11 THREE DIMENSIONAL GEOMETRY

MODEL QUESTIONS

Question 01 :

If the cartesian equation of a line is $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$, then write the vector equation for the line.

Solution :

Given cartesian equation, $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$

In standard form, $\frac{x-3}{-5}=\frac{y+4}{7}=\frac{z-3}{2}=\lambda$

$x=-5\lambda+3, y=7\lambda-4, z=2\lambda+3$

Now, $x\hat{i}+y\hat{j}+z\hat{k}=(-5\lambda+3)\hat{i}+(7\lambda-4)\hat{j}+(2\lambda+3)\hat{k}$

$$=\left(3\hat{i}-4\hat{j}+3\hat{k}\right)+\lambda\left(-5\hat{i}+7\hat{j}+2\hat{k}\right)$$

which is the vector equation for the line.

Question 02 :

Find the direction cosines of the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$,

Solution :

Given equation, $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$

In standard form, $\dfrac{x-4}{-2}=\dfrac{y}{6}=\dfrac{z-1}{-3}$

Here, direction ratios of the line are $-2, 6, -3$

and $\sqrt{(-2)^2+(6)^2+(-3)^2}=\sqrt{49}=7$

$\therefore$ Direction cosines are $-\dfrac{2}{7}, \dfrac{6}{7}, -\dfrac{3}{7}$

Question 03 :

If a line has direction ratios $2, -1, -2$, then what are its direction cosines?

Solution :

Given direction ratios are $2, -1, -2$

$\therefore$ Direction cosines of the line are

$$=\frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$$

$$=\frac{2}{\sqrt{4+1+4}}, \frac{-1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}}=\frac{2}{\sqrt{9}}, \frac{-1}{\sqrt{9}}, \frac{-2}{\sqrt{9}}$$

$\therefore$ Direction cosines are $\dfrac{2}{3}, -\dfrac{1}{3}, -\dfrac{2}{3}$

Question 04:

What are the direction cosines of a line which makes equal angles with the coordinate axes?

Solution:

A line makes equal angles with the coordinate axes, then

$l=m=n \qquad \{\because \alpha=\beta=\gamma\ ;\ l=\cos\alpha, m=\cos\beta, n=\cos\gamma\}$

We know that $l^2+m^2+n^2=1$

$\therefore \qquad l^2+l^2+l^2=1 \quad \Rightarrow \quad 3l^2=1 \quad \Rightarrow \quad l=\pm\dfrac{1}{\sqrt{3}}$

$\therefore$ Direction cosines are $\pm\dfrac{1}{\sqrt{3}}, \pm\dfrac{1}{\sqrt{3}}, \pm\dfrac{1}{\sqrt{3}}$.

Question 05 :

Write the vector equation of the line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$

Solution :

Given equation $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$

The point on the line is $(5,-4,6)$

Direction ratios are $(3,7,2)$

$\therefore$ The vector equation of the line is if point is $\vec{a}$ and direction of a line $\vec{b}$,is

$\vec{r}=\vec{a}-\lambda\vec{b}=\underline{(5\hat{i}-4\hat{j}+6\hat{k})+\lambda(3\hat{i}+7\hat{j}+2\hat{k})}$

Question 06 :

Find the direction cosines of the line $\frac{x+2}{2}=\frac{2y-7}{6}=\frac{5-z}{6}$, Also find the vector equation of the line through the point $(-1,2,3)$ and parallel to the given line.

Solution :

Given equation, $\frac{x+2}{2}=\frac{2y-7}{6}=\frac{5-z}{6}$

In standard form, $\frac{x+2}{2}=\frac{y-\frac{7}{2}}{3}=\frac{z-5}{-6}$

Here, direction ratios of the line are $2,3,-6$

and $\sqrt{(-2)^2+(6)^2+(-3)^2}=\sqrt{49}=7$

Now, direction cosines are

$$=-\frac{2}{\sqrt{2^2+3^2+(-6)^2}},\frac{3}{\sqrt{2^2+3^2+(-6)^2}},-\frac{6}{\sqrt{2^2+3^2+(-6)^2}}$$

$$=\frac{2}{7},\frac{3}{7},-\frac{6}{7}$$

The vector equation of the line through the point $(-1,2,3)$ and parallel to the given line is

$$\frac{x+1}{2}=\frac{y-2}{3}=\frac{z-3}{-6}$$

Question 07 :

Find the angle between the pair of lines

$$\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}\right)\text{and}$$

$$\vec{r}=7\hat{i}-6\hat{j}-6\hat{k}+\mu\left(\hat{i}+2\hat{j}+2\hat{k}\right)$$

Solution :

