3 MATRICES

Question :

Construct a 2×2 matrix where $a_{ij}=|-2i+3j|$.

Answer :

We have, $a_{ij}=|-2i+3j|$

$$a_{11}=|-2\times1+3\times1|=|1|=1$$
$$a_{12}=|-2\times1+3\times2|=|4|=4$$
$$a_{21}=|-2\times2+3\times1|=|-1|=1$$
$$a_{22}=|-2\times2+3\times2|=|2|=2$$

$$\therefore\ A=\begin{pmatrix}1 & 4\\ 1 & 2\end{pmatrix}$$

Question :

If $X=\begin{pmatrix}3 & 1 & -1\\ 5 & -2 & -3\end{pmatrix}$ and $X=\begin{pmatrix}2 & 1 & -1\\ 7 & 2 & 4\end{pmatrix}$, find $2X+Y$, $2X-Y$.

Answer :

We have, $X=\begin{pmatrix}3 & 1 & -1\\ 5 & -2 & -3\end{pmatrix}$ and $X=\begin{pmatrix}2 & 1 & -1\\ 7 & 2 & 4\end{pmatrix}$

$$2X+Y=2\begin{pmatrix}3 & 1 & -1\\ 5 & -2 & -3\end{pmatrix}+\begin{pmatrix}2 & 1 & -1\\ 7 & 2 & 4\end{pmatrix}$$

$$2X+Y=\begin{pmatrix}6 & 2 & -2\\ 10 & -4 & -6\end{pmatrix}+\begin{pmatrix}2 & 1 & -1\\ 7 & 2 & 4\end{pmatrix}$$

$$=\begin{pmatrix}6+2 & 2+1 & -2-1\\ 10+7 & -4+2 & -6+4\end{pmatrix}=\begin{pmatrix}8 & 3 & -3\\ 17 & -2 & -2\end{pmatrix}$$

$$2X-Y=\begin{pmatrix}6 & 2 & -2\\ 10 & -4 & -6\end{pmatrix}-\begin{pmatrix}2 & 1 & -1\\ 7 & 2 & 4\end{pmatrix}$$

$$=\begin{pmatrix}6-2 & 2-1 & -2+1\\ 10-7 & -4-2 & -6-4\end{pmatrix}=\begin{pmatrix}4 & 1 & -1\\ 3 & -6 & -10\end{pmatrix}$$

Question :

If $\begin{pmatrix}x-y & z\\ 2x-y & w\end{pmatrix}=\begin{pmatrix}-1 & 4\\ 0 & 5\end{pmatrix}$, then find the value of $x+y$.

Answer :

We have, $\begin{pmatrix} x-y & z \\ 2x-y & w \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ 0 & 5 \end{pmatrix}$

$$x-y=-1$$
$$2x-y=0$$

On solving, we get $x=1$ and $y=2$

$\therefore \quad x+y=1+2=\underline{\underline{3}}$

Question :

The elements a_{ij} of a 3×3 matrix are given by $a_{ij}=\frac{1}{2}|-3i+j|$.

Write the value of element a_{32}.

Answer :

We have, $a_{ij}=\frac{1}{2}|-3i+j|$

$$a_{32}=\frac{1}{2}|-3\times3+2|=\underline{\underline{\frac{7}{2}}}$$

Question :

If $(2x \quad 4)\begin{pmatrix} x \\ -8 \end{pmatrix}=0$, find the positive value of x.

Answer :

We have, $(2x \quad 4)\begin{pmatrix} x \\ -8 \end{pmatrix}=0$

$$2x^2-32=0$$

$$2x^2=32$$
$$x^2=16$$
$$\underline{\underline{x=4}}$$

Question :

If matrix $A=\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ and $A^2=kA$, then write the value of k.

Answer :

We have, $A=\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ and $A^2=kA$.

$$A^2 = A.A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} = 2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= 2A = kA$$

$$\therefore \qquad \underline{\underline{k = 4}}$$

Question :

If $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$, then find the value of $A^2 - 3A + 2I$.

Answer :

$$A^2 = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{pmatrix} = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$$

$$3A = \begin{pmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{pmatrix}, \qquad 2I = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$A^2 - 3A + 2I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 0 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 \\ 3 & -3 & -4 \\ -3 & 2 & 0 \end{pmatrix}$$

Question :

Express the following matrix as a sum of symmetric and skew symmetric matrices and verify the result.

$$\begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$$

Answer :

$$\text{Let } A = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$$

$$P = \frac{1}{2}(A + A') = \frac{1}{2}\left\{\begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 3 & -1 \\ 3 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}\right\}$$

$$= \frac{1}{2}\begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix} \quad ; \quad P' = \frac{1}{2}\begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix} = P$$

$$Q = \frac{1}{2}(A - A') = \frac{1}{2}\left\{\begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 3 & -1 \\ 3 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}\right\}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{pmatrix} \quad ; \quad Q' = \frac{1}{2}\begin{pmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{pmatrix} = -Q$$

$$P + Q = \frac{1}{2}\begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{pmatrix}$$

$$= \frac{1}{2}\left\{\begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix} + \begin{pmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{pmatrix}\right\} = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix} = A$$

Question :

Using elementary row transformation, find inverse of matrix

$$A = \begin{pmatrix} 6 & 5 \\ 5 & 4 \end{pmatrix}$$

Answer :

