Chapter– 2

Units and Measurement

1. What is meant by **Physical quantities?**

Ans: The quantities which can be measured directly or indirectly are called physical quantities.

2. Which are the two types of physical quantities? Explain.

Ans: There are two types of physical quantities:

a) **Fundamental Physical quantities** (Basic physical quantities)

The physical quantities which are independent of each other and cannot be expressed in terms of other physical quantities are called fundamental physical quantities.
There are seven fundamental or basic quantities. They are:

i. **Mass**
ii. **Length**
iii. **Time**
iv. **Electric current**
v. **Temperature**
vi. **Luminous intensity**
vii. **Amount of substance**.

b) **Derived Physical quantities**

The physical quantities which can be expressed in terms of fundamental physical quantities are called derived quantities.
Eg: - Velocity, Acceleration, Force, Momentum etc.

3. Define Physical unit.

Ans: The standard amount of a physical quantity chosen to measure the quantity of the same kind is called a physical unit.

4. What is meant by a **unit system**? Explain some commonly used unit systems.

Ans: A complete set of units which is used to measure all kinds of fundamental and derived quantities is called a system of units.
Some of the commonly used unit systems are as follows:

i.C. G. S. System
It was set up in France
Length – centimetre
Mass – gram
Time - second

ii.F. P. S. System
It is a British system
Length – foot
Mass – pound
Time - second

iii.M. K. S. System
It is also a French system
Length – metre
Mass – kilogram
Time – second

iv.S.I. System
It is the international system of units.
SI is the abbreviation for "Systeme Internationale d' Unites"
SI system is based on **7 fundamental units and two supplementary units.**

7 Fundamental units

i.Length – metre (**m**)
ii.Mass – kilogram (**Kg**)
iii.Time – second (**S**)
iv.Temperature – Kelvin (**K**)
v.Electric current – ampere (**A**)
vi.Luminous intensity – candela (**Cd**)
vii. Amount of substance – mole (**mol**)

Supplementary SI units

i. Plane angle – radian (**rad**)
ii. Solid angle – steradian (**Sr**)

5. Which are practical units used for measuring large distances? Compare them.

Ans:

(i) **Light year** (ℓy)

It is the distance travelled by light in vacuum in one year.

1 light year

= speed of light in vacuum × 1 year

$= 3 \times 10^8 \times 365.25 \times 24 \times 60 \times 60$

$= 9.4673 \times 10^{15} m$

$$1\ \ell y = 9.4673 \times 10^{15} m$$

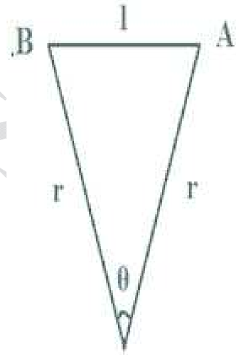
(ii) **Astronomical unit** (AU)

It is the defined as the mean distance of the earth from the sun.

$$1\ AU = 1.496 \times 10^{11} m$$

(iii) **Par sec** (Parallactic second)

It is the **largest** practical unit of distance used in astronomy. **It is defined as the distance at which an arc of length 1 astronomical unit subtends an angle of 1 second of arc.**



Angle=Arc/Radius

$\theta = \frac{\ell}{r} \quad \therefore \; r = \frac{\ell}{\theta}$

$1 \text{ parsec} = \frac{1 AU}{1''}$

$= \frac{1.496 \times 10^{11} m}{\frac{1}{3600} \times \frac{\pi}{180} rad}$

$= 3.08 \times 10^{16} m$

$$1 \text{ par sec} > 1 \ell y > 1 AU$$

6. Which are the **direct methods for the measurement of length**?

Ans: Length can be measured directly using

(i) Metre scale (Least count = 0.1cm)
(ii) Vernier callipers (Least count = 0.01cm)
(iii) Screw gauge (Least count = 0.001cm)

Indirect methods for Measuring Large Distances

7. What is meant by Parallax?

Ans: Parallax is the apparent shift in the position of an object with respect to another when we shift our eye side wise.

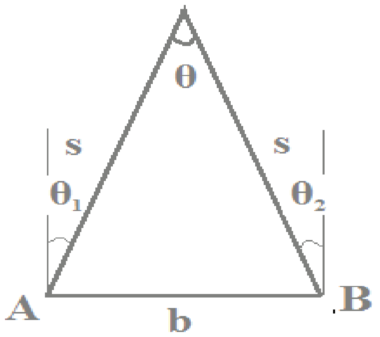
8. Explain the Parallax method to find the distance of the moon or any planet.

