## Chapter – 4

### Motion in a Plane

1. Distinguish between **Scalar** and **Vector quantities**.

**Ans**: The physical quantities which **have only magnitude** are called scalar quantities.

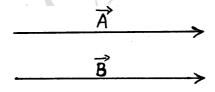
Eg: Mass, Time, Pressure, Volume, Temperature, Density, Work, Energy, Power, Speed, Pressure, Electric current, voltage, charge, etc.

The physical quantities which have both direction and magnitude and obey the laws of vector addition are called vector quantities.

- Eg: Displacement, Velocity, acceleration, Force, Momentum. Angular displacement, Angular velocity, Angular acceleration. Torque, Angular momentum, Area etc.
- 2. Define the different types of vectors.

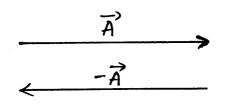
Ans: (i) Equal Vectors

Two vectors are said to be equal if they have the same magnitude and same direction.



### (ii) Negative of a Vectors

Negative of a vector is defined as another vector having the same magnitude but having opposite direction.



### (iii) Modulus of a vector

The modulus of a vector means the length or magnitude of that vector.

Modulus of vector  $\vec{A} = |\vec{A}| = A$ 

### (iv) Unit vector

Unit vector of  $\vec{A}$  is a vector of unit magnitude drawn in the direction of vector  $\vec{A}$ .

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

### (v) Zero vector Or Null vector

Zero vector is a vector having zero magnitude and an arbitrary direction, represented by  $\vec{0}$ .

- **3.** Give examples for zero vector. **Ans**:
- (i) The position vector of an object at the origin is zero vector.
- (ii) The velocity of an object at rest is zero vector.
  - (iii) The acceleration of an object moving with uniform velocity is zero vector.

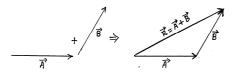
# Addition of Vectors (Graphical Method)

**4**. State and explain the Triangle law of vector addition.

#### Ans:

If two vectors can be represented by the two sides of a triangle taken in the same order, then their resultant is represented by the

third side of the triangle taken in the opposite order.

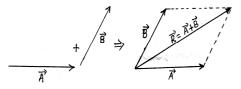


**5P.** A person walks **3km** towards North, then turns right and walks another **4km**. Find his displacement (Both magnitude and direction). Also find the total distance travelled.

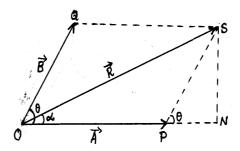
Ans:

**6.** State and explain the parallelogram law of vector addition.

Ans: If two vectors can be represented by the two adjacent sides of a parallelogram drawn from a common point, then their resultant is represented by the diagonal of the parallelogram passing through that point.



**7.** Give the analytical method Parallelogram law of vector addition. **Ans:** 



From

figure,

From 
$$\triangle$$
 PNS,  $\cos\theta = \frac{PN}{PS}$   
 $\Rightarrow PN = PS \cos\theta$   
and  $\sin\theta = \frac{SN}{PS}$   
 $\Rightarrow SN = PS \sin\theta$ 

Magnitude of R

$$R^{2} = ON^{2} + NS^{2}$$

$$R^{2} = (OP + PN)^{2} + NS^{2}$$

$$= OP^{2} + PN^{2} + 2OP.PN + NS^{2}$$

$$= OP^{2} + 2OP.PN + (PN^{2} + NS^{2})$$

$$= OP^{2} + 2OP.PN + PS^{2}$$

$$= OP^{2} + 2OP.PS\cos\theta + PS^{2}$$

$$= A^{2} + 2AB\cos\theta + B^{2}$$

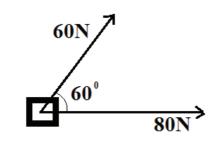
$$\therefore R = \sqrt{A^{2} + B^{2} + 2AB\cos\theta}$$

### Direction of R

$$\tan \alpha = \frac{SN}{OP + PN} = \frac{PS \sin \theta}{OP + PS \cos \theta}$$
$$= \frac{B \sin \theta}{A + B \cos \theta}$$

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

**8**. Find the magnitude and direction of resultant force acting on the body in the given figure.



Ans:

9. If the magnitude of two vectors and their resultant are the same, what is the angle between the two vectors?

Ans:

10P. A motor boat is racing towards north at 25km/hr and water current in that region is 10 km/hr in the direction  $60^{0}$  east of south. Find the resultant velocity of the boat.

