10 VECTOR ALGEBRA

MODEL QUESTIONS

Question 01 :

Write a unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$

Solution :

$$\vec{a} + \vec{b} = \left(2\hat{i} + 2\hat{j} - 5\hat{k}\right) + \left(2\hat{i} + \hat{j} - 7\hat{k}\right) = 4\hat{i} + 3\hat{j} - 12\hat{k}$$

$$\left|\vec{a} + \vec{b}\right| = \sqrt{(4)^2 + (3)^2 + (-12)^2} = \sqrt{16 + 9 + 144} = \sqrt{169} = 13$$

Required unit vector $= \dfrac{\vec{a} + \vec{b}}{\left|\vec{a} + \vec{b}\right|} = \dfrac{4}{13}\hat{i} + \dfrac{3}{13}\hat{j} - \dfrac{12}{13}\hat{k}$

Question 02 :

Find the value of p for which the vectors $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel.

Solution :

Direction ratios are proportional.

$$\therefore \quad \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$$

$$\frac{1}{-p} = \frac{3}{1} \quad \Rightarrow \quad p = -\frac{1}{3}$$

Question 03 :

(a) Write the value of cosine of the angle which the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ makes with Y axis.

(b) Find the angle between X-axis and the vector $\hat{i} + \hat{j} + \hat{k}$.

Solution :

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

(a) Cosine of the angle which the vector makes with Y axis is $\frac{1}{\sqrt{3}}$

(b) The angle between X-axis and the vector $\hat{i} + \hat{j} + \hat{k}$,

$$\cos\alpha = \frac{1}{\sqrt{3}} \quad \text{or} \quad \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Question 04 :

Write the value of the following :

$$\hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j})$$

Solution :

$$\hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j})$$

$$= \hat{i} \times \hat{j} + \hat{i} \times \hat{k} + \hat{j} \times \hat{k} + \hat{j} \times \hat{i} + \hat{k} \times \hat{i} + \hat{k} \times \hat{j}$$

$$= \hat{k} - \hat{j} + \hat{i} - \hat{k} + \hat{j} - \hat{i} = 0$$

Question 05 :

P and Q are two points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$, respectively. Write the position vector of a point R which divides the line segment PQ in the ratio 2:1 externally.

Solution :

Position vector of a point R

$$=\frac{2\times\left(\vec{a}+\vec{b}\right)-1\times\left(3\vec{a}-2\vec{b}\right)}{2-1}=2\vec{a}+2\vec{b}-3\vec{a}+2\vec{b}=4\vec{b}-\vec{a}$$

Question 06 :

Write the direction cosine of vector $-2\hat{i}+\hat{j}-5\hat{k}$

Solution :

The direction cosine of vector $a\hat{i}+b\hat{j}+c\hat{k}$ are

$$\frac{a}{\sqrt{a^2+b^2+c^2}},\ \frac{a}{\sqrt{a^2+b^2+c^2}},\ \frac{a}{\sqrt{a^2+b^2+c^2}}$$

Given $\vec{a}=-2\hat{i}+\hat{j}-5\hat{k}$

The direction cosine of vector $-2\hat{i}+\hat{j}-5\hat{k}$ are

$$\frac{-2}{\sqrt{(-2)^2+(1)^2+(-5)^2}},\frac{1}{\sqrt{(-2)^2+(1)^2+(-5)^2}},\frac{-5}{\sqrt{(-2)^2+(1)^2+(-5)^2}}$$

$$=\frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{-5}{\sqrt{30}}$$

Question 07 :

Find a vector of magnitude 5units and parallel to the resultant of $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$.

Solution :

Required unit vector $=\dfrac{\vec{a}+\vec{b}}{\left|\vec{a}+\vec{b}\right|}$

$$=\frac{\left(2\hat{i}+3\hat{j}-\hat{k}\right)+\left(\hat{i}-2\hat{j}+\hat{k}\right)}{\sqrt{(3)^2+(1)^2+(0)^2}}=\frac{3\hat{i}+\hat{j}}{\sqrt{10}}=\frac{3}{\sqrt{10}}\hat{i}+\frac{1}{\sqrt{10}}\hat{j}$$

Vector of magnitude 5units and parallel to the resultant of $\vec{a}$ and $\vec{b}$ is $5\left(\frac{3}{\sqrt{10}}\hat{i}+\frac{1}{\sqrt{10}}\hat{j}\right)$

Question 08 :

(a) Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $2\hat{i}-3\hat{j}+6\hat{k}$.

(b) Find the projection of the vector $\hat{i}+\hat{j}+\hat{k}$ along the vector $\hat{j}$.

