Chapter – 6

Work, Energy & Power

Scalar Product of Vectors

1. Define the scalar product (Dot product) of vectors.

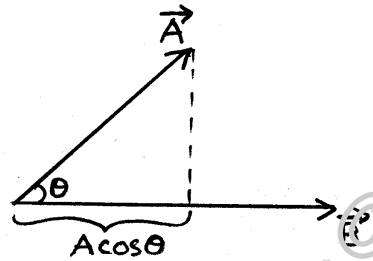
Ans:

Definition

$$\boxed{\vec{A} \bullet \vec{B} = |A||B|\cos\theta}$$

Here θ is the angle between $\vec{A}$ and $\vec{B}$.

Explanation



$$\vec{A} \bullet \vec{B} = (A\cos\theta)B$$

$A\cos\theta$ is the component of $\vec{A}$ along the direction of $\vec{B}$ or it is the projection of $\vec{A}$ on $\vec{B}$.

2. Is the scalar product of two vectors commutative?

Ans: Yes. Scalar product of two vectors is commutative.

That is, $\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$

3. If $\vec{A}$ and $\vec{B}$ are perpendicular, then what is the value of $\vec{A} \bullet \vec{B}$?

Ans: Here $\theta = 90^\circ$

$$\vec{A} \bullet \vec{B} = |A||B|\cos 90^\circ = 0$$

4. Obtain the component form of scalar product ($\vec{A} \bullet \vec{B}$)

Ans:

$$\vec{A} \bullet \vec{B} = |A||B|\cos\theta$$

If $\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors along the x, y, z directions, then

$$\hat{i} \bullet \hat{i} = |\hat{i}||\hat{i}|\cos 0^\circ = 1 \times 1 \times 1 = 1$$

Similarly, $\hat{j} \bullet \hat{j} = 1$ and $\hat{k} \bullet \hat{k} = 1$

$$\hat{i} \bullet \hat{j} = |\hat{i}||\hat{j}|\cos 90^\circ = 0$$

Similarly, $\hat{j} \bullet \hat{k} = 0$ and $\hat{k} \bullet \hat{i} = 0$

In component form,

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\begin{aligned}
&\vec{A} \bullet \vec{B} \\
&= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \bullet (B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\
&= A_xB_x(\hat{i} \bullet \hat{i}) + A_xB_y(\hat{i} \bullet \hat{j}) + A_xB_z(\hat{i} \bullet \hat{k}) \\
&+ A_yB_x(\hat{j} \bullet \hat{i}) + A_yB_y(\hat{j} \bullet \hat{j}) + A_yB_z(\hat{j} \bullet \hat{k}) \\
&+ A_zB_x(\hat{k} \bullet \hat{i}) + A_zB_y(\hat{k} \bullet \hat{j}) + A_zB_z(\hat{k} \bullet \hat{k}) \\
&= A_xB_x(1) + A_xB_y(0) + A_xB_z(0) \\
&+ A_yB_x(0) + A_yB_y(1) + A_yB_z(0) \\
&+ A_zB_x(0) + A_zB_y(0) + A_zB_z(1) \\
&= A_xB_x + A_yB_y + A_zB_z
\end{aligned}$$

$$\boxed{\vec{A} \bullet \vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

5. Give the expression to find the angle between A and B

Ans:

We have,

$$\vec{A} \bullet \vec{B} = |A||B|\cos\theta$$

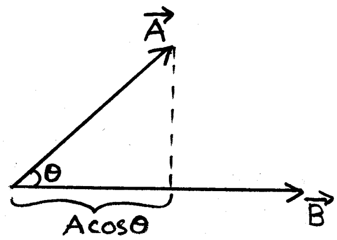
$$\Rightarrow \boxed{\cos\theta = \frac{\vec{A} \bullet \vec{B}}{|A||B|}}$$

Here $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

and $|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$

6. Give the expression to find the projection of $\vec{A}$ on $\vec{B}$

Ans: Projection of $\vec{A}$ on $\vec{B}$ is the component of $\vec{A}$ along the direction of $\vec{B}$



Projection of $\vec{A}$ on $\vec{B}$

$$= A\cos\theta == \frac{\vec{A} \bullet \vec{B}}{|\vec{B}|}$$

7P. If $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and

$\vec{B} = 4\hat{i} - 3\hat{j} + 2\hat{k}$. Find $\vec{A} \cdot \vec{B}$

Soln:

8P. Show that the given vectors

$2\hat{i} + 5\hat{j} + \hat{k}$ and $4\hat{i} - 3\hat{j} + 7\hat{k}$ are $\perp^r$.

