Chapter 8

GRAVITATION

(Prepared By Ayyappan C, HSST Physics, GHSS Udma, Kasaragod, *Mob: 9961985448*)

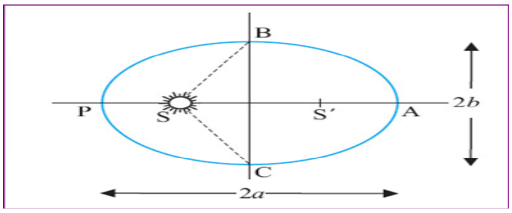
Introduction

- **Ptolemy** introduced the '**geocentric**' model to explain the motion of planets.
- **Copernicus** putforward the '**heliocentric theory'** in which the sun is at the centre of the planetary system.

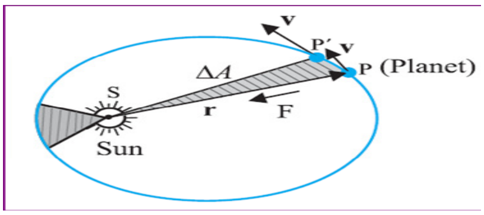
KEPLER'S LAWS OF PLANETARY MOTION

Law of orbits

- ***All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse.***



Law of areas



- **The line that joins any planet to the sun sweeps equal areas in equal intervals of time.**
- The **planets moves slower** when they are **away from the sun**
- The **planets moves faster** when they are **near to the sun**
- The law of areas is a consequence of conservation of angular momentum.

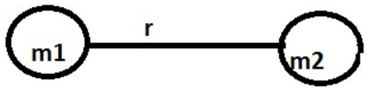
Law of periods

- **The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.**

$$T^2 \alpha\ a^3$$

UNIVERSAL LAW OF GRAVITATION

- ***Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.***



- Mathematically,

$$|\mathbf{F}| = G\frac{m_1 \ m_2}{r^2}$$

- where **G** is the***universal gravitational constant***
- The value of the gravitational constant G is experimentally determined by English scientist Henry Cavendish in 1798**.**

$$\mathbf{G = 6.67x10^{-11}\ N\ m^2/kg^2}$$

ACCELERATION DUE TO GRAVITY OF THE EARTH

Acceleration due to gravity on the surface

- The gravitational force acting on a body on the surface of earth is given by

$$F = \frac{GMm}{R^2}$$

Where G- gravitational constant, M- mass of earth, m- mass of the body, R- radius of the earth.

- The weight experience by the body is

F = mg, *where g – acceleration due to gravity*

- *Thus ,*

$$mg = \frac{GMm}{R^2}$$

- Therefore

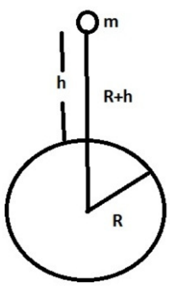
HSSLiVE.IN

$$g = \frac{GM}{R^2}$$

- The mass of the earth can be calculated using the values of acceleration due to gravity, G and radius of earth.

- This is the reason for the statement **"Cavendish weighed the earth".**

<u>Variation of acceleration due to gravity with height</u>



- The gravitational force on the mass *m at a height h above the* surface of the earth is

$$F = \frac{GMm}{(R+h)^2}$$

- The weight of the body at the height h is **mg_h**, where g_h is the acceleration due to gravity at height**.**
- Thus

$$\mathbf{mg_h} = \frac{GMm}{(R+h)^2}$$

Therefore , $\mathbf{g_h} = \dfrac{GM}{(R+h)^2}$

- **If R>>h**

$$\mathbf{g_h} = \frac{GM}{R^2(1+\frac{h}{R})^2}$$

- Ie , $\mathbf{g_h} = g(1+\frac{h}{R})^{-2}$,
- since $g = \dfrac{GM}{R^2}$
- Or

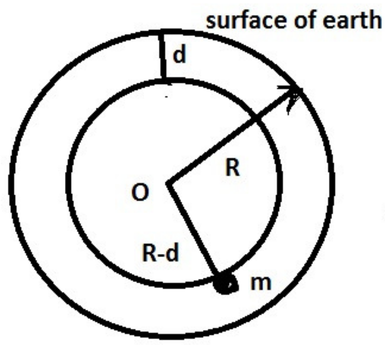
$$\mathbf{g_h} = g(1+\frac{h}{R})^{-2}$$