Given matrix, $A = \begin{pmatrix} 6 & 5 \\ 5 & 4 \end{pmatrix}$

Let $A = IA$

$$\Rightarrow \begin{pmatrix} 6 & 5 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{pmatrix} 1 & 1 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} A$$

Applying $R_2 \to R_2 - 5R_1$, we get

$$\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -5 & 6 \end{pmatrix} A$$

Applying $R_1 \to R_1 + R_2$, we get

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -4 & 5 \\ -5 & 6 \end{pmatrix} A$$

Applying $R_2 \to -R_2$, we get

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -4 & 5 \\ 5 & -6 \end{pmatrix} A$$

Hence $A^{-1} = \begin{pmatrix} -4 & 5 \\ 5 & -6 \end{pmatrix}$

Question :

Using elementary row transformation, find inverse of matrix $A = \begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$

Answer :

Given matrix $A = \begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$.

Let $A = IA$

$$\begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

Applying $R_2 \to R_2 + R_1$, $R_3 \to R_3 + 3R_1$, we get

$$\begin{pmatrix} -1 & 1 & 2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} A$$

Applying $R_1 \to -R_1$, we get

$$\begin{pmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} A$$

Applying $R_2 \to R_2 - R_3$, we get

$$\begin{pmatrix} 1 & -1 & -2 \\ 0 & -1 & -2 \\ 0 & 4 & 7 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{pmatrix} A$$

Applying $R_1 \to R_1 - R_2,\ \ R_3 \to R_3 + 4R_2$, we get

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ -2 & 1 & -1 \\ -5 & 4 & -3 \end{pmatrix} A$$

Applying $R_2 \to -R_2$, we get

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ -5 & 4 & -3 \end{pmatrix} A$$

Applying $R_2 \to R_2 + 2R_3$, we get

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ -5 & 4 & -3 \end{pmatrix} A$$

Applying $R_3 \to -R_3$, we get

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{pmatrix} A$$

Hence $$A^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{pmatrix}$$

HOME WORK QUESTIONS

Question : (March 2018)

Using elementary row operations, find the inverse of the matrix $\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}$

Hint or Answer :

Let $A = IA$

$$\Rightarrow \begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}A$$

$$\begin{pmatrix}1 & 2\\ 0 & 5\end{pmatrix} = \begin{pmatrix}1 & 0\\ -2 & 1\end{pmatrix}A \qquad \langle R_2 \to R_2 - 2R_1\rangle$$

$$\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\ \frac{2}{5} & -\frac{1}{5}\end{pmatrix}A \qquad \left\langle R_2 \to -\frac{1}{5}R_2\right\rangle$$

$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}\frac{1}{5} & \frac{2}{5}\\ \frac{2}{5} & -\frac{1}{5}\end{pmatrix}A \qquad \langle R_1 \to R_1 - 2R_2\rangle$$

Hence $A^{-1} = \begin{pmatrix}\frac{1}{5} & \frac{2}{5}\\ \frac{2}{5} & -\frac{1}{5}\end{pmatrix}$

Question : (March 2018)

(a) Find x and y if $x\begin{pmatrix}2\\ 3\end{pmatrix} + y\begin{pmatrix}-1\\ 1\end{pmatrix} = \begin{pmatrix}10\\ 5\end{pmatrix}$

(a) Express the matrix $\begin{pmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{pmatrix}$ as the sum of a symmetric and a skew symmetric matrices.

Hint or Answer :

$x = 3, y = -4$

Question : (Sept 2017)

(a) The number of all possible 2×2 matrices with entries 0 or 1 is (8 , 9 , 16 , 25)

(b) If the area of a triangle whose vertices are $(k,0),(5,0),(0,1)$ is 10 square units,then find k.

(c) Using elementary transformation find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

Hint or Answer :

(a) 16 (b) $\pm 10 = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 5 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$ or $k = 25$ or -15

(c) $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$

Applying $R_1 \leftrightarrow R_2$

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} A$$

Applying $R_2 \to -R_2$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} A$$

$$A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

Question : (March 2017)

(a) The value of k such that the matrix $\begin{pmatrix} 1 & k \\ -k & 1 \end{pmatrix}$ is symmetric is (0 , 1 , −1 , 2)

(b) If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, prove that $A^2 = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$

c) If $A = \begin{pmatrix} 1 & 3 \\ 4 & 1 \end{pmatrix}$, then then find $|3A'|$

Hint or Answer : (a) $k = 0$ (b) (c) -99

Question : (March 2017)

If $A = \begin{pmatrix} a & 1 \\ 1 & 0 \end{pmatrix}$ is such that $A^2 = I$ then a equals $(1, -1, 0, 2)$

Hint or Answer : $a = 0$

Question : (Imp 2016)

(a) If the matrix A is both symmetric and skew-symmetric, then A is a ....

(b) If $A = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$, then show that $A^2 - 5A + 10I = 0$

(c) Hence find A^{-1}.

Question : (March 2016)

(a) If $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, then BA.

(b) Write $A = \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$, as the sum of symmetric and a skew symmetric matrix.

(c) Find the inverse of $A = \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$.

Hint or Answer :

(a) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ (b) $A = \frac{1}{2}\begin{pmatrix} 6 & 6 \\ 6 & -2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix}$ (c) $A^{-1} = \frac{1}{2}\begin{pmatrix} -2 & 6 \\ -1 & 2 \end{pmatrix}$