Soln:

<u>Work</u>

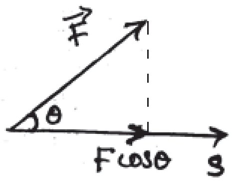
9. Define work.

Ans: Work = (component of force along the displacement) × displacement

W = (F cos θ) S

= F S cos θ

$= \vec{F} . \vec{S}$



$$\boxed{W = \vec{F} . \vec{S}}$$

10. What are the different types of work?

Ans:

<u>Positive work</u>

If the displacement is along the direction of force, the work done by the force is +ve

$W = \vec{F} . \vec{S}$

= FS cos 0

= FS

Negative work

If the displacement is opposite to the direction of force, the work done by the force is said to be –ve.

W = FS cos180

= FS (-1)

= -FS

Zero work

$W = \vec{F} \cdot \vec{S}$

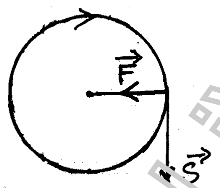
= FS cos 90

= 0

If the displacement is perpendicular to the force, the work done by the force is zero

11. What is the work done by a centripetal force?

Ans: -



The centripetal force is acting $\perp$ to the displacement of the particle.

$\therefore W = FS \cos 90^0 = 0$

Hence the work done by centripetal force is zero.

12P. If force $\vec{F} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ acting on a body produces a displacement of $\vec{d} = 5\hat{i} + 4\hat{j} - 3\hat{k}$, then find the work done by the force

Ans:

13P. A body constrained to move along the **z** axis of a coordinate system is subject to a constant force **F** given by $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N. What is the work done by this force on moving the body **4m** along the **z**-axis?

Soln:

14Q. a) A man tries to pull a rigid wall for a long time but fails to displace it. What is the external work done by him?

b) Suggest two conditions for the work done by a force to be zero.

Ans:

15. State and prove **Work-Energy Theorem.**

According to work energy theorem **'work done is equal to change in kinetic energy.'**

Ans: We have,

$$v^2 = u^2 + 2aS$$

$$2aS = v^2 - u^2$$

$$S = \frac{v^2 - u^2}{2a}$$

$$\text{Work}, W = FS$$
$$= m\cancel{a}\left(\frac{v^2 - u^2}{2\cancel{a}}\right)$$
$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$
$$= K_f - K_i$$
$$= \Delta K$$

$\therefore$ Work done = Change in K . E

16P. A body of mass **1 kg** travels in a straight line with a velocity $\mathbf{v=kx^{3/2}}$ where **k=5** SI units. Calculate the work done by the net force to displace from **x=0** to **x=2m**.

Soln:

17P. A bus and a car moving with the same kinetic energy are brought to rest by applying an equal retardation force by the breaking systems. Which one will come to rest at a shorter distance?

Ans: Change in KE = work

$$\Rightarrow \Delta K = F \times S$$

$\Delta K_{car} = F_{car} \times S_{car}$ ------------ (1)

$\Delta K_{bus} = F_{bus} \times S_{bus}$ ------------ (2)

(1) ÷ (2) →

$$\frac{\Delta K_{car}}{\Delta K_{bus}} = \frac{\cancel{F_{car}} \times S_{car}}{\cancel{F_{bus}} \times S_{bus}}$$
$$1 = \frac{S_{car}}{S_{bus}}$$
$$\Rightarrow S_{bus} = S_{car}$$

Both will travel equal distance.

18P. Consider a raindrop of mass **1g** falling from a height **1km** hits the ground with a speed of **50m/s**.

(a) What is the work done by the gravitational force?

(b) What is the work done by the unknown resistive force?