- Using binomial expression and neglecting the higher order terms we get

$$\mathbf{g_h} = g(1-\frac{2h}{R})$$

- Thus for small heights h above the value of g decreases

<u>Variation of g with depth</u>



- .If 'ρ' is the mean density of earth, then mass of earth is
- Mass = Volume x Density, ie

$$M = \frac{4}{3}\pi R^3 \rho$$

- Similarly mass of the small sphere of radius R-d is

$$M_S = \frac{4}{3}\pi (R-d)^3 \rho$$

- **Thus**

$$\frac{M_S}{M} = \frac{\frac{4}{3}\pi (R-d)^3 \rho}{\frac{4}{3}\pi R^3 \rho}$$

$$\frac{M_S}{M} = \frac{(R-d)^3}{R^3}$$

- **The acce**leration due to gravity on the surface of earth is

$$g = \frac{GM}{R^2}$$

- Thus the acceleration due to gravity on body at a depth d is

$$g_d = \frac{GM_S}{(R-d)^2}$$

- Thus dividing the two equations and substituting for M_S/M. we get

$$\frac{g_d}{g} = \frac{(R-d)}{R}$$

- Simplifying

$$g_d = g(1-\frac{d}{R})$$

- Thus, as we go down below earth's surface, the acceleration due gravity decreases.
- At the centre of the earth acceleration due to gravity is zero.

GRAVITATIONAL FIELD AND GRAVITATIONAL POTENTIAL

- The region around a mass where its gravitational force is experienced is called its **gravitational field**.
- The **gravitational potential** at point in the gravitational field is the work done in bringing a unit mass from infinity to that point in the field.
- The gravitational potential at a distance r from a mass M is given by

$$V_G = \int_{\infty}^{r} \frac{\mathrm{GM}}{r^2} \mathrm{dr}$$

$$V_G = -\frac{\mathrm{GM}}{r}$$

- If we are considering earth, M is the mass of the earth.

GRAVITATIONAL POTENTIAL ENERGY

- The **gravitational potential energy** at a point is the work done to bring a mass from infinity to that point.
- The gravitational potential energy associated with two particles of masses m_1 *and* m_2 *separated by distance by a* distance *r is given by*

$$\boxed{V = -\frac{Gm_1m_2}{r} \quad (\text{if we choose } V = 0 \text{ as } r \to \infty)}$$

ESCAPE SPEED

- **The minimum vertical velocity that has to be imparted to a body on the earth's surface , so that it escapes from the earth's gravitational pull and never returns to the earth is called escape velocity.**

Expression for escape velocity

- To escape from the earth's gravitational field the total energy of the body must be greater than or equal to zero.
- For the minimum velocity to escape the total energy is given by

$$\frac{1}{2}mv_e^{\,2} - \frac{GMm}{(R+h)} = 0$$

- Thus

$$\frac{1}{2}mv_e^{\,2} = \frac{GMm}{(R+h)}$$

- Therefore the velocity of escape is given by

$$v_e = \sqrt{\frac{2GM}{(R+h)}}$$

- If the body is thrown from near to the surface of earth, then h=0 , therefore

HSSLiVE.IN

$$v_e = \sqrt{\frac{2GM}{R}}$$

- We have $GM = gR^2$, thus

$$v_e = \sqrt{2gR}$$

- This is called **the *escape speed*,** sometimes loosely called **the *escape velocity.***
- On the surface of earth the escape speed is 11.2km/s approximately.
- The escape velocity **does not depend** on **mass of the body**.

Why moon has no atmosphere?

- The escape speed for the moon is **2.3 km/s**.
- The rms velocities of gas molecules on the surface of moon are greater than its escape speed.
- Therefore gas molecules escape from the moon and hence moon has no atmosphere.