Ans:

11P. On a certain day, rain was falling vertically with a speed of 35m/s. A wind started blowing after some time, with a speed of 12m/s in the east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella?

Ans:

12P. A man can swim with a speed of 4 km/hr in still water. How long does he take to cross a river 1km wide, if the river flows steadily at 3km/hr and

he makes his strokes normal to the river current? How far in the river does he go, when he reaches the other bank? **Ans**:

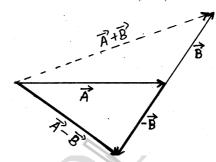
13P. A boatman can row with a speed of 10km/hr in still water. If the river flows steadily at 5km/hr, in which direction should the boatman rove in order to reach the point on the other bank, which is directly opposite to the point from where he started? Width of the river is 2km.

Ans:

**14.** How can you subtract a vector from another vector?

Ans: In order to subtract  $\vec{B}$  from  $\vec{A}$ , take the negative of  $\vec{B}$  and add it to  $\vec{A}$ .

i.e., 
$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$
.



**15.** Suppose  $\Lambda$  and  $\vec{B}$  are two vectors making an angle of  $\theta$  between them, then what is the magnitude of  $\vec{A} - \vec{B}$ ?

Ans: 
$$|\vec{A} \cdot \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

16. If  $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ , then what is the angle between  $\vec{A}$  and  $\vec{B}$ ?

Ans:

17P. Rain is falling vertically with a speed of 30m/s. A woman rides a bicycle with a speed of 10m/s in north to south direction. What is the direction in which she should hold her umbrella?

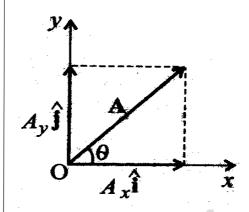
Ans:

**18P**. On a sunny day at 12 noon, you hold the umbrella vertically. If you run at certain speed, do you need to incline the umbrella? Justify your answer.

Ans:

19. Explain the resolution (splitting in to components) of vectors.

Ans:



By parallelogram law of vector addition,

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{OQ}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

 $A_x$ : and  $A_y$ ; are vector components of  $\overrightarrow{A}$  along X and Y axes and  $A_x$  and  $A_y$ 

are called scalar components of  $\overrightarrow{A}$ .

$$\cos \theta = \frac{OP}{OR},$$

i.e., 
$$\cos \theta = \frac{A_x}{A}$$

$$\Rightarrow \boxed{\mathbf{A}_{\mathbf{x}} = \mathbf{A} \cos \theta}$$

$$\sin\theta = \frac{PR}{OP}$$

i.e., 
$$\sin \theta = \frac{A_y}{A}$$

$$\Rightarrow A_y = A \sin\theta$$

Magnitude

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

Direction

$$\tan\theta = \frac{A_y}{A_x}$$

Scalar components of A:

 $A_x$ ,  $A_y$ 

<u>Vector components of</u>: -

$$A_x \hat{i}, A_y \hat{j}$$

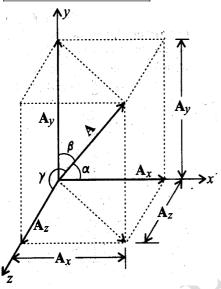
### <u>Rectangular Components of a Vector</u> <u>in Three Dimensions</u>

$$A_x = A\cos\alpha$$
,  $A_y = A\sin\beta$ ,  $A_z = A\cos\gamma$   
 $\overrightarrow{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ 

Magnitude

Ans:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



**20P.** Two forces  $\vec{F_i} = 3\hat{i} + 4\hat{j}$  and  $\vec{F_2} = 3\hat{j} + 4\hat{k}$  are acting simultaneously at a point. What is the magnitude of resultant force?

**21P.** Find whether the given vectors  $2\hat{i} + 3\hat{j} + 4\hat{k}$  and  $4\hat{i} + 6\hat{j} + 8\hat{k}$  are parallel or not.

Ans:

**22Q.** What are orthogonal unit vectors?

23P. Find the unit vector in the direction of the vector  $\hat{i} + 4\hat{j} - 2\hat{k}$ 

## **Motion in a Plane**

**24.** Define position vector.

Let P be the position of a particle.

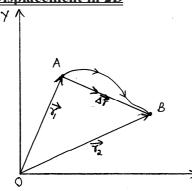
Then the vector  $\overrightarrow{r}$ , drawn from the origin to the point P is called the position vector of the particle

$$\vec{r} = x\hat{i} + y\hat{j}$$

- 25. Give the expressions for
- (i) Displacement (ii) Velocity and
- (iii) Acceleration in two dimensions.