Soln: (a)

Change in K.E,

$$\Delta K = \frac{1}{2}mv^2 - 0$$
$$= \frac{1}{2} \times 10^{-3} \times (50)^2$$
$$= \frac{10^{-3} \times 2500}{2}$$
$$= 1250 \times 10^{-3} = 1.250 \text{ J}$$

Work done by gravitational force,

W_g = FS= mgh = 10 × 9.8 × 1000

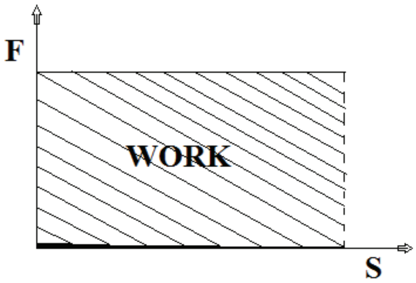
= 9.8 J

(b) Work done by resistive force on the rain drop is given by, $W_g + W_r = \Delta K$

$$W_r = \Delta K - W_g$$
$$= 9.87\text{J} – 1.250 \text{ J}$$
$$= 8.55 \text{ J}$$

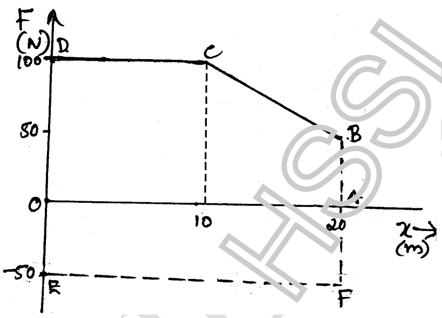
19. How can you find the work done from the **force – displacement graph**?

Ans: The area of force – displacement graph gives the work done.



20P. A woman pushes a trunk on a railway platform which has a rough surface. She applies a force of **100N** over a distance if **10m**. There after she gets progressively tired and her applied force reduces linearly with distance to **50N**. The total distance by which the trunk has been moved is **20m**. Plot the force applied by the women and the frictional force which is **50N**. Calculate the work done by the two forces over **20m**.

Soln: -



Work done by the woman

= area of OABCD

= Area of rectangle + Area of trapezium

= 100 × 10 + ½ (100 + 50) 10 = 1750J

Work done by the Friction

= -50 × 20 = -1000J

21Q. Ramesh lifts a body of mass ‘m’ to a height ‘h’ near the surface of the earth in a time ‘t’

a) Draw the force –displacement graph.
b) If ‘A’ is the area of the graph, what quantity does (A/t) indicate?

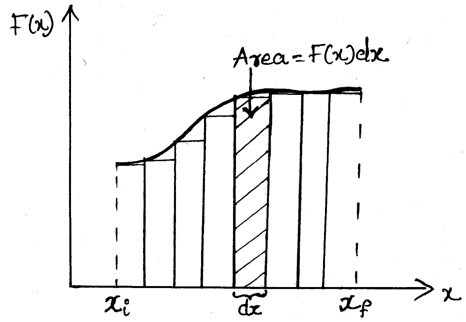
Ans:

22P. Calculate the work done in lifting a body of mass **10 Kg** to a height of **10 m** above the ground.

Soln:

23. How can you find work done by a variable force?

Ans:



For an infinitesimally small displacement **dx**, the force can be supposed to be a constant.

Work done to displace the body by a small amount 'dx' is given by,

$dW = F(x)dx$

Total workdone,

$$\boxed{W = \int_{x_i}^{x_f} F(x)dx}$$

24P. From the table given below.

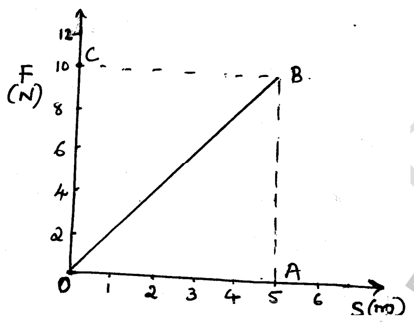
(a) Draw the force-displacement curve

(b) Analyse the graph and find the type of force involved.

(c) Estimate the work done.

