13 PROBABILITY

Summary

The conditional probability of an event E,given the occurrence of the event F is given by

$$P(E|F)=\frac{P(E\cap F)}{P(F)} \qquad [P(F)\neq 0,\ 0\leq P(E|F)\leq 1]$$

$$P(E'|F)=1-P(E|F)$$

$$P((E\cup F|G))=P(E|G)+P(F|G)-P((E\cap F|G))$$

$$P(E\cap F)=P(E)P(F|E), P(E)\neq 0$$

$$P(E\cap F)=P(F)P(E|F), P(F)\neq 0$$

If E and F are independent, then

$$P(E\cap F)=P(E)P(F)$$

$$P(E|F)=P(E),\ P(F)\neq 0$$

$$P(F|E)=P(F),\ P(E)\neq 0$$

Theorem of total probability:

Let $\{E_1, E_2, ..., E_n\}$ be a partition of a sample space and suppose that each of $E_1, E_2, ..., E_n$ has non-zero probability. Let A be any event associated with S, then

$$P(A)=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+...+P(E_n)P(A|E_n)$$

Bayes' theorem:

If $E_1, E_2, ..., E_n$ are events which constitute a partition of sample space S, i.e. $E_1, E_2, ..., E_n$ are pairwise disjoint and $E_1\cup E_2\cup ....E_n=S$ and A be any event with nonzero probability, then

$$P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{\sum_{j=1}^{n} P(E_j)P(A \mid E_j)}$$

A random variable is a real valued function whose domain is the sample space of a random experiment. The probability distribution of a random variable X is the system of numbers

$$\begin{array}{llllll} X & : x_1 & x_2 & x_3 & \ldots & x_n \\ P(X) & : p_1 & p_2 & p_3 & \ldots & p_n \end{array}$$

where $p_i > 0,\ \sum_{i=1}^{n} p_i = 1, i = 1, 2, 3, \ldots, n$

Let X be a random variable whose possible values x_1, x_2, ..., x_n occur with probabilities p_1, p_2, ... p_n respectively. The mean of X, denoted by $\sum_{i=1}^{n} x_i p_i$.

The mean of a random variable X is also called the expectation of X, denoted by E (X).

Let X be a random variable whose possible values x_1, x_2, ..., x_n occur with probabilities $p(x_1), p(x_2)$, ... $p(x_n)$ respectively.

Let $\mu = E(x)$ be the mean of X, The variance of X, denoted by $\mathrm{Var}(X)$ or σ_x^2, is defined as

$$\sigma_x^2 = \mathrm{Var}(X) = \sum_{i=1}^{n} (x_i - \mu)^2 p(x_i)$$

or equivalently

$$\sigma_x^2 = E(X - \mu)^2$$

The non-negative number:

$$\sigma_x = \sqrt{Var(X)} = \sqrt{\sum_{i=1}^{n}(x_i-\mu)^2 p(x_i)}$$

is called the standard deviation of the random variable X.

$$Var(X) = E(X)^2 - [E(X)]^2$$

Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

(i) There should be a finite number of trials.

(ii) The trials should be independent.

(iii) Each trial has exactly two outcomes : success or failure.

(iv) The probability of success remains the same in each trial.

For Binomial distribution

$$B(n,p), P(X=x) = {}^nC_x q^{n-x} p^x, x = 0,1,..n \quad ; \quad (q = 1-p)$$

MODEL QUESTIONS

Question :

The probability that at least one of the two events A and B occurs is 0.5. If A and B occures simultanously with probability 0.4,evaluate $P(\overline{A}) + P(\overline{B})$.

Solution :

$P(A \cup B) = 0.5,\ P(A \cap B) = 0.4$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) + P(B) = P(A \cup B) + P(A \cap B) = 0.5 + 0.4 = 0.9$

$P(\overline{A}) + P(\overline{B}) = 1 - P(A) + 1 - P(B) = 2 - [P(A) + P(B)] = \underline{1.1}$

Question :

A and B are two events such that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{4}$. Find $P(A/B)$, $P(B/A)$, $P(A'/B)$, $P(A'/B')$

Solution :

Given, $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{4}$

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{3}\right)}=\frac{3}{4}$$

$$P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)}=\frac{1}{4}$$

$$P(A'/B)=\frac{P(A'\cap B)}{P(B)}=\frac{P(B)-P(A\cap B)}{P(B)}$$

$$=\frac{\left(\frac{1}{3}-\frac{1}{4}\right)}{\left(\frac{1}{3}\right)}=\frac{1}{2}$$

$$P(A'/B')=\frac{P(A'\cap B')}{P(B')}=\frac{1-P(A\cup B)}{1-P(B)}$$

$$=\frac{1-\left[P(A)+P(B)-P(A\cap B)\right]}{1-P(B)}$$

$$=\frac{1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right]}{1-\frac{1}{3}}=\frac{1-\left(\frac{14}{24}\right)}{\left(\frac{2}{3}\right)}$$

$$=\frac{5}{8}$$

Question :

12 cards numbered 1 to 12 are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, then find the probability that it is an even number

Solution :

Given, $n(S)=12$.

$$A=\{4,5,6,7,8,9,10,11,12\},$$

$$B=\{2,4,6,8,10,12\}$$

$$A\cap B=\{4,6,8,10,12\}$$

$$n(A)=9,\ n(B)=6,\ n(A\cap B)=5.$$

$$P(A)=\frac{n(A)}{n(S)}=\frac{9}{12}=\frac{3}{4}$$

$$P(B)=\frac{n(B)}{n(S)}=\frac{6}{12}=\frac{1}{2}$$

$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}=\frac{5}{12}$$

Now

$$P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{\frac{5}{12}}{\frac{3}{4}}=\frac{5}{9}$$

Required probability is $\frac{5}{9}$.

