9 DIFFERENTIAL EQUATION

MODEL QUESTIONS

Question 01:

Write the degree of the differential equation $\left(\frac{dy}{dx}\right)^4 + 4y\frac{d^2y}{dx^2} = 0$

Solution:

Highest order derivative is $\frac{d^2y}{dx^2}$, whose degree is one.

So, the degree of the differential equation is one.

Question 02:

Form the differential equation representing family of curves given by $(x-a)^2 + 2y^2 = a^2$

Solution:

Given equation,

$$(x-a)^2 + 2y^2 = a^2$$

On differentiation,

$$2(x-a) + 4yy' = 0$$

$$2x - 2a + 4yy' = 0$$

$$2a = 2x + 4yy'$$

$$a = x + 2yy'$$

$$\therefore (x - x - 2yy')^2 + 2y^2 = (x + 2yy')^2$$

$$4y^2(y')^2 + 2y^2 = x^2 + 4xyy' + 4y^2(y')^2$$

$$2y^2 = x^2 + 4xyy'$$

Question 03:

Form the differential equation representing family of ellipses having foci on x – axis and centre at the origin.

Solution:

The equation of family of ellipse having foci on x – axis and centre at the origin is given by.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{, where, } a > b$$

On differentiating w.r.t x,

$$\frac{2x}{a^2}+\frac{2yy'}{b^2}=0$$

$$\frac{x}{a^2}+\frac{yy'}{b^2}=0$$

$$\frac{yy'}{x}=-\frac{b^2}{a^2}$$

Again, differentiating w.r.t x,

$$\frac{x\frac{d}{dx}(yy')-yy'}{x^2}=0$$

$$x\frac{d}{dx}(yy')-yy'=0$$

$$x[yy''+y'y']-yy'=0$$

$$xyy''+x(y')^2-yy'=0$$

Question 04 :

Find the particular solution of the differential equation $x\frac{dy}{dx}-y+x\operatorname{cosec}\left(\frac{y}{x}\right)=0$, given that $y=0$, when $x=1$.

Solution :

Given equation, $x\frac{dy}{dx}-y+x\operatorname{cosec}\left(\frac{y}{x}\right)=0$

$$\frac{dy}{dx}-\frac{y}{x}+\operatorname{cosec}\left(\frac{y}{x}\right)=0 \quad \text{or} \quad \frac{dy}{dx}=\frac{y}{x}-\operatorname{cosec}\left(\frac{y}{x}\right)$$

which is a homogeneous equation.

On putting $y=vx$,

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=\frac{vx}{x}-\operatorname{cosec}\left(\frac{vx}{x}\right)$$

$$v+x\frac{dv}{dx}=v-\operatorname{cosec} v$$

$$x\frac{dv}{dx}=-\operatorname{cosec} v$$

$$\frac{dv}{\operatorname{cosec} v}=-\frac{dx}{x}$$

$$\int\frac{dv}{\operatorname{cosec} v}=-\int\frac{dx}{x}$$

$$\int\sin v\, dv=-\int\frac{dx}{x}$$

$$-\cos v=-\log|x|+C$$

$$\cos\left(\frac{y}{x}\right)=\log|x|-C$$

$$y=x\cos^{-1}(\log|x|-C)$$

At $x=1, y=0$

$$0=1.\cos^{-1}(\log|1|-C)$$

$$0=\cos^{-1}(0-C)$$

$$\cos 0=\cos\left[\cos^{-1}(0-C)\right]$$

$$1=0-C$$

$$C=-1$$

$$y=x\cos^{-1}(\log|x|+1)$$

Question 05 :

Find the particular solution of the differential equation

$e^x\sqrt{1-y^2}dx+\frac{y}{x}dy=0$, given that $y=1$, when $x=0$.

Solution :

Given equation,

$$e^x\sqrt{1-y^2}dx+\frac{y}{x}dy=0$$

$$e^x\sqrt{1-y^2}dx=-\frac{y}{x}dy$$

$$xe^xdx=-\frac{y}{\sqrt{1-y^2}}dy$$

$$\int xe^xdx=-\int\frac{y}{\sqrt{1-y^2}}dy$$

On putting $1-y^2=t$

$$\Rightarrow\quad -ydy=\frac{dt}{2}$$

$$x\int e^xdx-\int\left[\frac{d}{dx}(1)\int e^xdx\right]dx$$

$$=\int\frac{dt}{2\sqrt{t}}$$

$$xe^x-\int e^xdx+C=\sqrt{t}$$

$$xe^x-e^x+C=\sqrt{1-y^2}$$

On putting $y=1, x=0$

$$0-e^0+C=\sqrt{1-1}$$

$$0-1+C=0$$

$$C=1$$

$\therefore$ The particular solution is

$$xe^x-e^x+1=\sqrt{1-y^2}$$

Question 06 :

Solve the differential equation

$$x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$$

Solution :

Given equation, $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$

$$\frac{x\log x\frac{dy}{dx}}{x\log x}+\frac{y}{x\log x}=\frac{\frac{2}{x}\log x}{x\log x}\quad\Rightarrow\quad\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x^2}$$

We have, $\frac{dy}{dx}+Py=Q$

On comparing equations, $P=\frac{1}{x\log x}$; $Q=\frac{2}{x^2}$

$$\therefore\qquad IF=e^{\int Pdx}=e^{\int\frac{1}{x\log x}dx}=e^{x\log x}=\log x$$

Now $y \times IF = \int (Q \times IF)\,dx + C$

$$y \times \log x = \int\left(\frac{2}{x^2} \times \log x\right)dx + C$$

$$y \log x = \log x \int \frac{2}{x^2}dx - \int\left(\frac{d}{dx}(\log x)\int \frac{2}{x^2}dx\right)dx + C$$

$$y \log x = \log x\left(-\frac{2}{x}\right) - \int\left(\frac{1}{x}\left(-\frac{2}{x}\right)\right)dx + C$$

$$y \log x = -\frac{2}{x}\log x + \int \frac{2}{x^2} + C$$

$$y \log x = -\frac{2}{x}\log x - \frac{2}{x} + C$$

Question 07 :

Find the general solution of the differential equation

$$(x-y)\frac{dy}{dx} = x + 2y.$$

Solution :

Given equation,

$$(x-y)\frac{dy}{dx} = x + 2y$$

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

which is a homogeneous equation.