Ans:

**Displacement in 2D** 



From figure,

$$\vec{r}_{2} = \vec{r}_{1} + \Delta \vec{r}$$

$$\Delta \vec{r} = \vec{r}_{2} - \vec{r}_{1}$$

$$= (x_{2}\hat{i} + y_{2}\hat{j}) - (x_{1}\hat{i} + y_{1}\hat{j})$$

$$= \hat{i}(x_{2} - x_{1}) + \hat{j}(y_{2} - y_{1})$$

$$= \Delta x \hat{i} + \Delta y \hat{j}$$

$$|\Delta \vec{r}| = \Delta x \hat{i} + \Delta y \hat{j}$$

$$|\Delta \vec{r}| = \sqrt{\Delta x^{2} + \Delta y^{2}}$$

**Velocity in 2D** 

$$\overrightarrow{v} = \lim_{\Delta t \to 0} \frac{\Delta \overrightarrow{r}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \left( \frac{\Delta x \hat{i} + \Delta y \hat{j}}{\Delta t} \right)$$

$$= \hat{i} \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} + \hat{j} \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$$

$$= \hat{i} \frac{dx}{dt} + \hat{j} \frac{dy}{dt}$$

$$= v_x \hat{i} + v_y \hat{j}$$

$$\overrightarrow{v} = v_x \hat{i} + v_y \hat{j}$$

$$v = |\overrightarrow{v}| = \sqrt{v_x^2 + v_y^2}$$

Acceleration in 2D

$$\hat{y} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \left( \frac{\Delta v_x \hat{i} + \Delta v_y \hat{j}}{\Delta t} \right)$$

$$= \hat{i} \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} + \hat{j} \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}$$

$$= \hat{i} \frac{dv_x}{dt} + \hat{j} \frac{dv_y}{dt}$$

$$= a_x \hat{i} + a_y \hat{j}$$

$$\Rightarrow a = a_x \hat{i} + a_y \hat{j}$$

26. A particle starts from a point

A (1, 2, -1) and reaches a point

B (3, 2, 2). Find the displacement vector  $\overrightarrow{AB}$  and its magnitude.

**27P.** The position of a particle is given by  $\vec{r} = 3\hat{ti} - 2t^2\hat{j} + 4\hat{k}$ , where t is in seconds and r is in meters.

- (a) Find the velocity  $\vec{v}$  and acceleration  $\vec{a}$  of the particle.
- (b) What is the direction of velocity and acceleration of the particle at t=2s?

Ans:

**28.** Write the kinematic equations for uniform acceleration for a two dimensional motion.

**Ans:** Motion in a plane can be treated as two separate simultaneous one-dimensional along two perpendicular directions.

### Along the x-direction

$$v_x = u_x + a_x t$$

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$v_x^2 = u_x^2 + 2a_x S_x$$

### Along the y-direction

$$v_v = u_v + a_v t$$

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$v_y^2 = u_y^2 + 2a_y S_y$$

# **Projectile Motion**

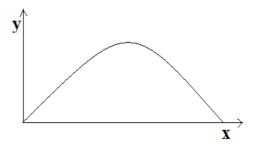
**29.** What is a projectile? Give examples.

Ans: Projectile is a body which is thrown in to space with some initial velocity and thereafter moves only under the effect of gravity.

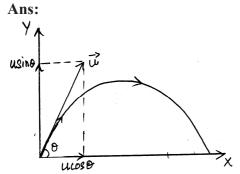
### Examples of projectile motion

- (i) A javelin thrown by an athlete.
- (ii) A bullet fired from a rifle.
- (iii) A jet of water coming from the side hole of a vessel.
- (iv) A stone thrown horizontally from the top of a building.
- (v) An object dropped from an aeroplane.
- **30.** What is the path (trajectory) of a projectile?

Ans: Parabolic path



**31.** What are the components of initial velocity of the projectile?



Horizontal component,

$$u_x=u\cos\theta$$

Vertical component,

- 32. What is the force on the projectile?

  Ans: F=-mg (downwards)
- **33.** What is the acceleration of the projectile?

Ans: a=-g (downwards)

**34.** What is the acceleration of the projectile at the highest position?

Ans: a=-g (downwards)

- 35. How the initial velocity changes during the motion of the projectile?