Force (N)	2	4	6	8	10
Displacement(m)	1	2	3	4	5

Soln:



(a) OB is the force-displacement graph.

(b) The graph is a straight line.
The force is a variable force.

(c) Work done = area of Δ OAB
= ½OA × AB
= ½ × 5 × 10 = **25J**

Kinetic energy

25. Define kinetic energy.

Ans: Energy possessed by a body by virtue of its motion is called its kinetic energy.

26. Derive the expression for Kinetic energy.

Ans: Consider a body of mass **m** which is initially at rest. When a force is applied, it gets an acceleration **a** and its velocity increases to **v** while travelling a distance **S.**

$$v^2 = u^2 + 2aS$$

$$\Rightarrow v^2 = 2aS$$

$$\Rightarrow S = \frac{v^2}{2a}$$

We have, Work done, $W = \vec{F} \cdot \vec{S}$

$= F\,S$

$= ma \cdot \dfrac{v^2}{2a}$

$= \dfrac{1}{2} m v^2$

This work appears as the Kinetic energy of the body.

$$\Rightarrow \boxed{K.E = \frac{1}{2} mv^2}$$

Potential energy

27. Define potential energy.

Ans: The energy possessed by a body by virtue of its position or state of strain is called its potential energy.

28. Derive the expression to find the **gravitational potential energy** near the surface of earth.

Ans: Work done to lift a body to A height **h** from the surface of earth is given by,

$W = F\,S$

$= mg \times h$

$= mgh$

This work appears as the gravitational potential energy of the body.

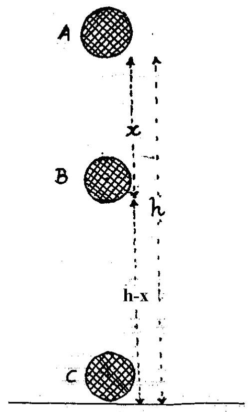
$\therefore$ Potential energy, $\boxed{U = mgh}$

29. What is meant by mechanical energy?

Ans: Kinetic energy and potential energy are together called as mechanical energy.

30. Show that the mechanical energy is conserved for a ball dropped from a height.

Ans:



<u>**At position A**</u>

Potential energy, **U = mgh**

Kinetic energy, **K = 0**

Total energy, **E = U + K = mgh**

<u>**At position B**</u>

U = mg (h – x)

= **mgh – mgx**

$v^2 = u^2 + 2as$

$v^2 = 0 + 2gx$

$v^2 = \mathbf{2gx}$

K = ½mv^2

= ½m (2gx)

= **mgx**

∴**E** = U + K

= mgh – mgx + mgx

= **mgh**

<u>**At position C**</u>

U = 0

$v^2 = u^2 + 2as$

= 0 + 2gh = **2gh**

$\therefore\ K = \frac{1}{2} mv^2$

$= \frac{1}{2} m(2gh) = mgh$

E = U + K

= 0 + mgh = **mgh**

The total mechanical energy is conserved at the three positions A, B and C

31P. A bob of a pendulum is released from a horizontal position. If the length of the pendulum is **1.5m**, what is the speed with which the bob arrives at the lowermost point, given that it dissipated **5%** of its initial energy against air resistance?

Ans:

32. Derive an expression for the potential energy of a spring (**Elastic potential energy**).

Ans: Consider a block attached to a spring and resting on a smooth horizontal surface. The other end of the

spring is attached to a rigid wall. Suppose that we pull the block outwards through a displacement x_m, then the restoring force developed in the spring is given by **Hooke's law** is,
Restoring force, **F = -kx, where**
k is called the spring constant (or stiffness constant)