Ans: To measure the distance (s) of the moon (or a planet), we observe it simultaneously from two different positions (observatories) A and B on earth.

Let $\theta = \theta_1 + \theta_2$

We have, Angle $= \frac{Arc}{Radius}$

$\therefore \; \theta = \frac{b}{s} \quad \Rightarrow \quad s = \frac{b}{\theta}$



9. 1micron =--------m?

Ans: 10^{-6}m

10. One angstrom unit=------m?

Ans: 10^{-10}m

11. Kilowatt-hour is the unit of **------**

Ans: Energy **(**Commercial unit of electric energy**)**

Sub Multiples	
Centi	**10^{-2}**
Milli	**10^{-3}**
Micro	**10^{-6}**
Nano	**10^{-9}**
Pico	**10^{-12}**
Femto	**10^{-15}**

Dimensions of Physical Quantities

Physical Quantity		**Dimensions**	**Unit**
Fundamental Physical Quantities			
1	Length	**[L]**	M
2	Mass	**[M]**	Kg
3	Time	**[T]**	S
4	Temperature	**[K]**	K
5	Electric current	**[A] or [I]**	A
6	Luminous Intensity	**[Cd]**	Cd
7	Amount of substance	**[mol]**	mol
Derived Physical Quantities			
8	Velocity/ speed	**$[LT^{-1}]$**	m/s
9	Acceleration $\left(a = \frac{dv}{dt}\right)$	**$[LT^{-2}]$**	m/s^2
10	Force (F = ma)	**$[MLT^{-2}]$**	$Kgms^{-2}$ (N)
11	Momentum (P = mv)	**$[MLT^{-1}]$**	Kgm/s
12	Work/Energy (W=F× S)	**$[ML^2T^{-2}]$**	Kgm^2s^{-2} (J)
13	Electric power $\left(p = \frac{w}{t}\right)$	**$[ML^2T^{-3}]$**	Kgm^2s^{-3} (W)
14	Area	**$[L^2]$**	m^2
15	Volume	**$[L^3]$**	m^3
16	Density $\left(\rho = \frac{m}{V}\right)$	**$[ML^{-3}]$**	Kgm^{-3}
17	Pressure $\left(P = \frac{F}{A}\right)$	**$[ML^{-1}T^{-2}]$**	$Kgm^{-1}s^{-2}$ (Pa)
18	Gravitational constant $\left(G = \frac{F\,r^2}{m_1 m_2}\right)$	**$[M^{-1}L^3T^{-2}]$**	Kgm^3s^{-2}
19	Angular momentum (L = mvr)	**$[ML^2T^{-1}]$**	Kgm^2s^{-1} (J.S)
20	Torque $\left(\vec{\tau} = \vec{r} \times \vec{F}\right)$	**$[ML^2T^{-2}]$**	Kgm^2s^{-2}
21	Coefficient of viscosity $\left(\eta = \frac{F\ell}{Av}\right)$	**$[ML^{-1}T^{-1}]$**	$Kgm^{-1}s^{-1}$

12. Define Dimensional formula and Dimensional Equations

Ans: Dimensional formula: - The expression which shows the relation of a physical quantity with the fundamental physical quantities is called dimensional formula.

Eg : The dimensional formula of volume is $[M^0L^3T^0]$

The dimensional formula of velocity is $[M^0L^1T^{-1}]$

Dimensional equation of volume is, $[V] = [M^0L^3T^0]$

Dimensional equation of velocity is, $[v] = [M^0L^1T^{-1}]$

13. State the **Principle of Homogeneity of Dimensions**

Ans: It states that "**the dimensions of fundamental physical quantities on both sides of an equation should be the same**."

Applications of Dimensional Analysis

14. What are the applications of dimensional analysis?

Ans: The applications are:

- **i.** To check the correctness of an equation.
- **ii.** To derive the equations for various physical quantities.

Application # 1

15P. Check the equation $V = KA^2ut$, where '**A**' is the area of cross section of the pipe, **u** is the speed of flow, **t** is the time, **V** is the volume of water flowing through the pipe and **K** is a dimensionless constant. State whether the equation is correct or not.