  Ans: During the motion of a projectile, the horizontal component of velocity  $(\mathbf{v_x} = \mathbf{ucos}\theta)$  remains same but the vertical component decreases first, reaches zero value and then increases in the downward direction.
- **36.** What is the velocity of the projectile at its highest position?

Ans: ucosθ

**Explanation** 

At the highest position, Horizontal component,

$$v_x = u\cos\theta$$

Vertical component,

$$\mathbf{v}_{\mathbf{v}} = \mathbf{0}$$

Resultant velocity,

$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(u\cos\theta)^2 + 0^2}$$
$$= u\cos\theta$$

**37**. What will be the magnitude and direction of velocity, 't' time after projection?

#### Ans:

Horizontal component of velocity,

$$v_x = u\cos\theta$$

Vertical component,

$$\mathbf{v_y} = \mathbf{u_y} + \mathbf{at}$$

$$= \mathbf{usin}\theta - \mathbf{gt}$$

Resultant velocity,

$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$$

Direction of resultant velocity,

$$\tan\theta = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

**38.** Derive expressions for (i) Time of flight (ii) Maximum Height (iii) Horizontal range of a projectile.

#### Ans:

### Time of flight (t<sub>f</sub>)

It is the time taken by the projectile to reach back to the horizontal plane of the point of projection.

To derive the expression for time of flight we must consider the **vertical motion** of the projectile.

Vertical Displacement 
$$S_y = 0$$
,  
time  $t \rightarrow t_f$ 

Initial velocity  $u_v = u \sin\theta$ 

Acceleration  $a_v = -g$ 

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$0 = u \sin\theta \times t_f + \frac{1}{2} \times -g \times t_f^2$$

$$0 = u \sin\theta t_f - \frac{1}{2}g t_f^2$$

$$\Rightarrow \frac{1}{2}g t_f^2 = u \sin\theta t_f$$

$$\frac{1}{2}g\,t_f = u\,sin\theta$$

$$\Rightarrow \boxed{t_f = \frac{2u \sin\theta}{g}}$$

Time of flight is maximum when, Sin  $\theta = 1$  i.e., when  $\theta = 90^{\circ}$ 

$$\therefore \left[ \left( t_{f} \right)_{\text{max}} = \frac{2u}{g} \right]$$

#### Maximum height (H)

It is the maximum vertical distance attained by the projectile above the horizontal plane of projection.

To derive expression for maximum height reached, let us consider the **vertical motion** of the projectile.

Displacement,  $S_y = H$ Initial velocity,  $u_y = u \sin \theta$ Final velocity,  $v_y = 0$ Acceleration,  $a_y = -g$  $v_y^2 = u_y^2 + 2a_yS_y$ 

$$0^{2} = (u \sin \theta)^{2} + 2 \times -g \times H$$

$$\Rightarrow 0 = u^{2} \sin^{2}\theta - 2g H$$

$$\Rightarrow 2g H = u^{2} \sin^{2}\theta$$

$$H = \frac{u^{2} \sin^{2}\theta}{2g}$$

For 
$$\theta = 90^{\circ}$$
,  $H_{max} = \frac{u^2}{2g}$ 

#### Horizontal Range (R)

It is the horizontal distance covered by the projectile during its time of flight.

To find the expression for horizontal range, let us consider the horizontal motion of the projectile.

There is no acceleration along the horizontal direction. So the horizontal motion is a uniform motion.

Forizontal range = Horizontal velocity × time of flight

$$S_{x} = u_{x} \times t_{f}$$

$$R = u \cos\theta \times \frac{2u \sin\theta}{g}$$

$$= \frac{u^{2} 2 \sin\theta \cos\theta}{g} = \frac{u^{2} \sin2\theta}{g}$$

$$R = \frac{u^{2} \sin2\theta}{g}$$

Horizontal range is maximum when,  $\sin 2\theta = 1 \implies 2\theta = 90$ 

$$\Rightarrow \theta = 45^{\circ} \qquad \boxed{R_{max} = \frac{u^2}{g}}$$

**39.** Give the relation between the maximum height and maximum horizontal distance for a given initial velocity.

Ans: We have,

$$H_{\text{max}} = \frac{u^2}{2g} \text{ and } R_{\text{max}} = \frac{u^2}{g}$$

$$\Rightarrow \boxed{R_{\text{max}} = 2H_{\text{max}}}$$

**40P**. A boy can throw a ball to a maximum horizontal distance of **100m**. With the same speed, how much high above the ground can he throw the same ball?

Ans:

**41P**. Ceiling of a hall is **25m** high. What is the maximum horizontal distance that a ball thrown with a speed of **40m/s** can go without hitting the ceiling of the hall?

Ans:

**42P.** A ball thrown by a player reaches another player in 2s. What is the maximum height attained by the ball above the point of projection?

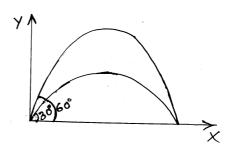
(Take 
$$g=10m/s^2$$
)

Ans:

**43Q**. The horizontal range of a projectile thrown at an angle  $\theta$  is R. Suggest another angle for which the range is the same.

**Ans:** 90-θ

**Eg**: For  $60^{\circ}$  and  $30^{\circ}$  the horizontal range is the same.



- **44P**. A ball is thrown horizontally from the top of a tower with a velocity 40m/s. Take  $g=10\text{m/s}^2$ .
  - a) Find the horizontal and vertical displacement after 1,2,3,4,5 seconds, then plot the path of motion of the ball.
- b) If the ball reaches the ground in 4 seconds, find the height of the tower.

Ans:

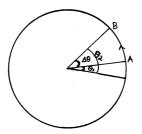
# **Circular Motion**

**45.** Define angular displacement and angular velocity.

Ans:

#### Angular displacement ( $\Delta\theta$ )

It is the angle swept by the radius vector in a given interval of time.



Angular displacement

$$\Delta\theta = \theta_2 - \theta_1$$

S.I unit of angular displacement is radian. But it has no dimension.

#### Angular velocity (ω)

The rate of change of angular displacement is called angular velocity.

Average angular velocity,

$$\frac{\omega}{\omega} = \frac{\sqrt{2}}{\sqrt{2}} \frac{\partial}{\partial t}$$

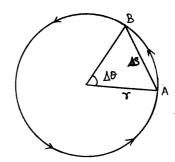
Instantaneous angular velocity

$$\mathfrak{D} = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d \theta}{d t}$$

S.I wait of angular velocity is radian/second. The dimensional formula is [M<sup>0</sup>L<sup>0</sup>T<sup>-1</sup>].

**46**. Derive the relation between linear displacement and angular displacement.

Ans:



We have,

Angle = 
$$\frac{Arc}{R \text{ adius}}$$
  
 $\Rightarrow \Delta \theta = \frac{\Delta S}{r} (\text{for small } \Delta \theta)$   
 $\Rightarrow \Delta S = r \Delta \theta$ 

 $\Delta S$  is the linear displacement and  $\Delta \theta$  is the angular displacement.

**47**. Derive the relation between linear velocity and angular velocity.

Ans: From the above figure,

$$\Delta\theta = \frac{\Delta S}{r} (\text{for small } \Delta\theta)$$

$$\Delta S = r \Delta\theta$$
Dividing by  $\Delta t$ 

$$\frac{\Delta S}{\Delta t} = r \frac{\Delta \theta}{\Delta t}$$

Taking limit  $\Delta t \rightarrow 0$  on both sides

$$\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$$

$$\Rightarrow v = r\omega$$

$$v = r\omega$$

48. Define time period.

Ans: The time taken by a particle to complete one circular path is called its time period of revolution.

49. Define frequency of a volution.

Ans: The no. of revolutions completed by the particle in one recond is called the frequency.

$$\upsilon = \frac{1}{T}$$

**50**. Give the relations between  $\omega$ , v and T.

Ans: When a particle completes one circular path, angular velocity,  $\omega$ 

$$= \frac{\text{Angular displacement}}{\text{Time}}$$

$$\boxed{\omega = \frac{2\pi}{T}} \Rightarrow \boxed{\omega = 2\pi \upsilon}$$

**51P**. Calculate the angular speed of

- (i) Second hand
- (ii) Minute hand and
- (iii) Hour hand of a watch.

Ans:

**52P.**An insect travels in a circle. It travels 6 revolutions in an anticlockwise direction for a time of 31.4 sec.

- a) Find the angular velocity of the insect.
- b) If the insect travels another 4 revolutions in the clockwise direction for a time of 8.6 sec, what will be the **angular speed** averaged over the total time?

**53P**. A ball, trapped in a circular path of radius 10 cm moves steadily and completes 10 revolutions in 100 s. What is the angular velocity and what is the linear velocity of its motion?

#### Ans:

$$r = 10cm = 0.1m, frequency, v = \frac{10 rev}{100 s}$$

$$= 0.1 rps = 0.1 Hz$$
Angular frequency,  $\omega = 2\pi v$ 

$$= 2 \times 3.14 \times 0.1 = 0.628 \ rad \ / s$$
Linear velocity,  $v = r\omega = 0.1 \times 0.628$ 

$$= 6.28 \times 10^{-2} \ m \ / s$$

### **Uniform Circular Motion**

**54**. What is meant by uniform circular motion? Give examples

**Ans:** If a particle moves along a circular path with a **constant speed**, then its motion is said to be a uniform circular motion.

Eg: (i) Motion of the tip of the second hand of a clock.

- (ii) The motion of a point on the run of a wheel rotating uniformly.
- (iii) The motion of the end of a leaf of a fan.
- (iv) Motion of a satellite.
- **55**. In uniform circular motion, which physical quantity remains constant?

**Ans**: Speed remains constant.

**56**. Is the velocity in uniform circular motion a constant vector?

**Ans**: No. Even though the magnitude of velocity remains constant, the direction continuously changes.

**57.** What is the direction of velocity in uniform circular motion?

**Ans**: Velocity at any point is tangential to the circular path.

**58**. Is there any acceleration in uniform circular motion?

**Ans**: Yes. Since the direction of velocity changes, there is acceleration.

**59.** Is the acceleration in uniform circular motion, a constant vector?

Ans: No. The magnitude of acceleration remains constant but the direction continuously changes.

**60.** What are the magnitudes and directions of (i) Acceleration and (ii) Force in uniform circular motion?

Ans: (1) 
$$a = \frac{v^2}{r}$$

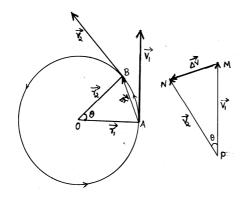
his directed towards the centre of the circular path. So it is called **centripetal** acceleration.

(ii) 
$$F = \frac{mv^2}{r}$$

It is directed towards the centre of the circular path. So it is called **centripetal force**.

**61.** Derive an expression for centripetal acceleration.

**Ans:** Consider a particle executing uniform circular motion around a circle of radius 'r'. Let v be its speed.



Since the speed of the particle is

$$\left|\overrightarrow{\mathbf{v}_{1}}\right| = \left|\overrightarrow{\mathbf{v}_{2}}\right| = \mathbf{v}$$

i.e., PM = PN = v

$$\left| \overrightarrow{r_1} \right| = \left| \overrightarrow{r_2} \right| = r$$

uniform,

$$\Rightarrow$$
 OA = OB = r,

radius of the circle.

$$\angle AOB = \angle MPN = \theta$$

Thus triangles AOB and

MPN are similar.

Hence 
$$\frac{MN}{AB} = \frac{MP}{AO}$$

$$\Rightarrow \frac{\Delta v}{\Delta r} = \frac{v}{r}$$

$$\Delta v = \frac{v}{r} \Delta r$$

Dividing both sides by  $\Delta$  t

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \frac{\Delta r}{\Delta t}$$

Taking limits  $\Delta t \rightarrow 0$ 

on both sides,

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v}{r} \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$$

$$\Rightarrow a = \frac{v}{r} \cdot v$$

$$\Rightarrow \overline{\left| a = \frac{v^2}{r} \right|}$$

But we have  $v = r\omega$ 

$$\therefore a = \frac{(r\omega)^2}{r} = \frac{\chi^2 \omega^2}{\chi}$$
 Direction of 
$$\Rightarrow \boxed{a = \omega^2 r}$$

Centripetal acceleration is towards the centre of the circle along the radius.

**62.** Obtain the expression for centripetal acceleration.

**Ans:** It is the force acting towards the centre of the circular path.

Centripetal force,

$$F = m \times a$$

$$= m \times \frac{v^2}{r}$$

$$F = \frac{mv^2}{r}$$

$$F = m\omega^2 r$$

**63**Q. Can an object be accelerated without speeding up? Illustrate with an example.

Ans: Yes. A body executing uniform circular motion has constant speed. But it has an acceleration towards the centre of the circular path, which is called the centripetal acceleration. This acceleration is produced by the centripetal force, which changes the direction of velocity at every instant.

**64P**. A stone tied to the end of a string **80cm** long is whirled in a horizontal circle with a constant speed. If the stone makes **14 revolutions in 25 seconds**, what is its acceleration?