$$W = \int_0^{x_m} F\,dx$$

$$= \int_0^{x_m} -kx\,dx \qquad \boxed{\int x^n dx = \frac{x^{n+1}}{n+1}}$$

$$= -k \int_0^{x_m} x\,dx$$

Work done by the spring force is

$$= -k \left[\frac{x^2}{2}\right]_0^{x_m}$$

$$= -k\left[\frac{x_m^2}{2} - \frac{0^2}{2}\right]$$

$$= -\frac{1}{2} k x_m^2$$

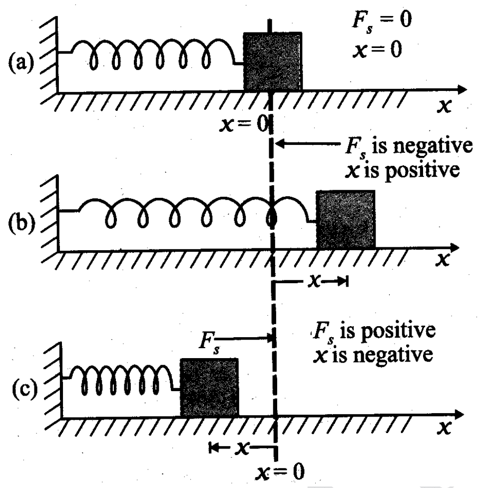
This is the work done by the spring force and is negative.
But the work done by the external pulling force is positive. This work done is stored in the form of elastic potential energy in the spring.

P . E of spring,

$$\boxed{U = \frac{1}{2} k x_m^2}$$

33. Explain the **conservation of mechanical energy** in the case of a spring.

Ans:



When the spring is stretched to a position $x = x_m$ and then released it **oscillates between positions** $x = -x_m$ **and** $x = x_m$

At the Stretched position $x = x_m$

PE is maximum, which is $U = \frac{1}{2}kx_m^2$

And KE, **K=0**

At the equilibrium position x = 0

KE is maximum, $K = \frac{1}{2}kx_m^2$

And PE, **U=0**

At the compressed position $x = -x_m$

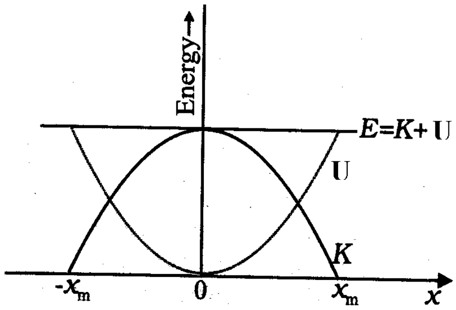
P.E is maximum, $U = \frac{1}{2}kx_m^2$

And KE = 0

At any position x between $-x_m$ and x_m

$$\text{Total energy, } E = U + K$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$



Expression for Maximum Velocity of the block of mass

The max. KE is $\frac{1}{2}kx_m^2$

$$\therefore \frac{1}{2}mv_{max}^2 = \frac{1}{2}kx_m^2$$

$$v^2_{max} = \frac{kx_m^2}{m}$$

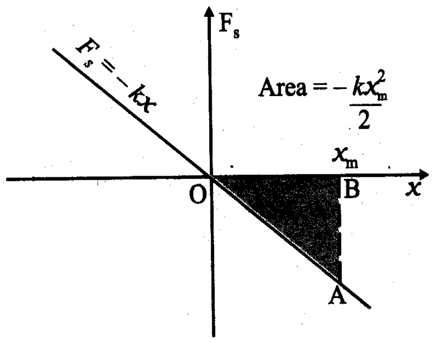
$$v_{max} = \sqrt{\frac{kx_m^2}{m}}$$

$$= \sqrt{\frac{k}{m}}\,x_m$$

$$\boxed{v_{max} = \sqrt{\frac{k}{m}}\,x_m}$$

34. Obtain the expression for work done by the spring force from the force displacement graph.

Ans: The spring force is given by Hooke's law **F = -kx**



Area between x axis and the line **F = -kx** gives the work done by the spring force.

$$W = Area = \frac{1}{2}F \times x_m$$

$$= \frac{1}{2}(-kx)x$$

$$= -\frac{1}{2}kx^2$$

35Q. An arrow shot from a bow has kinetic energy. How does it get this kinetic energy?

Ans: When the string of the bow is stretched with the arrow, the work done on the string is stored in it, in the form of elastic potential energy. When the arrow is released this potential energy is converted as kinetic energy.

36P. To simulate car accidents, auto manufactures study the collisions of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass **1000kg** moving with a speed of **18.0 km/h** on a smooth road and colliding with a horizontally mounted spring of spring constant **6.25 × 10³ N/m**. What is the maximum compression of the spring?