Soln:

16P. Check the dimensional validity of the equation.

$$\mathbf{P = Fv + Av^3\rho}$$

Soln:

17P. Check by the method of dimensions whether the following equations are correct.

i) $T = 2\pi\sqrt{\frac{\ell}{g}}$

[T is the time period; l is the length of the string]

ii) $v = \sqrt{\frac{P}{\rho}}$

[P= pressure and ρ is the density]

iii) $v = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$

[**l** = length of the string, **T** = tension in the string and **m** = mass per unit length and v=velocity]

Soln:

18P. Check whether the equation is dimensionally correct

$$T = 2\pi\sqrt{\frac{m}{g}}$$

19P. Check the correctness of the equation $KE = \frac{1}{2} mv^2$

<u>**Ans: -**</u> [LHS] = [KE] = $[ML^2T^{-2}]$

[RHS] = $\left[\frac{1}{2} mv^2\right]$ = [M] $[LT^{-1}]^2$

= [M] $[L^2T^{-2}]$

= $[ML^2T^{-2}]$

[LHS] = [RHS]

Therefore the given equation is dimensionally correct.

20P. Check the correctness of the equation $S = ut + \frac{1}{2} at^2$

<u>**Ans: -**</u> [LHS] = [S] = [L]

[RHS] = [ut] + $\left[\frac{1}{2} at^2\right]$

= $[LT^{-1}]$ [T] + $[LT^{-2}]$ $[T^2]$

= $[LT^{-1+1}]$ + $[LT^{-2+2}]$

= $[LT^{-1+1}]$ + $[LT^{-2+2}]$

= $[LT^0]$ + $[LT^0]$

= [L] + [L]

= [L]

[LHS] = [RHS]

Therefore the given equation is dimensionally correct.

21P. Check the correctness of the equation **E = mc**

Ans: - [LHS] = [E] = $[ML^2T^{-2}]$

[RHS] = [mc] = [M] $[LT^{-1}]$

= $[MLT^{-1}]$

[LHS] ≠ [RHS]

Therefore the given equation is not dimensionally correct.

22P. Check whether the following equation is dimensionally correct.

$$\frac{1}{2} mv^2 = mgh$$

Soln:

$$[LHS] = \left[\frac{1}{2} mv^2\right] = [M] [LT^{-1}]^2$$

$$= [M] [L^2T^{-2}]$$

$$= [ML^2T^{-2}]$$

$$[RHS] = [mgh] = [M][LT^{-2}][L]$$

$$= [ML^2T^{-2}]$$

[LHS] = [RHS]

Therefore the given equation is dimensionally correct.

23P. The Vander waal's equation for a gas is $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

Determine the dimensions of **a** and **b.** Hence write the SI units of **a** and **b**.

Soln:

241P. If $x = a + bt + ct^2$, where **x** is in metre and **t** is in seconds; then what are the units and dimensions of **a, b** and **c**?

Ans:

25P. **x = A sin (ω t + c)**, where **x** is displacement and **t** is time. What are the units and dimensions of **A, ω** and **c**?

Soln:

26P. $\mathbf{y = a \sin \frac{2\pi v t}{k}}$, where **v** is the velocity at instant **t**. For the equation to be dimensionally correct, what should be dimensions of **k**?

Ans:

27P. A famous relation in physics relates moving mass m to the 'rest mass' m_0 of a particle in terms of its speed υ and the speed of light C. (This relation first arose as a consequence of special

Relativity by Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant C. He writes:

$$m = \frac{m_0}{(1 - v^2)^{1/2}}$$

Guess where to put the missing **c**.

Soln:

Application # 2

28P. Derive an expression for the centripetal force **F** acting on a particle of mass **m**, moving with velocity **v** in a circle of radius **r**.

Ans:

29P. The viscous force (**F**) acting on a spherical ball falling through a viscous medium depends on

(i) Coefficient of viscosity (**η**)

(ii) Radius of the ball (**r**)

(iii) Velocity of the ball (**v**)

Derive an expression for viscous force **F**

(Dimension of **η** = **[ML⁻¹T⁻¹]**)

Soln:

30P. Let the time period (**T**) of a simple pendulum depends on its length (**l**), mass of the bob (**m**) and acceleration due to gravity (**g**). Derive the expression for its time period using method of dimensions.