Given lines : $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$

$\vec{b}_1=3\hat{i}+2\hat{j}+6\hat{k},\quad \vec{b}_2=\hat{i}+2\hat{j}+2\hat{k}$

$$\cos\theta=\frac{\left|\vec{b}_1.\vec{b}_2\right|}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}=\frac{\left|\left(3\hat{i}+2\hat{j}+6\hat{k}\right).\left(\hat{i}+2\hat{j}+2\hat{k}\right)\right|}{\sqrt{3^2+2^2+6^2}\sqrt{1^2+2^2+2^2}}$$

$$=\frac{\left|\left(3\hat{i}+2\hat{j}+6\hat{k}\right).\left(\hat{i}+2\hat{j}+2\hat{k}\right)\right|}{\sqrt{3^2+2^2+6^2}\sqrt{1^2+2^2+2^2}}=\frac{3\times1+2\times2+6\times2}{\sqrt{49}\times\sqrt{9}}=\frac{19}{7\times3}$$

$$\cos\theta=\frac{19}{21}\quad\Rightarrow\quad\theta=\cos^{-1}\left(\frac{19}{21}\right)$$

Question 08 :

Show that the line $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7},\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ are intersect. Also,find their point of intersection.

Solution :

The given lines are,

$$\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}=\lambda$$

$$\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}=\mu$$

Then, any point on the lines are

$P(3\lambda-1, 5\lambda-3, 7\lambda-5)$

$P(\mu+2, 3\mu+4, 5\mu+6)$

If lines intersect, then these points coincide.

$$3\lambda-1=\mu+2 \quad \Rightarrow \quad \mu=3\lambda-3 \qquad (i)$$

$$5\lambda-3=3\mu+4 \quad \Rightarrow \quad 5\lambda-7=3\mu \qquad (ii)$$

$$7\lambda-5=5\mu+6 \quad \Rightarrow \quad 7\lambda-5\mu=11 \qquad (iii)$$

From (i) and (ii),

$$5\lambda-7=3(3\lambda-3) \quad \Rightarrow \quad 9-7=9\lambda-5\lambda \quad \Rightarrow \quad \lambda=\frac{1}{2}$$

and $\qquad \mu=3\lambda-3=3\frac{1}{2}-3 = -\frac{3}{2}$

Put $\lambda=\frac{1}{2}, \mu=-\frac{3}{2}$ in (iii)

$7\frac{1}{2}-5\left(-\frac{3}{2}\right)=11 \quad \Rightarrow \quad 11=11$, which is true.

The point of intersection is

$$P\left(3\times\frac{1}{2}-1, 5\times\frac{1}{2}-3, 7\times\frac{1}{2}-5\right)=P\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$$

Question 09 :

Find the shortest distance between the lines

$$\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k}) \text{ and}$$

$$\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$$

Solution :

The given equation of lines are,

$$\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k}) \text{ and}$$

$$\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$$

On comparing with $\vec{r}=\vec{a}+\lambda\vec{b}$

$\vec{a}_1 = \hat{i} + \hat{j}$, $\vec{b}_1 = (2\hat{i} - \hat{j} + \hat{k})$ and

$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b}_2 = (3\hat{i} - 5\hat{j} + 2\hat{k})$

The shortest distance $= \left| \dfrac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3) = 3\hat{i} - \hat{j} - 7\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{3^2 + (-1)^2 + (-7)^2} = \sqrt{59}$$

$$\vec{a}_2 - \vec{a}_1 = (2\hat{i} + \hat{j} - \hat{k}) - (\hat{i} + \hat{j}) = \hat{i} - \hat{k}$$

$$\therefore \quad d = \left| \frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \frac{(3\hat{i} - \hat{j} - 7\hat{k}).(\hat{i} - \hat{k})}{\sqrt{59}} = \frac{3+7}{\sqrt{59}} = \frac{10}{\sqrt{59}}$$

Shortest distance (d) :

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| \text{ ,If } \vec{b}_1 \times \vec{b}_2 = 0, \quad d = \left| \frac{\vec{b}.(\vec{a}_2 - \vec{a}_1)}{|\vec{b}|} \right|$$

Question 10 :

Find the shortest distance between the lines

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and

$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

Solution :