EARTH SATELLITES

- **Earth satellites are objects which revolve around the earth.**
- The natural satellite of earth is moon (Time period 27.3 days aprox.).
- Examples for artificial (manmade) satellites are Sputnik, Aryabatta, INSAT etc.
- Artificial satellites are used in fields like telecommunication, geophysics and meteorology.

ORBITAL VELOCITY OF A SATELLITE

- **The velocity of a satellite in its orbit is called orbital velocity.**

Expression for orbital velocity

- The centripetal force required for the orbital motion of the satellite is provided by the gravitational force between satellite and the planet.
- Thus

HSSLiVE.IN

$$\frac{mv^2}{R+h} = \frac{GMm}{(R+h)^2}$$

Where m- mass of the satellite, v –velocity

- Therefore the orbital velocity is given by

$$v = \sqrt{\frac{GM}{R+h}}$$

- A satellite very close to earth's surface (h=0) is referred to as an ***earth satellite.***
- Thus the orbital velocity of an earth satellite is given by

$$v_o = \sqrt{\frac{GM}{R}}$$

- We have $GM = gR^2$, thus

$$v_o = \sqrt{gR}$$

RELATION CONNECTING ESCAPE VELOCITY AND ORBITAL VELOCITY

- The escape velocity is given by

$$v_e = \sqrt{2gR}$$

- The orbital velocity is

$$v_o = \sqrt{gR}$$

- Therefore

$$v_e = \sqrt{2}v_o$$

TIME PERIOD OF A SATELLITE

- Time period of satellite is the time for it to go once fully in its orbit.
- Period , T = (Distance travelled in one revolution) / (Orbital velocity)
- Thus

$$T = \frac{2\pi(R+h)}{\sqrt{\frac{GM}{R+h}}}$$

- **Or**

$$\boxed{T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}}$$

- Here $GM = gR^2$,.
- For a satellite very close to the surface of earth *h can* be neglected in comparison to *radius.*

- The period when h=0 is given by

$$\boxed{T = 2\pi\sqrt{\frac{R}{g}}}$$

- Substituting the numerical values , we get T= 85 minutes approx.

ENERGY OF AN ORBITING SATELLITE

- The kinetic energy of the satellite in a circular orbit with speed *v is*

$$KE = \frac{1}{2}mv^2$$

- Substituting for orbital velocity , we get

$$KE = \frac{GMm}{2(R+h)}$$

- The potential energy at distance (R+h) from the center of the earth is

$$PE = \frac{-GMm}{(R+h)}$$

- The K.E is positive whereas the P.E is negative.
- The total energy is

E = KE+PE

$$E = \frac{GMm}{2(R+h)} - \frac{GMm}{(R+h)}$$

$$E = -\frac{GMm}{2(R+h)}$$

- The total energy of an circularly orbiting satellite is negative.

GEOSTATIONARY SATELLITES

- **Satellites in a circular orbits around the earth in the equatorial plane with *T = 24 hours are called* Geostationary Satellites.**

- They appears stationary with respect to earth is called geo-stationary satellite.
- The direction of motion of the satellite is same as that of earth, i.e., from west to east.
- The height of the orbit of geostationary satellite is **36,000km approx.**
- These satellites are mainly used for communication purpose like T V and radio broadcasting and weather forecasting.

POLAR SATELLITES

- These are low altitude (*h is approximately equal to* 500 to 800 km) satellites, but they go around the poles of the earth in a north-south direction.

- Since its time period is around 100 minutes it crosses any altitude many times a day.

- These satellites are used for studying the weather, environment and spying.
- Polar satellites are extremely useful for remote sensing, meteorology as well as for environmental studies of the earth.

WEIGHTLESSNESS

- Weight of an object is the force with which the earth attracts it.

- In a satellite around the earth, every part and parcel of the satellite has acceleration towards the center of the earth which is exactly the value of earth's acceleration due to gravity at that position.

- Thus in the satellite everything inside it is in a state of free fall.

- Thus, in a manned satellite, people inside experience no gravity.

- When an object is in free fall, it is weightless and this phenomenon is usually called the phenomenon of **weightlessness.**

******

HSSLiVE.IN