Soln:

Let $T \propto m^a l^b g^c$

$$T = K m^a l^b g^c \text{ ------------(1)}$$

Where K is a dimensionless constant.

Taking dimensions on both sides of eqn. (1), we get

$$[T] = [M]^a [L]^b [LT^{-2}]^c$$

$$[M^0L^0T^1] = [M^aL^bL^c T^{-2c}]$$

$$[M^0L^0T^1] = [M^a L^{b+c} T^{-2c}]$$

By the Principle of Homogeneity of dimensional equations,

a = 0, b + c = 0, -2c = 1

$$\Rightarrow c = \frac{1}{-2} = \frac{-1}{2}$$

$$b + \frac{-1}{2} = 0$$

$$b = \frac{1}{2}$$

$$\therefore a = 0,\ b = \frac{1}{2},\ c = \frac{-1}{2}$$

∴ substituting in eqn (1) we get

$$T = K m^0 l^{1/2} g^{-1/2}$$

$$T = K \frac{l^{1/2}}{g^{1/2}}$$

$$T = K \frac{\sqrt{l}}{\sqrt{g}}$$

$$T = K \sqrt{\frac{l}{g}}$$

From experiments, K = 2π

$$\therefore T = 2\pi\sqrt{\frac{l}{g}}$$

31. What are the limitations of dimensional analysis?

Ans:

i. If an equation contains **two or more terms** it cannot be derived using dimensional analysis.

ii. Expressions for **dimensionless** physical quantities (such as numbers) cannot be derived using dimensional analysis.

iii. If a physical quantity is **related to more than three quantities**, its relation cannot be derived using dimensional analysis.

iv. The proportionality constant in an equation cannot be found using dimensional analysis.

32. Do all physical quantities have dimensions? If no, name three physical quantities which are dimensionless?

Ans: No, all physical quantities do not have dimensions. The physical quantities like **angle, strain** and **relative density** are dimensionless.

33. Name two physical quantities having the dimensions $[ML^2T^{-2}]$?
Ans: Work and torque

34. Can a physical quantity has dimensions but still has no units?
Ans: No, a quantity having dimensions must have some units of its measurement.

35. Can a quantity have different dimensions in different systems of units?
Ans: No, a quantity has same dimensions in all system of units.

36. Can a quantity have units but still be dimensionless?
Ans: Yes. For example, a plane angle has no dimensions but has unit **radian** for its measurement.

37. Give an example for a unit less, dimensionless physical quantity.
Ans: Strain is a physical quantity that has no units and dimensions

38. Write pairs of physical quantities having same dimensions

Ans:

1. Distance & displacement – $[L]$
2. Speed & velocity – $[LT^{-1}]$
3. Frequency & angular velocity – $[T^{-1}]$
4. Momentum & Impulse – $[MLT^{-1}]$
5. Work (Energy) & Torque– $[ML^2T^{-2}]$
6. Pressure & Stress - $[ML^{-1}T^{-2}]$

39. Distinguish between **accuracy and precision** of instruments

Ans:

Accuracy: The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.

Precision :- The precision tells us to what resolution or limit the quantity is measured.

Explanation

For example, suppose the true value of certain length is near 3.678cm. In one experiment using a measuring instrument of resolution 0.1cm, the measured value is found to be 3.5 cm, while in another experiment using a measuring device of greater resolution, say 0.01cm, and the length is determined to be 3.38cm.

The first measurement has more accuracy but less precision, while the second measurement is less accurate but more precise.

40. Which measurement is most precise?

(i) Vernier calipers having **5** divisions on sliding scale.

(ii) Vernier calipers having **10** divisions on sliding scale.

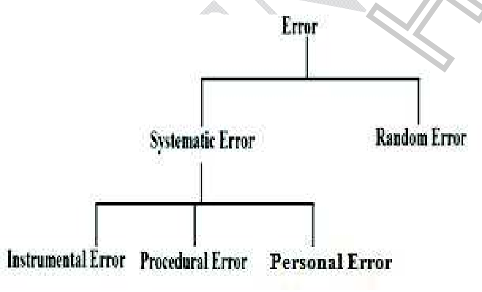
(iii) Vernier calipers having **20** divisions on sliding scale.