Ans:

Conservative and Non-conservative forces

37. Define **conservative force**. Give Examples. What are its properties?

Ans: If the work done by or against a force is independent of the path followed by the object, then the force is said to be conservative.

Eg:- **Electrostatic force, Gravitational force, Magnetic force, Lorentz force, Elastic force** etc.

Explanation- The work done by a conservative force does not depend on distance but only on displacement.

Properties of conservative force

(i) The work done for a round trip is zero.

(ii) They are central forces.

(iii)Work done by or against conservative force is completely recoverable.

38. Define **non - conservative** force

If the work done by or against the force depends on the path followed by the object, then the force is said to be non - conservative

Eg: - **Frictional force, Viscous force**

Explanation- The work done by a conservative force depends on distance.

Properties of non-conservative forces

(i) The work done for a round trip is not zero

(ii) They are not central forces.

(iii)Work done by or against conservative force is not completely recoverable.

39. Which are the various forms of energy?

Ans: Heat energy, Chemical energy, electrical energy, sound energy, light energy, nuclear energy are various forms of energy.

40. Explain the equivalence of mass and energy.

Ans: Albert Einstein showed that the mass and energy are equivalent and are related by the equation $\mathbf{E=mc^2}$. Here **c** is the speed of light. $\mathbf{c=3\times10^8 m/s}$. The equation tells us that a very large amount of energy is associated with a small amount of mass.

41P. Calculate the energy equivalent of 1kg mass.

Ans:

42. State the law of conservation of energy.

Ans:The law states that " **the energy can neither be created nor be destroyed but it can transform from one form to another**".

OR

"The total energy of the universe is a constant."

Power (P)

43. Define power.

Ans: Power is the rate of doing work

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{W}{t}$$

$$\text{Dimension} = \frac{ML^2T^{-2}}{T}$$

$$= ML^2T^{-3}$$

S.I unit of power is watt (W)

$$\boxed{\mathbf{1hp = 746\ W}}$$

44. Derive an expression for power in terms of force and velocity.

Ans:

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

$$P = \frac{\vec{F}\,.\,\vec{S}}{t}$$

$$= \vec{F}\,.\,\frac{\vec{S}}{t}$$

$$= \vec{F}\,.\,\vec{v}$$

$$\boxed{P = \vec{F}\,.\,\vec{v}}$$

45P. A pump on the ground floor of a building can pump up water to fill a tank of volume **30 m^3** in **15** minutes. If the tank is **40m** above the ground and the efficiency of the pump is **30%** how much electric power is consumed by the pump?

Ans:

46P. An elevator can carry a maximum load of **1800 kg** (elevator + passengers) is moving up with a constant speed of **2m/s**. The frictional force opposing the motion is **4000N**. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power.

Ans:

47P. A body is initially at rest. It undergoes one dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

(i) $t^{\frac{1}{2}}$ (ii) t (iii) $t^{\frac{3}{2}}$ (iv) t^2

Ans:

Collisions

48. Which physical quantities are conserved during all types of collisions?

Ans: The **total energy** and the **total linear momentum** are conserved during all types of collisions.

49. Distinguish between **elastic** and **inelastic** collisions.

(i) **Elastic collision**

If the total kinetic energy is conserved during a collision, then the collision is said to be elastic

Eg: The collision of subatomic particles is nearly elastic.

(ii) **Inelastic collision**

If the total kinetic energy is not conserved during a collision, then the collision is said to be inelastic

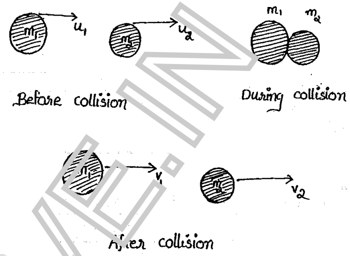
Eg: collision of vehicles, collision of pebbles.

Note: Almost all collisions in nature are inelastic.