Question :

A bag contains 3 red and 7 black balls. Two balls are selected at random one by one without replacement. If the second selected ball happens to be red, what is the probability that the first selected ball is also red?

Solution :

Given, $P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{7}{10}$

$\therefore \quad P(A/E_1) = \frac{2}{9}$ and $P(A/E_2) = \frac{3}{9}$

By Bayes theorem,

$$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2)}$$

$$= \frac{\frac{3}{10} \times \frac{2}{9}}{\frac{3}{10} \times \frac{2}{9} + \frac{7}{10} \times \frac{3}{9}} = \frac{6}{27} = \frac{2}{9}$$

Question :

Out of a group of 30 honest people, 20 always speak the truth. Two persons are selected at random from the group. Find the probability distribution of the number selected persons who speak the truth. Also, find the mean of the distribution. What values are described in this questions?

Solution :

Given, $n(S) = 30$,

Number of people speaking truth $= 20$

Number of people not speaking truth $= 10$

$\therefore P(\text{none speak truth})$

$$= \frac{{}^{10}C_2}{{}^{30}C_2} = \frac{\frac{10 \times 9}{2 \times 1}}{\frac{30 \times 29}{2 \times 1}} = \frac{9}{87}$$

$P(\text{only one of them speak truth})$

$$= \frac{{}^{20}C_1 \times {}^{10}C_1}{{}^{30}C_2} = \frac{20 \times 10}{\frac{30 \times 29}{2 \times 1}} = \frac{40}{87}$$

P(none speak truth)

$$= \frac{^{20}C_2}{^{30}C_2}$$

$$= \frac{\frac{20 \times 19}{2 \times 1}}{\frac{30 \times 29}{2 \times 1}} = \frac{38}{87}$$

Probability distribution is given

x	0	1	2
$p(x)$	$\frac{9}{87}$	$\frac{40}{87}$	$\frac{38}{87}$

$$\text{Mean} = \sum x_i p_i$$

$$= x_1p_1 + x_2p_2 + x_3p_3$$

$$= 0 \times \frac{9}{87} + 1 \times \frac{40}{87} + 2 \times \frac{38}{87}$$

$$= \frac{40}{87} + \frac{76}{87}$$

$$= \frac{116}{87}$$

It clearly depicted the value of truthfulness and morality.

HOME WORK QUESTIONS

Question :(Imp2017)

A man is known to be speak truth 3 out of 4 times.

(a) Write the probability that he speaks truth.

(b) He throws a die.

(i) Find the probability that he reports that "it is a six".

(ii) If he reports "it is six". Find the probability that it is actually not a six.

OR

Let x denote the number of study hours of a person during a randomly selected day.

The probability distribution is given below:

x	: 0	1	2	3	4
P(x)	: 0.1	k	2k	2k	k

(a) Find k.

(b) Find the probability that he studies at least two hours.

Answer : (a) $\frac{3}{4}$ (b) $\frac{3}{8}$ OR (a) $\frac{3}{20}$ (b) $\frac{3}{4}$

Question :(March 2017)

(a) If A and B are two events such that $A \subset B$ and $P(A) \neq 0$, then $P(A/B)$ is

(b) There are two identical bags. Bag I contains 3 red and 4 black balls while Bag II contains 5 red and 4 black balls.

One ball is drawn at random from one of the bags

(i) Find the probability that the ball drawn is red.

(i) If the ball drawn is red what is the probability that it was drawn from bag I?

OR

Consider the following probability distribution of a random variable X

x	: 0	1	2	3	4
P(x)	: $\frac{1}{6}$	$\frac{1}{16}$	K	$\frac{5}{16}$	$\frac{1}{16}$

(a) Find the value of K.

(b) Determine the Mean and Variance of X.

Answer :

(a) $\frac{P(A)}{P(B)}$ (b) $\frac{31}{63}, \frac{27}{62}$ OR (a) $K = \frac{7}{16}$ (b) 0.902

Question :(Imp 2016)

(a) If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, then

$P(A/B)$ is $\left(\frac{9}{4}, \frac{16}{13}, \frac{4}{9}, \frac{11}{13}\right)$

(b) Probability of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, then

(i) Find the probability that the problem is solved.

(ii) Find the probability that exactly one of them solves the problem

OR

A die is thrown 6 times. If getting an odd number is a success

(i) Find probability of success and failure.

(ii) Find the probability of 5 success.

(iii) Find the probability of atleast 5 successes.

Answer :

(a) $\frac{4}{9}$

(b) (i) $1-P(A').P(B')=1-\left(\frac{1}{2}\times\frac{2}{3}\right)=\frac{2}{3}$

(ii) $\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{2}{3}=\frac{1}{2}$

OR (i) $\frac{1}{2}$ (ii) $\frac{3}{32}$ (iii) $\frac{7}{64}$

Question :(March 2016)

(a) If $P(A)=0.3, P(B)=0.4$ then the value of $P(A\cup B)$ where A and B independent events is

$(0.48,\ 0.51,\ 0.52,\ 0.58)$

(b) A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be diamonds. Find the probability of the lost card being a diamond.

OR

A pair of dice is thrown 4 times. If getting a doublet is considered as a success,

(1) find the probability of getting a doublet.

(2) hence, find the probability of two successes.

Answer : (a) 0.58 (b) $\frac{11}{50}$ OR (1) $\frac{1}{6}$ (2) $\frac{25}{216}$

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