Ans:

Errors

41. Define error.

Ans: Error is the difference between the true value and measured value



Calculation of Errors

42. Let a_1, a_2, a_3 -----------------a_n be a set of measured values. Write the expressions for

i. Mean value
ii. Mean Absolute Error
iii. Relative Error
iv. Percentage Error

Ans: i) Mean value

$$\bar{a} = \frac{a_1 + a_2 + a_3 + \cdots\cdots + a_n}{n}$$

ii) Absolute errors

$$\Delta a_1 = |\bar{a} - a_1|$$

$$\Delta a_2 = |\bar{a} - a_2|$$

$$\Delta a_3 = |\bar{a} - a_3|$$

$$\Delta a_n = |\bar{a} - a_n|$$

Mean absolute error

$$\overline{\Delta a} = \frac{\Delta a_1 + \Delta a_2 + \Delta a_3 + \cdots\cdots + \Delta a_n}{n}$$

iii) Relative error = $\frac{\overline{\Delta a}}{\bar{a}}$

iv) Percentage error = $\frac{\overline{\Delta a}}{\bar{a}} \times 100\,\%$

43P. The period of oscillation of a simple pendulum measured are 2.63 Sec, 2.56 Sec, 2.42 Sec, 2.71 Sec, 2.80 Sec.

Find,

a) True period of oscillation
b) Mean absolute error
c) Fractional error (relative error)
d) Percentage error
e) Time period of the simple pendulum with error limits.

Ans: a_1 = 2.63 S

a_2 = 2.56 S

a_3 = 2.42 S

$a_4 = 2.71$ S

$a_5 = 2.80$ S

a) True value = mean value

$$\vec{a} = \frac{a_1 + a_2 + a_3 + a_4 + a_5}{5}$$

$$= \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$= 2.624$

$= 2.62$

b) $\Delta a_1 = |\bar{a} - a_1| = |2.62 - 2.63|$

$= |-0.01| = 0.01$

$\Delta a_2 = |\bar{a} - a_2| = |2.62 - 2.56|$

$= |0.06| = 0.06$

$\Delta a_3 = |\bar{a} - a_3| = |2.62 - 2.42|$

$= |0.2| = 0.2$

$\Delta a_4 = |\bar{a} - a_4| = |2.62 - 2.71|$

$= |-0.09| = 0.09$

$\Delta a_5 = |\bar{a} - a_5| = |2.62 - 2.80|$

$= |-0.18| = 0.18$

Mean absolute error,

$$\overline{\Delta a} = \frac{\Delta a_1 + \Delta a_2 + \Delta a_3 + \Delta a_4 + \Delta a_5}{5}$$

$$= \frac{0.01 + 0.06 + 0.20 + 0.09 + 0.18}{5}$$

$= 0.108$

$= 0.11$

$$\text{Fractional error} = \frac{\overline{\Delta a}}{\bar{a}} = \frac{0.11}{2.62}$$

$= 0.04$

$$\text{Percentage error} = \frac{\overline{\Delta a}}{\bar{a}} \times 100$$

$= 0.04 \times 100 = 4\,\%$

Period of simple pendulum with error limit,

$a = \bar{a} \pm \Delta \bar{a}$

$= (2.62 \pm 0.11)$s

Combination of errors

Error in Sum or Difference

If Δ**a** and Δ**b** are the errors in 'a & b' the **error in a ± b = ± (Δa + Δb)**

44. The sides of a rectangular lamina are (8.5 ± 0.2) cm and (5.6 ± 0.1) cm. Calculate the perimeter of the lamina with error limits.

Ans: l = 8.5 cm

Δ l = 0.2 cm

b = 5.6 cm

Δ b = 0.1 cm

Perimeter = 2(l + b)

= 2(8.5 + 5.6)

= 28.2 cm

Δ P = ± 2((Δ l + Δ b)

= ±2 (0.2 + 0.1)

= ± 0.6 cm

$\therefore$ Perimeter = (28.2 ± 6) cm

45P. Two resistance R_1 = 100 ± 3 Ω and R_2 = 200 ± 4 Ω are connected in series. What is their equivalent resistance?

Soln:

46P. If $l_1 = (10.0 \pm 0.1)$ cm and, $l_2 = (9.0 \pm 0.1)$ cm, find their sum, difference and error in each?