Elastic Collision in One Dimension

50. Derive the expressions for final velocities of two bodies colliding **elastically in one dimension**.

Ans: Consider two masses $\mathbf{m_1}$ and $\mathbf{m_2}$ moving along a straight line with velocities $\mathbf{u_1}$ and $\mathbf{u_2}$ collide elastically with each other and their velocities changes to $\mathbf{v_1}$ and $\mathbf{v_2}$.



Since the linear momentum is conserved, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)\cdots(1)$$

Since the K.E also is conserved,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\frac{1}{2}(m_1u_1^2 + m_2u_2^2) = \frac{1}{2}(m_1v_1^2 + m_2v_2^2)$$

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

$$m_1u_1^2 - m_1v_1^2 = m_2v_2^2 + m_2u_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 - v_1)(u_1 + v_1)$$
$$= m_2(v_2 - u_2)(v_2 + u_2)\cdots(2)$$

Dividing (2) by (1)

$$\Rightarrow u_1 + v_1 = v_2 + u_2 \cdots\cdots(3)$$

$$\Rightarrow u_1 - u_2 = v_2 - v_1$$

$$\Rightarrow \quad \boxed{u_{12} = v_{21}}$$

i.e., **the relative velocity of the 1st body with respect to the 2nd before collision is equal to relative velocity of the 2nd body w.r.t the 1st after collision**.

Eqn. (3) →

$v_2 = u_1 + v_1 - u_2$

Substituting this value of v_2 in eqn. (1), we get,

$$m_1 (u_1 - v_1) = m_2 ((u_1 + v_1 - u_2) - u_2)$$

$$m_1 (u_1 - v_1) = m_2 (u_1 + v_1 - 2u_2)$$

$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - 2m_2u_2$$

$$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$$

$$(m_1 - m_2)u_1 + 2m_2u_2 = v_1(m_1 + m_2)$$

$$\boxed{v_1 = \frac{(m_1 - m_2)\, u_1 + 2m_2u_2}{(m_1 + m_2)}} \text{----}(5)$$

Eqn. (3) →

$v_1 = v_2 + u_2 - u_1$

Substituting this value of v_1 in eqn. (1)

$$m_1 (u_1 - (v_2 + u_2 - u_1)) = m_2 (v_1 - u_2)$$

$$m_1 (u_1 - v_2 - u_2 + u_1) = m_2 (v_2 - u_2)$$

$$m_1 (2u_1 - v_2 - u_2) = m_2 (v_2 - u_2)$$

$$2m_1u_1 - m_1v_2 - m_1u_2 = m_2v_2 - m_2u_2$$

$$2m_1u_1 - m_1u_2 + m_2u_2 = m_2v_2 + m_1v_2$$

$$2m_1u_1 + (m_2 - m_1)\, u_2 = (m_1 + m_2)\, v_2$$

$$v_2 = \frac{2m_1u_1 + (m_2 - m_1)\, u_2}{(m_1 + m_2)}$$

$$\boxed{v_2 = \frac{(m_2 - m_1)\, u_2 + 2m_1u_1}{(m_1 + m_2)}} \text{---}(6)$$

Special Cases

Case I

If the two colliding bodies have the same mass i.e., $\mathbf{m_1 = m_2 = m}$

From equations (4) and (5), we get

$\mathbf{v_1 = u_2}$ and $\mathbf{v_2 = u_1}$

Thus the two bodies interchange their velocities after collision.

Case II

If the two colliding bodies have the same mass i.e., $\mathbf{m_1 = m_2 = m}$. of the 2nd body is at rest i.e., $\mathbf{u_2 = 0}$

$v_1 = u_2 = 0$

$v_2 = u_1$

That is after collision, the first body comes to rest and the second body move with the initial velocity of the first.

Case III

If $\mathbf{u_2 = 0}$ and $\mathbf{m_1 >> m_2}$

i.e., $\mathbf{m_2 \approx 0}$, then from equations (4) and (5)

$$v_1 = \frac{m_1u_1}{m_1} = u_1$$

$$v_2 = \frac{2m_1u_1}{m_1} = 2u_1$$

Case IV

$\mathbf{u_2 = 0}$ and $\mathbf{m_1 << m_2}$

i.e., $\mathbf{m_1 \approx 0}$

$$v_1 = \frac{-m_2u_1}{m_2} = -u_1$$

$$v_2 = 0$$

Eg: - The collision of gas molecules with the walls of the container.