The given equation of lines are,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right) \text{and}$$

$$\vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$$

On comparing with $\vec{r} = \vec{a} + \lambda\vec{b}$

$$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k} \quad , \quad \vec{b}_1 = \left(\hat{i} - 3\hat{j} + 2\hat{k}\right) \text{and}$$

$$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k} \quad , \quad \vec{b}_2 = \left(2\hat{i} + 3\hat{j} + \hat{k}\right)$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}(-2-6) - \hat{j}(1-4) + \hat{k}(3+6) = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-9)^2 + 3^2 + 9^2} = \sqrt{81 + 9 + 81} = \sqrt{171}$$

$$\vec{a}_2 - \vec{a}_1 = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\therefore \quad d = \left|\frac{\left(\vec{b}_1 \times \vec{b}_2\right).\left(\vec{a}_2 - \vec{a}_1\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$$

$$= \frac{\left(-9\hat{i} + 3\hat{j} + 9\hat{k}\right).\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)}{\sqrt{171}} = \frac{-27 + 9 + 27}{\sqrt{59}} = \frac{9}{\sqrt{59}} \text{units}$$

Question 11:

Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}, \quad \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Solution:

The given equation of lines are,

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}, \quad \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

On comparing with one point form of equation of line,

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$a_1=1,\ b_1=-2,\ c_1=1 \quad ; \quad x_1=3,\ x_1=5,\ x_1=7$

$a_2=7,\ b_2=-6,\ c_2=1 \quad ; \quad x_2=-1,\ x_2=-1,\ x_2=-1$

$$d=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$$

$$d=\left|\frac{\begin{vmatrix} -4 & -6 & -8 \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}}{\sqrt{(-2+6)^2+(7-1)^2+(-6+14)^2}}\right|=\left|\frac{-4(4)+6(-6)-8(8)}{\sqrt{4^2+6^2+8^2}}\right|$$

$$=\left|\frac{-4(4)+6(-6)-8(8)}{\sqrt{16+36+64}}\right|=\frac{116}{\sqrt{116}}=\sqrt{116}\text{units}$$

Question 12 :

Find the angle between the pair of lines

$$\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3} \quad , \quad \frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$$

Solution :

Given equations of lines are

$$\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3} \quad , \quad \frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$$

In standard form,

$$\frac{x-2}{2}=\frac{y-1}{7}=\frac{z+3}{-3} \quad , \quad \frac{x+2}{-1}=\frac{y-4}{2}=\frac{z-5}{4}$$

On comparing with one point form of equation of line,

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$a_1=2,\ b_1=7,\ c_1=-3 \quad ; \quad a_2=-1,\ b_2=2,\ c_2=4$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{(2)(-1)+(7)(2)+(-3)(4)}{\sqrt{2^2+7^2+(-3)^2}\sqrt{(-1)^2+2^2+4^2}} = \frac{-2+14-12}{\sqrt{4+49+9}\sqrt{1+4+16}}$$

$$\cos\theta = \frac{0}{\sqrt{62}\sqrt{21}} = 0 \quad \Rightarrow \quad \theta = \cos^{-1}(0) = \frac{\pi}{2}$$

$\therefore$ The lines are perpendicular to each other

Question 13 :

Find the equation of perpendicular from point $(3,-1,11)$ to line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also, find the coordinates of foot of perpendicular and the length of perpendicular.

Solution :

Given equations of line is $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

$\Rightarrow \quad x = 2\lambda,\ y = 3\lambda + 2,\ z = 4\lambda + 3$

$\therefore$ Any point on the given line $= (2\lambda, 3\lambda+2, 4\lambda+3)$

Let P be the foot of $\perp$ from the point $Q(3,-1,11)$.

$QP = (2\lambda-3, 3\lambda+2+1, 4\lambda+3-11)$

Direction ratio of line $QP = (2\lambda-3, 3\lambda+3, 4\lambda-8)$

Here, $a_1 = 2\lambda-3, b_1 = 3\lambda+3, c_1 = 4\lambda-8$; $a_2 = 2, b_2 = 3, c_2 = 4$

We have $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(2\lambda-3)(2)+(3\lambda+3)(3)+(4\lambda-8)(4) = 0$$

$$4\lambda - 6 + 9\lambda + 9 + 16\lambda - 32 = 0$$

$$29\lambda - 29 = 0 \quad \text{or} \quad \lambda = 1$$

Foot of perpendicular $= (2, 3+2, 4+3) = (2,5,7)$

Equation of perpendicular QP, where $Q(3,-1,11)$ and $P(2,5,7)$ is given by

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-3}{-2}=\frac{y+1}{6}=\frac{z-11}{-4}$$

Distance between P and Q

$$=\sqrt{(2-3)^2+(5+1)^2+(7-11)^2}=\sqrt{53}$$

The distance from point (x_1, y_1, z_1) to the plane

$Ax+By+Cz+D=0$ is $\dfrac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}$

Question 14 :

Find the length of the perpendicular drawn from the origin to the plane $2x-3y+6z+21=0$

Solution :

Given equation of plane is

$\therefore$ Length of the perpendicular drawn from the origin to the plane

$$=\left|\frac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|=\left|\frac{2.0-3.0+6.0+21}{\sqrt{2^2+((-3))^2+6^2}}\right|=\left|\frac{21}{\sqrt{49}}\right|=3\text{units}$$

Question 15 :

Write the intercept cut-off by plane $2x+y-z=5$

Solution :

Given equation of plane is $2x+y-z=5$

$$\frac{2x}{5}+\frac{y}{5}-\frac{z}{5}=\frac{5}{5}$$

$$\frac{x}{\left(\frac{5}{2}\right)}+\frac{y}{5}+\frac{z}{-5}=1$$

On Comparing with the intercept form of equation of plane

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

The intercept cut-off on X-axis $=\dfrac{5}{2}$

HOME WORK QUESTIONS

Question : (Imp2017)

(a) Which of the following is a plane perpendicular to

$$x + 3y + 4z = 7\ ?$$

$$(4x + 3y + z = 7, 4x - 4z = 7, 3x + 4y + z = 0, x + y + z = 0)$$

(b) Find the shortest distance between the lines

$$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t\left(-\hat{i} - 2\hat{j} - 2\hat{k}\right) \text{and}$$

$$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s\left(\hat{i} - \hat{j} - \hat{k}\right)$$

Answer :

(a) $4x - 4z = 7$ (b) $\dfrac{5}{\sqrt{2}}$

Question : (March 2017)

(a) Distance of the point $(1, 0, 0)$ from the plane $x + 2y + 2z = 0$ $\left(\dfrac{2}{3}, \dfrac{1}{3}, \dfrac{1}{2}, 1\right)$

(b) Find the Cartesian equation of a line passing through $(1, 2, -4)$ and perpendicular to the lines

$$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{z-1}{1} \text{ and } \frac{x-5}{1} = \frac{y}{1} = \frac{z-2}{1}$$

Answer :

(a) $\dfrac{1}{3}$ (b)

Question : (March2017)

(a) The line $x - 1 = y = z$ is perpendicular to the line

(a) $\dfrac{x-2}{1} = \dfrac{y-1}{2} = \dfrac{z}{-3}$ (b) $x - 2 = y - 2 = z$

(c) $\dfrac{x-2}{1} = \dfrac{y-1}{2} = \dfrac{z}{3}$ (b) $x = y = \dfrac{z}{2}$

(b) Find the shortest distance between the lines

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$ and $\vec{r} = \hat{i} + \hat{j} + \hat{k} + \mu(\hat{i} + \hat{j} + \hat{k})$

Answer :

(a) a

(b) $SD = \sqrt{2}$

Question :(March 2017)

(a) Distance of the point (0, 0, 1) from the plane $x + y + z = 3$ $\left(\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \sqrt{3}, \frac{\sqrt{3}}{2}\right)$

(b) Find the equation of a plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to $x - y + z = 0$

Answer :

(a) $\frac{2}{\sqrt{3}}$ (b) $x - z + 2 = 0$

Question :(Imp 2016)

(a) The equation of the line which passes through the points $(1,2,3)$ and parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

$(3\hat{i} + 2\hat{j} - 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$(2\hat{i} - 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$

$(\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-2\hat{i} + 4\hat{j} - 2\hat{k})$

$(\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$

(b) Find the angle between the pair of lines

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

Answer :

(a) $\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$

(b) $\theta=\cos^{-1}\left(\frac{19}{21}\right)$

Question :(imp 2016)

(a) The distance of the plane $x+y+z+1=0$ from the point $(1,1,1)$ $\left(4\text{units},\ \frac{1}{\sqrt{3}}\text{units},\ \frac{4}{\sqrt{3}}\text{units},\ \frac{1}{4\sqrt{3}}\text{units}\right)$

(b) Find the equation of the plane passing through $(1,\ 0,-2)$ and perpendicular to each of the planes $2x+y-z=2$ and $x-y-z=3$

Answer :

(a) $\frac{4}{\sqrt{3}}$units

(b) $2x-y+3z+4=0$

Question :(March 2016)

Find the shortest distance between the lines

$\vec{r}=\hat{i}+\hat{j}+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$ and $\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu\left(3\hat{i}-5\hat{j}+2\hat{k}\right)$

Answer : $\frac{10}{\sqrt{59}}$

Question :(March 2016)

(a) Equation of the plane with intercepts 2,3,4 on the x,y and z respectively is

(i) $2x+3y+4z=1$

(ii) $2x+3y+4z=12$

(iii) $6x+4y+3z=1$

(iv) $6x+4y+3z=12$

(b) Find the Cartesian equation of the plane passing through the points $A(2, 5, -3)$, $B(-2, -3, 5)$ and $C(5, 3, -3)$.

Answer :

(a) $6x + 4y + 3z = 12$

(b) $2x + 3y + 4z = 7$