Soln:

Error in product or quotient

Percentage error in **ab** or $\frac{a}{b}$

= % error in a + % error in b

$$= \frac{\Delta a}{a} \times 100\,\% + \frac{\Delta b}{b} \times 100\,\%$$

47P. If the percentage error in the measurement of voltage is 5% and that in current is 2%, then what is the percentage error in

(i) Resistance?

(ii) Power?

Soln:

48P. The resistance $R = \frac{V}{I}$, where V = 100 ± 3V and I = 10 ± 0.2A. Find the percentage error in R.

Soln:

49P. If displacement of a body, **S** = (200 ± 5) meters and time taken by it **t** = (20 ± 0.2) seconds, then find the percentage error in the calculation of velocity?

Ans:

$$v = \frac{S}{t}$$

Percentage error in v

$$= \frac{\Delta S}{S} \times 100 + \frac{\Delta t}{t} \times 100$$

$$= \frac{5}{200} \times 100 + \frac{0.2}{20} \times 100$$

$$= 2.5 + 1 = 3.5\%$$

Error in power (Exponent)

Let $Z = \frac{x^a}{y^b}$

% error in Z = a (% error in x) + b (% error in y)

50P. A physical quantity P is related to four observable a, b, c and d as $P = \frac{a^3 b^2}{\sqrt{c}\, d}$. The percentage errors of the measurements in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in P?

Soln:

51P. A physical quantity Q is given by

$$Q = \frac{A^2 B^{3/2}}{C^4 D^{1/2}}$$

The percentage errors in A, B, C and D are 1%, 2%, 4%, 2% respectively. Find the percentage error in Q?

Soln:

Percentage error in Q

$$= 2(\%\text{error in A}) + \frac{3}{2}(\%\text{error in B})$$

$$+4(\%\text{error in C}) + \frac{1}{2}(\%\text{error in D})$$

$$= 2(1\%) + \frac{3}{2}(2\%) + 4(4\%) + \frac{1}{2}(2\%)$$

$$= 2\% + 3\% + 16\% + 1\% = 22\%$$

52P. The error in the measurement of radius of a sphere is 0.4%. Find the permissible error in the surface area.

Soln: -

53P. If the percentage error in the measurement of radius 'R' of a sphere is 2%, then what is the percentage error in its volume?

Soln:

54P. If the length and time period of an oscillating pendulum have errors of 1% and 2% respectively, what is the percentage error in **g**?

Soln:

55P. The percentage error in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimate of kinetic energy obtained by measuring mass and speed?

Soln:

$$KE = \frac{1}{2} m v^2$$

$$\%\ \text{error KE} = \%\ \text{error m} + 2(\%\ \text{error v})$$

$$= 2 + 2(3\%) = 8\%$$

Significant Figures

Rules for finding the Number of Significant Figures (NSF)

i. All the non-zero digits are significant.

Eg: 12345 NSF = 5

ii. All the zeros between two non-zero digits are significant.

Eg: 1005 NSF = 4

12003 NSF = 5

iii. If a number is less than 1, the zeros on the right of the decimal point before a non-zero digit are not significant.

Eg: 0.000123 NSF= 3

1.000123 NSF= 7

512.004100 NSF= 9

iv. For a number greater than 1 without any decimal the trailing zeros are not significant.

Eg: 512000 NSF= 3

v. For a number with a decimal the trailing zeros are significant.

Eg: 512.000 NSF= 6

5.10×10^3 NSF= 3

5.1000 NSF = 5

56P. State the number of significant figures in the following:

(i) $0.007 m^2$

(ii) 2.64×10^{24} kg

(iii) $0.2370 gcm^{-3}$

(iv) 6.320 J

(v) $6.032 Nm^{-2}$

(vi) $0.0006032 m^2$

(vii) 2.000m

(viii) 5100 kg

(ix) 0.050

Ans:

Rules for Arithmetic Operations

Significant Figures in Product or Quotient

In multiplication or division, the final result should retain as many significant figures as are there in the original number with least significant figures.

57P. The mass of a body is **4.237g** and volume **2.51 cm^3**. Calculate the density of the body to correct no. of significant figures?

Soln:

58P. What is the area of a square of side **1.4cm** in proper significant figures?

Ans:

Significant figures in Sum or Difference

In addition, or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.

Eg: 1) 436.32 + 227.2 + 0.301
= 663.8

2) 0.307 – 0.12 = 0.187
= 0.19