Solution :

(a) $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$; $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$

The projection of the vector $\vec{a}$ on the vector $\vec{b}$ is given by

$$\frac{\vec{a}.\vec{b}}{\left|\vec{b}\right|}=\frac{(1\times 2)+(3\times -3)+(7\times 6)}{\sqrt{(2)^2+(-3)^2+(6)^2}}=\frac{35}{\sqrt{49}}=\frac{35}{7}=5$$

(b) $\vec{a}=\hat{i}+\hat{j}+\hat{k}$; $\vec{b}=\hat{j}$

The projection of the vector $\vec{a}$ on the vector $\vec{b}$ is given by

$$\frac{\vec{a}.\vec{b}}{\left|\vec{b}\right|}=\frac{(1\times 0)+(1\times 1)+(1\times 0)}{\sqrt{(0)^2+(1)^2+(0)^2}}=\frac{1}{\sqrt{1}}=1$$

Question 09 :

Write the value of λ, so that the vectors $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$ are perpendicular to each other.

Solution :

$\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$ are perpendicular

$$\therefore \quad \vec{a}.\vec{b}=0$$

$$\vec{a}.\vec{b}=\left(2\hat{i}+\lambda\hat{j}+\hat{k}\right).\left(\hat{i}-2\hat{j}+3\hat{k}\right)=0$$

$$(2\times 1)+(\lambda\times -2)+(1\times 3)=0$$

$$2+-2\lambda+3=0$$

$$\lambda=\frac{5}{2}$$

Question 10 :

If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}\right|$, then prove that $2\vec{a}+\vec{b}$ is perpendicular to vector $\vec{b}$.

Solution :

Given $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}\right|$

Squaring both sides,

$$\left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}\right|^2$$

$$\left|\vec{a}\right|^2 + 2\vec{a}.\vec{b} + \left|\vec{b}\right|^2 = \left|\vec{a}\right|^2$$

$$2\vec{a}.\vec{b} + \vec{b}.\vec{b} = 0$$

$$\left(2\vec{a}+\vec{b}\right).\vec{b} = 0$$

$$\therefore \qquad 2\vec{a}+\vec{b} \perp \vec{b}$$

Question 11 :

For what value of λ are the vectors $\vec{a} = \hat{i} + 2\lambda\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ perpendicular?

Solution :

If $\vec{a} = \hat{i} + 2\lambda\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ are perpendicular,then

$$\vec{a}.\vec{b} = 0$$

$$\vec{a}.\vec{b} = \left(\hat{i} + 2\lambda\hat{j} + \hat{k}\right).\left(2\hat{i} + \hat{j} - 3\hat{k}\right) = 0$$

$$(1\times 2) + (2\lambda \times 1) + (1\times -3) = 0$$

$$2 + 2\lambda - 3 = 0$$

$$\lambda = \frac{1}{2}$$

Question 12 :

Find the angle between vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively, having $\vec{a}.\vec{b} = \sqrt{6}$

Solution :

$$\left|\vec{a}\right| = \sqrt{3} \quad ; \quad \left|\vec{b}\right| = 2 \quad \vec{a}.\vec{b} = \sqrt{6}$$

The angle between vectors $\vec{a}$ and $\vec{b}$ is given by,

$$\cos\theta = \frac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|} = \frac{\sqrt{6}}{\sqrt{3}\times 2} = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$$

$\therefore \qquad \theta = \frac{\pi}{4}$

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HOME WORK QUESTIONS

Question :(Imp2017)

(a) The value of $\left|\vec{a}\right|$, if b is a unit vector and

$\left(2\vec{x}-2\vec{b}\right).\left(\vec{x}+\vec{b}\right)=30$ $\left(\sqrt{6},\ 6,\ 4,\ 2\right)$

(b) If $\vec{a}=\hat{i}+3\hat{j}$ and $\vec{b}=3\hat{j}+\hat{k}$, then find a unit vector which is perpendicular to both $\vec{a}$ and $\vec{b}$

Answer :

(a) 4 (b) $\dfrac{3\hat{i}-\hat{j}+3\hat{k}}{\sqrt{19}}$

Question :(Imp2017)

(a) Cosine of the angle between the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is $\left(\frac{1}{3},\ \frac{2}{3},\ \frac{1}{2},\ 1\right)$

(b) If $\vec{a},\ \vec{b},\ \vec{c}$ are three vectors $\left|\vec{a}\right|=3,\ \left|\vec{b}\right|=4,\ \left|\vec{c}\right|=4$ such that and each one of them is perpendicular to the sum of other two,then find $\left|\vec{a}+\vec{b}+\vec{c}\right|$

Answer :

(a) $\frac{1}{3}$

(b) $\vec{a}.\left(\vec{b}+\vec{c}\right)=0,\vec{b}.\left(\vec{c}+\vec{a}\right)=0,\vec{c}.\left(\vec{a}+\vec{b}\right)=0$

$$\vec{a}.\left(\vec{b}+\vec{c}\right)+\vec{b}.\left(\vec{c}+\vec{a}\right)+\vec{c}.\left(\vec{a}+\vec{b}\right)=0$$

$$2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$$

$$\left|\vec{a}+\vec{b}+\vec{c}\right|=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)$$

$$=9+16+16+0=\sqrt{41}$$

Question :(March 2017)

(a) The projection of the vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\hat{i}+\hat{j}+\hat{k}$ is

(b) Find the area of parallelogram whose adjacent sides are the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}$.

Answer :

(a) $\frac{7}{\sqrt{3}}$ (b) $A=\left|\vec{a}\times\vec{b}\right|=\sqrt{11}$

Question :(March 2017)

(a) The angle between the vectors $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is

$(60°,\ 30°,\ 45°, 90°)$

(b) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors $\vec{a}+\vec{b}+\vec{c}=0$, find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

Answer :

(a) $60°$

(b)
$$\left|\vec{a}+\vec{b}+\vec{c}\right|=\left(\vec{a}+\vec{b}+\vec{c}\right)^2=0$$

$$\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$$

$$1+1+1+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$$

$$\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=-\frac{3}{2}$$

Question :(Imp 2016)

(a) The projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$ is

$(1, 0, 2, -1)$

(b) Find the area of parallelogram whose adjacent sides are given by the vectors $3\hat{i}+\hat{j}+4\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$.

Answer :

(a) 0 (b) $A = \left|\vec{a}\times\vec{b}\right| = \sqrt{42}$

Question :(Imp 2016)

(a) $\left(\vec{a}-\vec{b}\right)\left(\vec{a}+\vec{b}\right)$ is equals to

Answer : $2\left(\vec{a}\times\vec{b}\right)$

(b) If $\vec{a}$ and $\vec{b}$ are any two vectors, then prove that

$$\left(\vec{a}\times\vec{b}\right)^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$$

Answer :

$$\left(\vec{a}\times\vec{b}\right)^2 = \left(\vec{a}\times\vec{b}\right)\left(\vec{a}\times\vec{b}\right) = (ab\sin\theta)(ab\sin\theta) = a^2b^2\sin^2\theta$$

$$= a^2b^2\left(1-\cos^2\theta\right) = a^2b^2 - a^2b^2\cos^2\theta$$

$$= (a^2)(b^2) - (ab\cos\theta)(ab\cos\theta) = \left(\vec{a}.\vec{a}\right)\left(\vec{b}.\vec{b}\right) - \left(\vec{a}.\vec{b}\right)^2$$

$$= \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$$

(c) Using vectors, show that the points $A(1,2,-1), B(2,6,3), A(3,10,-1)$ are collinear.

Answer :

$\overrightarrow{AC} = 2\overrightarrow{AB}$

Question :(March 2016)

(a) The angle between the vectors $\vec{a}$ and $\vec{b}$ such that $\left|\vec{a}\right| = \left|\vec{b}\right| = \sqrt{2}$ and $\vec{a}.\vec{b} = 1$ is $\left(\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, 0\right)$

(b) Find the unit vector along $\vec{a}-\vec{b}$ where $\vec{a}=\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=3\hat{i}+2\hat{j}+\hat{k}$.

Answer :

(a) $\frac{\pi}{3}$ (b) $\frac{1}{3}\left(-2\hat{i}+\hat{j}-2\hat{k}.\right)$

Question :(March 2016)

(a) If the points A and B are $(1, 2, -1)$ and $(2, 1, -1)$ respectively, then $\overrightarrow{AB}$ is

$$\left(\hat{i}+\hat{j}\ ,\ \hat{i}-\hat{j}\ ,\ 2\hat{i}+\hat{j}-\hat{k}\ ,\ \hat{i}+\hat{j}+\hat{k}\right)$$

(b) Find the value of λ for which the vectors $2\hat{i}-4\hat{j}+5\hat{k}$, $\hat{i}-\lambda\hat{j}+\hat{k}$ and $3\hat{i}+2\hat{j}-5\hat{k}$ are coplanar.

(c) Find the angle between the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$

Answer :

(a) $\hat{i}-\hat{j}$ (b) $\frac{26}{25}$ (c) $\theta=\cos^{-1}\left(\frac{-1}{3}\right)$

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