51P. The bob of a pendulum released from **30^0** to the vertical hits on another bob of equal mass at rest. How high does the first bob rise after the collision?

(Assume the collision is elastic and the sizes of the bobs are negligible)

Soln:

Perfectly Inelastic collision

52. Define perfectly inelastic collision.

Ans: In these types of collisions the K.E is not conserved and after the collision the colliding bodies stick together and move as a single mass.

Eg: A fired bullet embedded in a wooden block, mud thrown in to a wall.

53. (i) Deduce the expression for final velocity of the combined mass, when two masses collide inelastically in one dimension.

(ii) Also derive the expression for change in the kinetic energy during perfectly in elastic collision.

Ans: (i) By the law of conversion of momentum, we have

$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$

$$\boxed{v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}}$$

(ii) Loss of kinetic energy

$$\Delta K = K_f - K_i$$

$$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1+m_2)v^2$$

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

$$-\frac{1}{2}(m_1+m_2)\left(\frac{m_1u_1 + m_2u_2}{m_1 + m_2}\right)^2$$

After simplification we get,

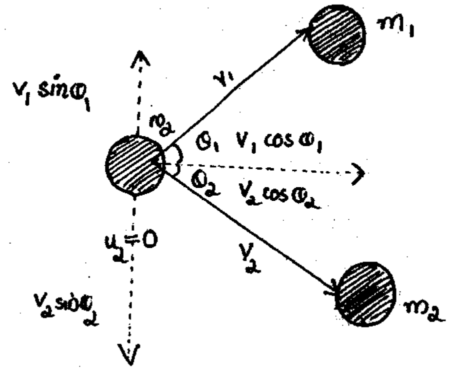
$$\boxed{\Delta K = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 - u_2)^2}$$

From the equation it is clear that the change in KE is always **+ve** which shows that there will be always a loss in K.E.

Collision in two dimensions

54. Explain the collision in two dimensions.

Ans: Consider a mass **m_1** moving with a velocity **u_1** collides with another body of mass **m_2**, which is initially at rest, and both move in two different directions.



For the conservation of momentum **along x – direction**,

$$m_1u_1 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2 \cdots (1)$$

For the conservation of momentum **along y –direction,**

$$0 = m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2 \cdots (2)$$

If the collision is elastic, the kinetic energy is conserved and we have one more equation,

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \cdots (3)$$

There are only 3 equations, and 4 unknown [v_1, v_2, θ_1, θ_2] if we know the value of any one of the unknowns (say θ_1), then all the other unknowns are calculated.

55. Derive the relation between linear momentum and kinetic energy.

Ans: We have Kinetic energy,

$$K = \frac{1}{2}mv^2$$

$$= \frac{mv^2 \times m}{2 \times m}$$

$$= \frac{m^2v^2}{2m}$$

$$= \frac{(mv)^2}{2m}$$

$$= \frac{p^2}{2m}$$

$$\therefore \boxed{K = \frac{p^2}{2m}}$$

$$\Rightarrow p^2 = 2mK$$

$$\Rightarrow \boxed{p = \sqrt{2mK}}$$

56Q. Two masses one lighter and other heavier have the same momentum. Which one will have greater K.E?

Ans:

$$\text{We have, } K = \frac{P^2}{2m}$$

$$\Rightarrow K \,\alpha\, \frac{1}{m}$$

Therefore, lighter body will have more K.E.

57. Two masses one lighter and other heavier have the same K.E. Which are will have greater momentum?

Ans:

$$p = \sqrt{2mK}$$

$$\Rightarrow p \,\alpha\, \sqrt{m}$$

So heavier body will have greater momentum.

58P. If the kinetic energy of a body is doubled, what is the percentage change in its linear momentum?

Soln:

59P. If the KE of a body is increased by **4** times, how will the momentum change?

Ans:

60P. Suppose an electron and a proton are projected with equal kinetic energy, what will be the ratio of their linear momenta if the proton is **1830** times heavier than an electron?

Ans: