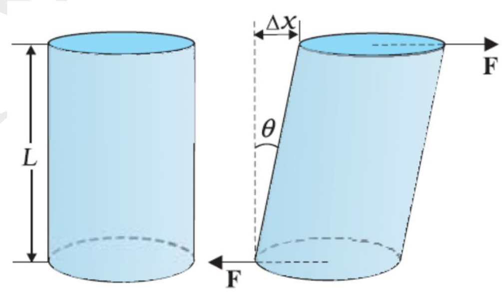
$$= \frac{\Delta V}{V}$$

2. Shearing Strain



If the deforming force produces a change in shape of the body without changing volume, the strain produced is called shearing strain.

$$\text{Shearing Strain} = \frac{\Delta x}{L} = \tan\theta \approx \theta$$



Robert Hooke
(1635 – 1703 A.D.)

An English Scientist

Hooke's Law

According to Hooke's law, "**with in the elastic limit stress is directly proportional to stain**".

i.e., Stress α strain

Stress = K × Strain,

Where **K** is the proportionality constant called the **modulus of elasticity**.

$$K = \frac{\text{Stress}}{\text{Strain}}$$

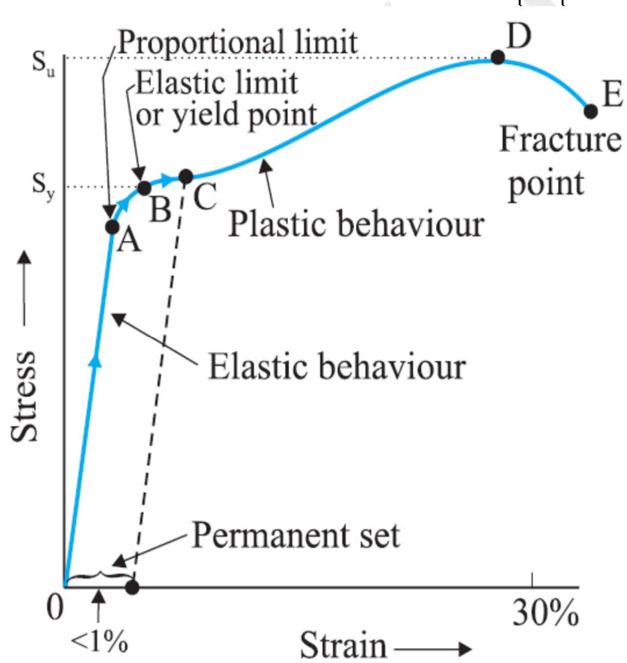
Note: - Modulus of elasticity **depends on**:

(i) Nature of the material of the body and (ii) Temperature.

It **is independent** of the dimensions (i.e., length, volume etc) of the body.

S.I unit of 'k' is Nm^{-2} or Pascal [P_a]

Stress-Strain Curve for a metal



The above curve is a typical stress-strain curve for a metal. The stress-strain curves vary from material to material.

In the **region OA**, the curve is linear. In this region Hooke's law is obeyed. The body regains its original dimensions, when the applied force is removed. In this region the solid behaves as an elastic body.

In the **region AB**, stress and strain are not proportional. Still the body returns to its original dimension when the load is removed.

The point B in the curve is known as **elastic limit** [**yield point**] and the corresponding stress is known as **yield strength** (**S_y**) of the material. If the load is increased further (beyond elastic limit) the body cannot regain its original dimension.

In the portion of the curve between C and D, if the load is increased, strain increases rapidly even for a small change in the stress. When the load is removed at some point, say at C between B and D, the body doesn't regain its original dimension. The material is said to have a permanent set. The deformation is said to be plastic deformation. The point D on the graph corresponds to the **ultimate tensile strength (S$_u$)** of the material.

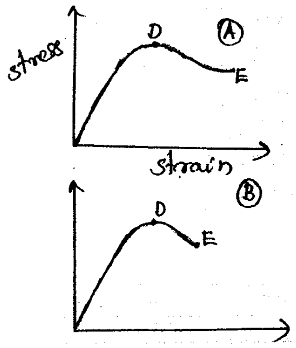
Beyond the point D, additional strain is produced even by a reduced applied force. And fracture occurs at E.

Note: -

(i) If the ultimate strength and fracture points D and E are close, the material is to be brittle.

(ii) If D and E are far apart, the material is said to be ductile.

Question1: The stress strain curves for two materials A and B are given



(i) Which is more ductile

(ii) Which is more brittle

Ans: -

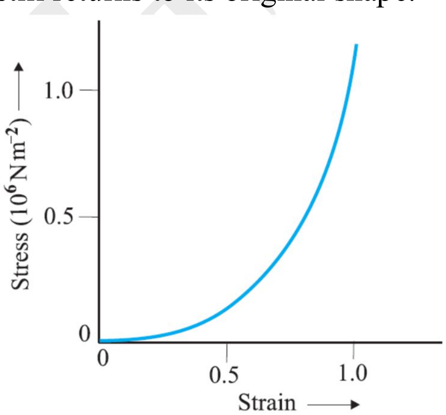
(i) A is more ductile as the fracture point (E) and ultimate stress point (D) are away from each other.

(ii) B is more brittle as the fracture point (E) and ultimate stress point(D) are close to each other.

Elastomers

Substances like tissue of aorta, rubber, etc., which can be stretched to cause large strains are called elastomers.

These substances can be pulled to several times the original length and still returns to its original shape.



Although elastic region is very large the material doesn't obey Hook's law over most of the region. There is no well-defined plastic region.

Moduli of elasticity

There are three types of moduli of elasticity. They are

(i) Young's modulus

(ii) Bulk modulus

(iii) Shear modulus or Rigidity modulus.

Young's modulus (Y) – The ratio of longitudinal stress to the longitudinal strain is defined as the Young's modulus and is denoted by Y.

$$\text{i.e., } Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$$

$$= \frac{F/A}{\Delta L/L} = \frac{F\,L}{A\,\Delta L}$$

Since strain is a dimensionless quantity, the unit of Young's modulus is same as that of stress.

i.e., N/m^2 or Pascal.

If the material has circular cross-section, then $A = \pi r^2$

$$\therefore \quad Y = \frac{F\,L}{(\pi r^2)\Delta L}$$

Substance	Density ρ (kg m^{-3})	Young's modulus Y (10^9 N m^{-2})
Aluminium	2710	70
Copper	8890	110
Iron (wrought)	7800-7900	190
Steel	7860	200
Glass#	2190	65
Concrete	2320	30
Wood#	525	13
Bone#	1900	9
Polystyrene	1050	3

Pr.1: a) Young's modulus for a perfectly rigid body is -----

b) One end of a rope of 4.5m and diameter 6mm is fixed to the branch of a tree. A monkey weighing 100N jumps to catch the free end and stays there. Find the elongation of the rope (Young's modulus=4.8×10^{11}N/m^2)

Ans:

Shear modulus – The ratio of shearing stress to the shearing strain is called the shear modulus of the material and is denoted by 'G'. It is also called rigidity modulus.

$$\text{i.e.,}\quad G = \frac{\text{Shearing Stress}}{\text{Shearing Strain}}$$

$$= \frac{F/A}{\theta} = \frac{F}{A\,\theta}$$

The unit of rigidity modulus is N/m^2 or Pa.

Dimensional formula is $ML^{-1}T^{-2}$

For most of the materials $G \approx \frac{Y}{3}$

Table 9.2 Shear moduli *(G)* of some common materials

Material	*G* (10^9 Nm^{-2} or GPa)
Aluminium	25
Brass	36
Copper	42
Glass	23
Iron	70
Lead	5.6
Nickel	77
Steel	84
Tungsten	150
Wood	10

Pr.2: An edge of an aluminium cube is **10cm** long. One face of the cube is firmly fixed to a vertical wall. A mass of **100kg** is then attached to the opposite face of the cube. The shear modulus of aluminium is **25GPa**. What is the vertical deflection of the face?

Ans:

Bulk modulus (B)– The ratio of the volume stress to the corresponding volume strain is defined as bulk modulus. It is denoted by 'B'.

i.e., $B = \dfrac{\text{Volume Stress}}{\text{Volume Strain}}$

$$= \frac{F/A}{\Delta V/V}$$

$$= \frac{\text{Pressure}}{\Delta V/V} = \frac{-PV}{\Delta V}$$

Table 9.3 **Bulk moduli *(B)* of some common Materials**

Material Solids	B (10^9 N m^{-2} or GPa)
Aluminium	72
Brass	61
Copper	140
Glass	37
Iron	100
Nickel	260
Steel	160
Liquids	
Water	2.2
Ethanol	0.9
Carbon disulphide	1.56
Glycerine	4.76
Mercury	25
Gases	
Air (at STP)	1.0×10^{-4}

Pr3: Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compression strain of each column. Bulk modulus of mild steel is 160GPa.

Soln:

Pr4: The average depth of Indian Ocean is 3000m. Calculate the fractional compression, $\frac{\Delta V}{V}$, of water at the bottom of the ocean, given that the bulk modulus of water is 2.2×10^9 Nm^{-2}. Take g=10ms^{-2}.

Solution:

Compressibility

The reciprocal of bulk modulus is called compressibility and is denoted by **K**.

i.e., $K = \dfrac{1}{B} = \dfrac{-\Delta V}{PV}$

S I unit :- Pa^{-1} or M^{2}N^{-1}

Dimensional formula: - $M^{-1}LT^2$

The bulk modulus for solids is much larger than that for liquids, which is again larger than the bulk modulus for gases.

Thus solids are least compressible where as gases are most compressible.

Question2: Fluids possess volume elasticity. Which is more elastic, air or water? Why?

Ans:

Pr.5: If the bulk modulus of water is $2x10^9 N/m^2$, find its compressibility.

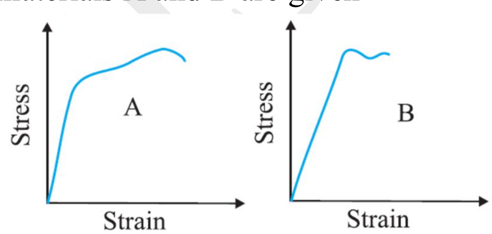
Ans:

Pr.6: A 10 KN force stretches a wire and decreases its radius by 2%. Find the change in the Young's modulus of the material of the wire.

Solution: -

There is no change in Young's modulus of the material of the wire because it is independent of the dimensions (size) of the wire.

Pr.7: The stress-strain graph for two materials A and B are given



(a) Which is more elastic?

(b) Which is stronger?

Answer:

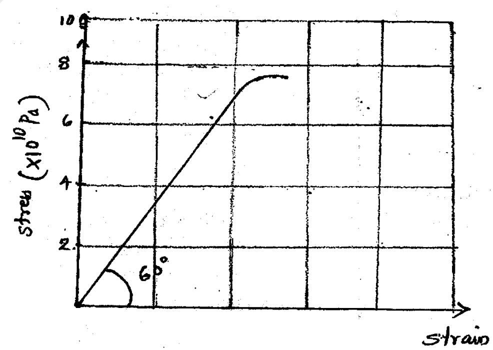
(a) The slope of stress-strain graph gives the Young's modulus of the material.

Slope of (A) > Slope of (B)
So (A) has greater Young's modulus
So (A) is more elastic

(b) Since Young's modulus of (A) is greater, it is stronger.

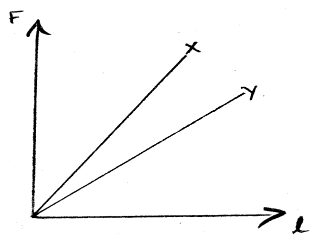
Question3: Which is more elastic, steel or rubber?
Answer: Steel is more elastic because it has greater Young's modulus than rubber.
Question4: Between steel and diamond, which is more elastic?
Answer: Diamond is more elastic. Diamond is almost a rigid body. Hence its elasticity is extremely large.

Pr.8: What is the Young's modulus and approximate yield strength of the material from the figure?



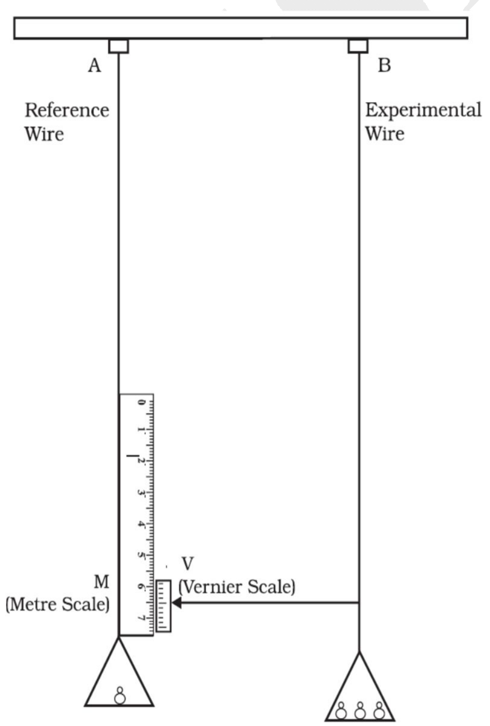
Soln:

Pr.9: Figure shows the variation of F, the load applied to the wire X and Y and their extension 'l'. Both the wires are of iron and have the same length. Which wire has smaller cross section? Why?



Ans:

Determination of Young's modulus of the material of a wire



The apparatus consist of two long straight wires of same length and equal radius suspended side by side from a rigid support. Since both the reference and experimental wires are of the same material, their thermal expansion will be the same.

The weights placed in the pan exert a downward force and stretch the experimental wire under a tensile stress. The elongation of the wire (increase in length) is measured by the vernier arrangement.

Let r and L be the initial radius and length of the experimental wire, respectively. Let M be the mass that produced an elongation ΔL in the wire. The Young's modulus of the material of the experimental wire is given by,

$$Y = \frac{\text{Linear Stress}}{\text{Linear Strain}}$$

$$= \frac{F/A}{\Delta L/L} = \frac{Mg/\pi r^2}{\Delta L/L}$$

$$= \frac{Mg\,L}{\pi r^2\,\Delta L}$$

Using the above formula, the Young's modulus of the material of the experimental wire can be calculated.

Some Practical Applications of elasticity

Application-1: To find the thickness required for a metal rope, to be used in cranes to pull up heavy objects.

Suppose we want to make a crane which has a lifting capacity of 10 tonnes or one metric tonne [10000Kg]. Here the condition is that the load shouldn't deform the rope permanently. Therefore the extension shouldn't exceed the elastic limit.

Steel has yield strength of $3\times 10^8 N/m^2$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$\therefore \quad \text{Area} = \frac{\text{Force}}{\text{Stress}}$$

$$= \frac{Mg}{\text{Stress}}$$

$$= \frac{10000 \times 10}{3 \times 10^8}$$

$$= \frac{10^5}{3 \times 10^8}$$

$$= \frac{1}{3} \times 10^{-3}$$

$$= 3.33 \times 10^{-4} m^2$$

$$\Rightarrow \pi r^2 = 3.33 \times 10^{-4}$$

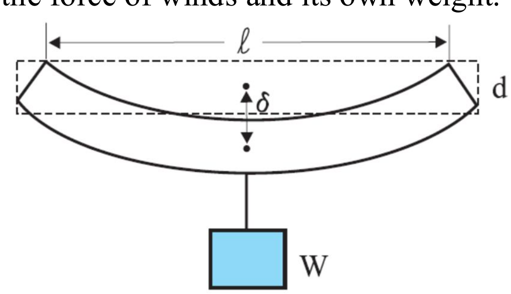
$$\Rightarrow \quad r^2 = \frac{3.33}{3.14} \times 10^{-4} = 1.06 \times 10^{-4}$$

$$\Rightarrow \quad r = 1.03 \times 10^{-2}\ m \approx 1\ cm$$

Generally a large margin for safety is provided. Thus a thicker rope of radius 3 cm is recommended. A single wire of this radius would practically be a rigid rod (no flexibility). So the ropes are always made of a number of thin wires braided together for flexibility and strength.

<u>Application-2</u>: - <u>To design a bridge for maximum safety.</u>

A bridge has to be so designed that it can with stand the load of the traffic, the force of winds and its own weight.

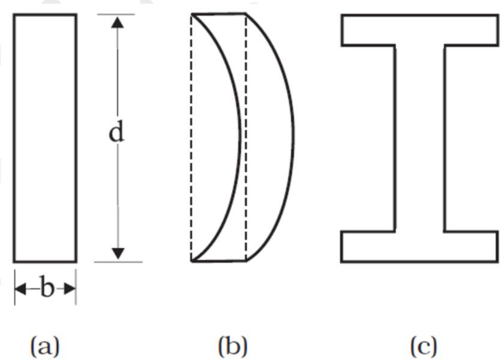


A bar of length 'ℓ', breath 'b' and depth 'd' supported at the ends, when loaded at the centre by a load 'W' sags (depresses) by an amount given by

$$\delta = \frac{W\ell^3}{4Ybd^3}$$

Hence to reduce bending for a given load, Y of the material of the beam should be large and 'ℓ' should be as small as possible. Since δ is inversely proportional to d^3, the depression can be reduced more effectively by increasing the thickness 'd' rather than increasing the breadth 'b' of the beam.

But on increasing the depth, unless the load is exactly at right place the bar may bend sideways. This is called **buckling**.



To avoid this, large load bearing surface as shown in figure (**c**) is used. This is called **I-sation** of the beam.

This shape reduces the weight of the beam, without sacrificing the strength. Hence reduces the cost.

<u>Application-3:</u> - <u>To answer the question why maximum height of a mountain on earth is limited to approximately 10 Km.</u>

At the bottom of the mountain of height 'h', the force per unit area due to the weight of the mountain is hρg, where 'ρ' is the density of the material of the mountain and 'g' the acceleration due to gravity.

Thus material at the bottom experiences this force in the vertical direction. Now the elastic limit for a typical rock is $3 \times 10^8 N/m^2$. Equating this to **hρg** with $\rho = 3 \times 10^3 Kg/m^3$ (density of rock)

$3 \times 10^8 = h\rho g$

$$h = \frac{3 \times 10^8}{\rho g} = \frac{\cancel{3} \times 10^8}{\cancel{3} \times 10^3 \times 10}$$

$$= \frac{10^8}{10^4} = 10^4 m$$

$$= 10 \text{ Km},$$

which is more than the height of Mount Everest.