On putting $y = vx$,

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{x+2vx}{x-vx}$$

$$v + x\frac{dv}{dx} = \frac{1+2v}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+2v}{1-v} - v$$

$$x\frac{dv}{dx} = \frac{1+2v-v(1-v)}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+2v-v+v^2}{1-v}$$

$$\frac{1-v}{1+v+v^2}dv = \frac{dx}{x}$$

On integrating,

$$\int \frac{1-v}{1+v+v^2}dv = -\int \frac{dx}{x}$$

$$I = -\int \frac{dx}{x}$$

$$I = -\log|x| + C$$

$$I = -\log|x| + C$$

$$I = \int \frac{1-v}{1+v+v^2}dv$$

$$1 - v = A\frac{d}{dv}\left(1+v+v^2\right) + B$$

$$1-v=A(2v+1)+B$$
$$1-v=2Av+A+B$$
On comparing coefficients,
$$2A=-1 \quad \text{or} \quad A=\frac{-1}{2}$$
$$\frac{-1}{2}+B=1 \quad \text{or} \quad B=\frac{3}{2}$$

$$\therefore I=\int\frac{-\frac{1}{2}(2v+1)+\frac{3}{2}}{1+v+v^2}dv$$
$$I=-\frac{1}{2}\int\frac{(2v+1)dv}{1+v+v^2}+\frac{3}{2}\int\frac{dv}{1+v+v^2}$$
$$\left\{\begin{array}{l}\because \text{Put } t=1+v+v^2,\\ dt=(2v+1)dv\end{array}\right\}$$

$$I=-\frac{1}{2}\int\frac{dt}{t}+\frac{3}{2}\int\frac{dv}{v^2+v+\frac{1}{4}+1-\frac{1}{4}}=-\frac{1}{2}\log t+\frac{3}{2}\int\frac{dv}{\left(v^2+v+\frac{1}{4}\right)-\left(\frac{3}{4}\right)}$$
$$=-\frac{1}{2}\log t+\frac{3}{2}\int\frac{dv}{\left(v+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}=-\frac{1}{2}\log t+\frac{3}{2}\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\tan^{-1}\left(\frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+C$$
$$=-\frac{1}{2}\log t+\frac{6}{2\sqrt{3}}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right)+C$$
$$=-\frac{1}{2}\log\left(1+v+v^2\right)+\sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right)+C$$
$$=-\frac{1}{2}\log\left(1+\frac{y}{x}+\frac{y^2}{x^2}\right)+\sqrt{3}\tan^{-1}\left(\frac{2\frac{y}{x}+1}{\sqrt{3}}\right)+C$$
$$=-\frac{1}{2}\log\left(\frac{x^2+xy+y^2}{x^2}\right)+\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right)+C$$
$$\therefore\quad -\frac{1}{2}\log\left(\frac{x^2+xy+y^2}{x^2}\right)+\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right)+C=\log|x|+C$$
which is the required solution.

HOME WORK QUESTIONS

Question :(Imp2017)

The degree of the differential equation $2x^2\frac{d^2y}{dx^2}+x^4\left(\frac{dy}{dx}\right)^3+7=0$

Answer :

Degree is 1

Question :(Imp2017)

Solve the differential equation $\left(1+y^2\right)\frac{dy}{dx}+x=e^{-\tan^{-1}y}$

Answer :

$xe^{\tan^{-1}y}=\tan^{-1}y+C$

Question :(March 2017)

The order of the differential equation $x^4\frac{d^2y}{dx^2}=1+\left(\frac{dy}{dx}\right)^3$

Answer :

Order is 1

Question :(March 2017)

Find the particular solution of the differential equation

$\left(1+x^2\right)\frac{d^2y}{dx^2}+2xy=\frac{1}{\left(1+x^2\right)}$, $y=0$ when $x=1$.

Answer :

Out of syllabus (Solving second order diff.eqn)

Question :(Imp2016)

(a) The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$$

Answer :

Not defined

(b) Solve $\frac{dy}{dx}+2y\tan x=\sin x, y=0,\text{when } x=\frac{\pi}{3}$

Answer :

$y=\cos x-2\cos^2 x$

Question :(March 2016)

(a) $y=a\cos x+b\sin x$ is the solution of differential equation.

(i) $\frac{d^2y}{dx^2}+y=0$ (ii) $\frac{d^2y}{dx^2}-y=0$

(iii) $\frac{dy}{dx}+y=0$ (iv) $\frac{dy}{dx}+x\frac{dy}{dx}=0$

Answer :

$\frac{d^2y}{dx^2}+y=0$

(b) Find the solution of the differential equation

$$x\frac{dy}{dx}+2y=x^2,(x\neq 0)\text{given that, } y=0,\text{when } x=1$$

Answer :

$yx^2=\frac{x^4}{4}-\frac{1}{4}$

-●●●-

Prepared By

Fassal Peringolam

Calicut

www.sciencetablet.in